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Introduction:-

1.1 Types of structural System:-

→ A structure is a body made up of by the comb^{n} of bars, walls, plates etc. such that they can resist applied load without appreciable deformatio

→ structure transfer the load from one member to another member & finally to the soil.

→ while transferring the loads acting on the structure, the member of structure are subjected to internal forces like axial force, shearing force, bending & torsional moment.

→ structural Analysis or Theory of structure deals about these internal forces developed in member of structure & effect of these internal force.

→ The effect of internal force is the deflection of structural member.

→ In structural analysis I, we calculate the deflection in beam & frame (only statically determinate) by various methods. Also, we study about the three hinged arches, cables & suspension bridges.

(a) On the basis of stress:-
Uniform stress:- cables, arches, shells, truss
Non-uniform stress:- beam, slab

(b) On the basis of analysis:-
1. Determinate structure
↳ can be analysed with eqn of static eqb
↳ Total number of unknown = total no. of aviable eqn.

2. Indeterminate structure:-
↳ cannot be analysed with the help of eqn of static eqb
↳ Total number of unknown > total no. of available eqn.

※ Structural Members:-

① Beam:- Structural member subjected to transverse loading & usually placed in horizontal positions.



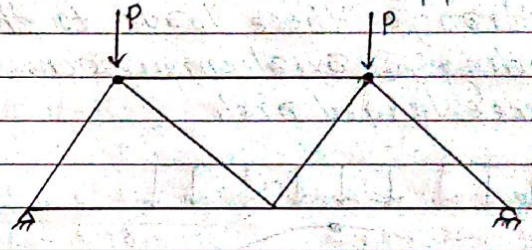
② Tie:- The member which is subjected to only axial tension.



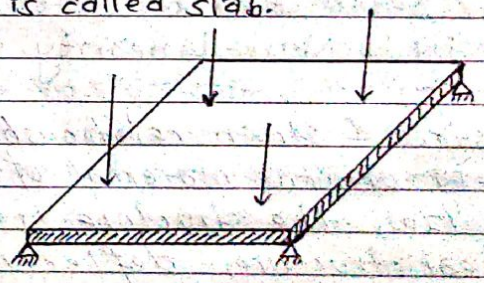
③ Strut / column / post:- The member which is subjected to only axial compression.



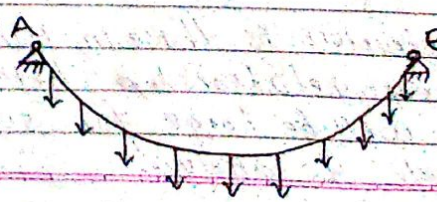
④ Truss:-
It is group of tie & strut connecting in triangular form & external load are applied at its joints.



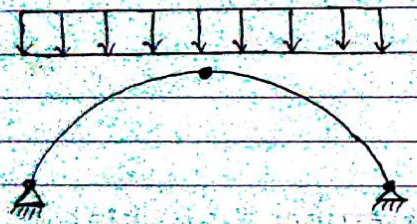
⑤ Slab:-
The structural member in which plate is supported loaded (transversely) & subjected to shear, bending & twisting is called slab.



⑥ Cable:-
Structural member which cannot resist compression, shear & bending, only tensile force is developed under the application of loads.



④ Arch:-
Arch are the curved beam which may be subjected to vertical, horizontal or inclined load & transfer these loads to the abutments by bending & axial compression.
Eg:- Three hinged arch



* Linearity & Non-Linearity in structural analysis

→ When stress & strain relationship for a structure is linear or the material of structure obeys Hook's law, the structure is called linear structure
→ In linear structure, deflection will be small & simple for analysis.
→ It is assumed that when a section of a structure attends its yield stress, the structure fails. But actually, the structure doesnot fail at that stress level.

→ Non linear structure is those in which there will be non-linear relationship beth stress & strain
→ Deflection will be large for such structure & difficult to analyse.

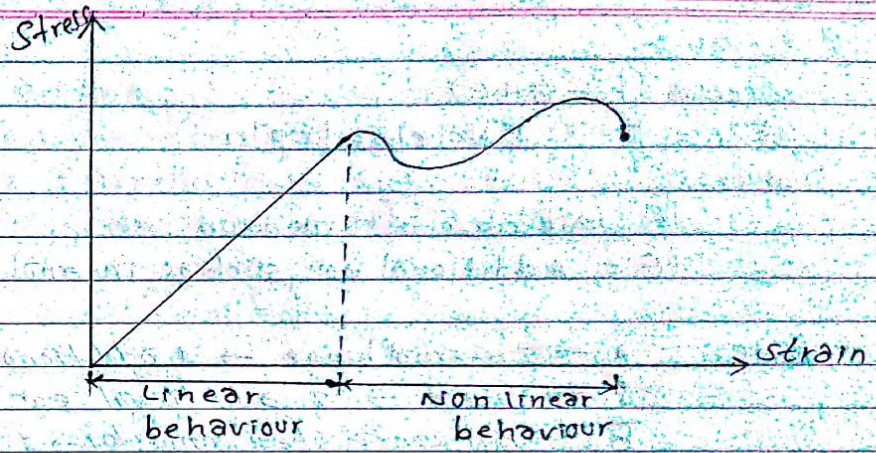


Fig:- Stress-strain curve for mild steel.

* Degree of static indeterminancy: (D_s)
It is the number of exceed unknown reaction with respect to available eqn of static equilibrium.

ⓐ Degree of static indeterminancy due to external (D_{se})
↳ due to exceed of support rxn

ⓑ Degree of static indeterminancy due to internal (D_{si})
↳ due to exceed of member.

$$D_s = D_{se} + D_{si}$$

① Beam:-

$D_{se} = R - (3+e)$

$D_{si} = 0$ [No close loop]

where,

R = Number of unknown rxn

e = Additional eqn such as internal hinge

Note: • → Internal hinge → 1 additional eqn

→ Link → 2 additional eqn

→ Horizontal shear connector → 1 additional eqn

② Frame:-

$D_{se} = R - 3$

$D_{si} = 3c$ where, c = Number of close loop.

③ Truss:-

$D_{se} = r - 3$ where r = total no. of unknown

$D_{si} = m - (2j - 3)$ m = total no. of member

j = total no. of joint

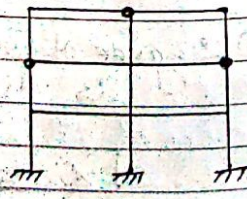
④ For hybrid

ⓐ $D_{se} = r - 3$

$D_{si} = 3c - \Sigma(m_i - 1)$

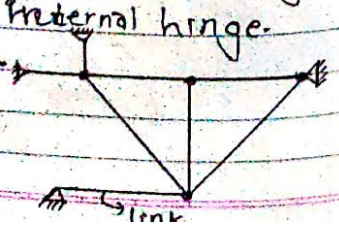
where,

m_i = number of ~~internal~~ member meeting at internal hinge.



ⓑ In truss, link is not a member. Support connected with link gives (n-1) rxn & joint is not considered.

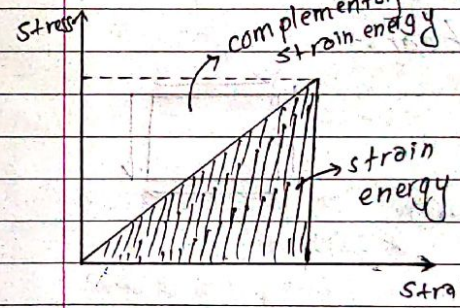
[Here, m=5, r=5, j=4



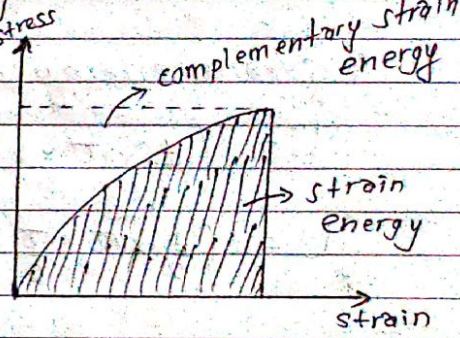
2. Strain energy Method:-

* Strain Energy:-

The internal workdone which is stored as energy due to straining of the material is called strain energy. It is denoted by 'U'.



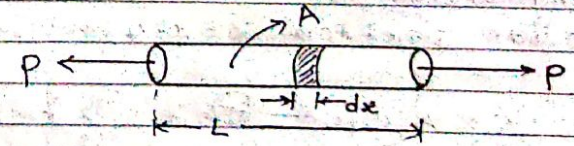
stress-strain curve for linear elastic material



Stress-strain curve for non-linear elastic material

Strain energy (U) = $\int_{vol} \frac{1}{2} \times \sigma \times \epsilon \times dv$

* Strain energy due to axial force:-



$U = \int \frac{1}{2} \times \sigma \times \epsilon \times dv$

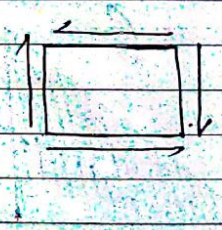
$= \int \frac{1}{2} \times \sigma \times \frac{\sigma}{E} \times A \times dx$

$= \int \frac{\sigma^2}{2E} \times A dx = \int \frac{P^2}{2AE} \times dx$

$$= \int_0^L \frac{P^2}{2AE} \times dx$$

$$\boxed{U = \frac{P^2 L}{2AE}}$$

* Strain energy due to shear:-



$$U = \int \frac{1}{2} \times \tau \times \gamma \times dV$$

$$U = \int \frac{1}{2} \times \tau \times \frac{\tau}{G_1} \times dV \quad \text{—— (i)}$$

$$U = \int \frac{1}{2} \times \frac{V^2}{A^2 \times G} \times A\,dx \quad \text{—— (ii)}$$

$$U = k \int \frac{V^2}{2AG} \times dx \quad \text{—— (iii)}$$

where, k is constant. Its value depends upon the shape of section.
For rectangular section, $k = 1.2$
For circular section, $k = 1.4$

Here, $\tau = \frac{V}{Ib} \times A'\bar{y}$ — (iv)

put eqn (iv) in (i) to get eqn (iii).

* Strain energy due to Torsion:-

$$U = \int \frac{T^2}{2G} \times dV$$

From torsion eqn, $\frac{T}{J} = \frac{\tau}{\rho} \Rightarrow \tau = \frac{T\rho}{J}$

Then,

$$U = \int_{vol} \frac{T^2 \rho^2}{J^2 \times 2G} \times dV$$

$$U = \int_L \frac{T^2}{2GJ^2} \left(\iint_A \rho^2 dA \right) dx$$

$$U = \int_0^L \frac{T^2 dx}{2GJ}$$

$$U = \frac{T^2 L}{2GJ}$$

* Strain energy due to Bending:-



we know,

$$U = \int \frac{1}{2} \times \sigma \times e \times \text{volume}$$

$$U = \int \frac{1}{2} \times \sigma \times \frac{\sigma}{E} \times \text{volume}$$

From flexural eqn,

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\Rightarrow \sigma = \frac{My}{I}$$

$$U = \int \frac{1}{2E} \times \frac{M^2 y^2}{I^2} \times \text{volume}$$

$$= \int_L \frac{M^2}{2EI^2} \left(\int_A y^2 dA \right) dx$$

$$= \int_L \frac{M^2}{2EI} dx$$

$$= \frac{M^2 L}{2EI}$$

→ Total strain energy,

$$U_T = U_{axial} + U_{shear} + U_{bending} + U_{torsion}$$

→ While calculating deflection, the strain energy due to axial, shear & torsion are neglected but only due to bending is considered.

$$\therefore U_{total} = U_{bending}$$

→ Workdone by external load = $\frac{P}{2} \times \delta$

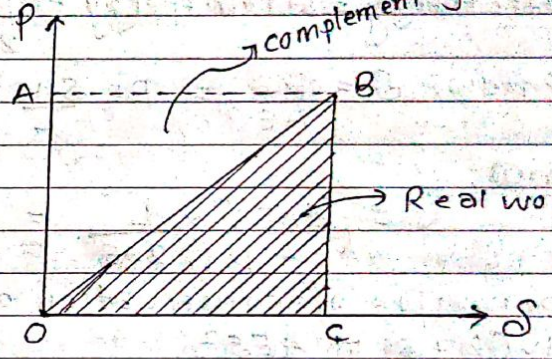
→ While calculating deflection, we use following formula,

$$\int \frac{M^2 dx}{2EI} = \frac{P}{2} \times \delta$$

where, δ = deflection.

※ Work & Complementary work:-

When load is applied, the structure deforms. The product of applied load & deformation gives the workdone by force.



→ The area under/below line OB is real work whereas the area above line OB is complementary work.

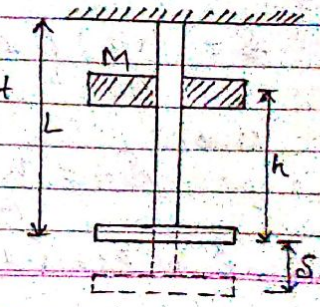
→ For linear elastic material, the real work & complementary work will be equal.

※ Impact load:-

The load suddenly applied from a certain height to the structure is called impact load.

Eg: traffic movement, water waves etc.

For the analysis of impact load, let us consider a simple arrangement shown in figure. A collar of mass 'M', initially at rest falls from height 'h' on flange at the lower end of bar.



When the collar falls down to the flange, its potential energy gets converted into K.E.

potential energy of collar = $W(h+\delta)$

kinetic energy = strain energy = $\frac{P^2 L}{2AE}$ —— ①

we know, $\delta = \frac{PL}{AE} \Rightarrow P = \frac{\delta AE}{L}$

From ①,

$$U = \left(\frac{\delta AE}{L}\right)^2 \times \frac{L}{2AE} = \frac{AE\delta^2}{2L}$$

By principle of conservation of energy,

$$W(h+\delta) = \frac{AE\delta^2}{2L}$$

$$\Rightarrow \left(\frac{AE}{2L}\right)\delta^2 - W\delta - Wh = 0$$

which is quadratic in δ, so,

$$\delta = \frac{W \pm \sqrt{W^2 + 4\times\left(\frac{AE}{2L}\right)\times Wh}}{2\times\frac{AE}{2L}}$$

$$\delta = \frac{WL}{AE} \pm \sqrt{\left(\frac{WL}{AE}\right)^2 + 2\left(\frac{WL}{AE}\right)\times h}$$

$$\delta = \frac{WL}{AE}\left(1 + \sqrt{1 + 2h\left(\frac{AE}{WL}\right)}\right)$$

put, $\delta_{st} = \frac{WL}{AE}$,

$$\delta = \delta_{st}\left(1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right)$$

$$\Rightarrow \delta_d = \delta_{st}\left(1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right)$$

Here, $\delta = \delta_d$ = dynamic deflection.

$$\Rightarrow \boxed{\delta_d = \delta_{st} \times I.F}$$

where, I.F = Impact factor = $\left(1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right)$

※ Note:-

Resilience:- The strain energy stored in a body within elastic limit.

Proof Resilience:- The maximum strain energy stored in a body.

Modulus of Resilience:- Maximum strain energy stored per unit volume.

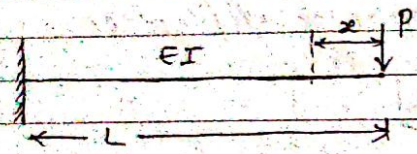
※ Limitation of strain Energy Method:- (for deflection)

→ It is applicable at that condition when only one point load is acting on structure.

→ We can calculate the deflection only at point of application of concentrated load in the direction of applied load.

Numericals:-

① Find the deflection of following beam at free end



→ Here, $M = -P \times x$

We know,

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$= \int_0^L \frac{P^2}{2EI} x^2 dx$$

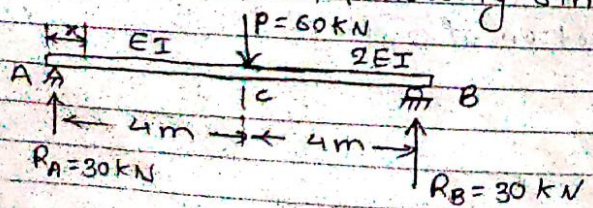
$$= \frac{P^2 L^3}{6EI}$$

Also, External workdone $= \frac{P}{2} \times \delta$

Then, $\frac{P}{2} \times \delta = \frac{P^2 L^3}{6EI}$

$$\boxed{\delta = \frac{PL^3}{3EI}}$$

② Find the deflection of following simply supported beam.



→ Here,

$M = 30 \times x$

$$U = \int \frac{M^2 dx}{2EI} + \int \frac{M^2 dx}{4EI}$$

AC BC

$$U = \int \frac{(30x)^2 dx}{2EI} + \int \frac{(30x)^2 dx}{4EI}$$

$$= \frac{450}{EI}\int_0^4 x^2 dx + \frac{225}{EI}\int_0^4 x^2 dx$$

$$= \frac{450}{EI} \times \frac{4^3}{3} + \frac{225}{EI} \times \frac{4^3}{3}$$

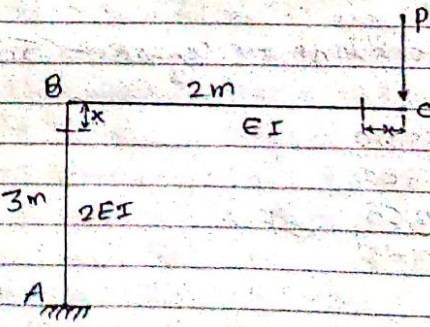
$$= \frac{14400}{EI}$$

Also, External workdone $= \frac{P}{2} \times \delta = \frac{60}{2} \times \delta$

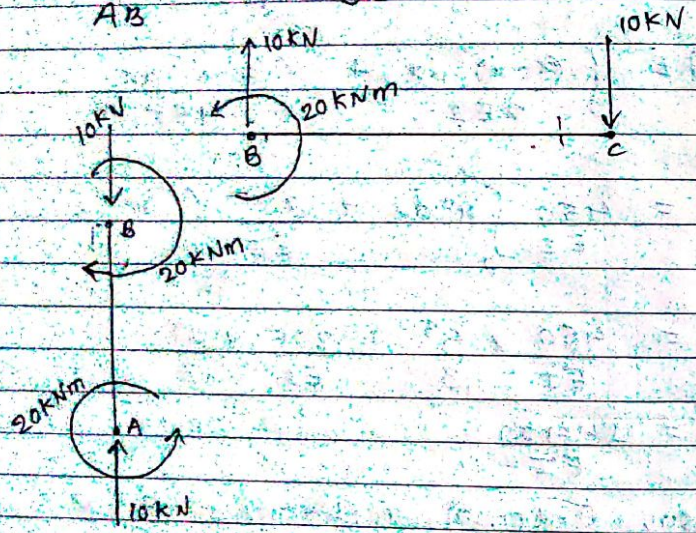
$$\delta \times 30 = \frac{14400}{EI}$$

$$\delta = \frac{480}{EI}$$

③ Find deflection at 'c' of cantilever frame.



$$U = \underbrace{\int \frac{M^2 dx}{4EI}}_{AB} + \underbrace{\int \frac{M^2 dx}{2EI}}_{BC}$$



$$U = \int_0^3 \frac{(20)^2 dx}{4EI} + \int_0^2 \frac{(-10x)^2}{2EI} dx$$

$$= \frac{400}{4EI} \times 3 + \frac{100}{2EI} \times \frac{2^3}{3} = \frac{1300}{3EI}$$

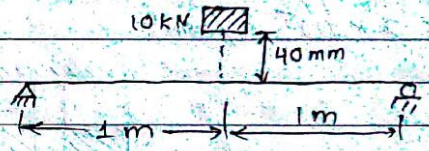
Also,

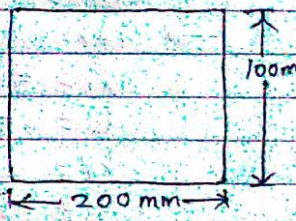
External Workdone $= P/2 \times \delta = 5\delta$

So, $5\delta = \frac{1300}{3EI}$

$\delta = \frac{260}{3EI}$ Ans.

4. A rectangular beam (200 x 100) mm is simply supported over a span of 2m. A load of 10kN is drop on the middle of beam from the height of 40mm. Find the maximum instantaneous deflection in the beam & stress induced in beam. Take $E = 1.1 \times 10^4$ GPA





Here,

$$\delta_{st} = \frac{PL^3}{48EI}$$

$$= \frac{10 \times 10^3 \times 2^3 \times 12}{48 \times 1.1 \times 10^4 \times 10^9 \times 0.2 \times 0.1^3}$$

$$= 9.09 \times 10^{-6} m$$

$$I.f = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$$

$$= 1 + \sqrt{1 + \frac{2 \times 40 \times 10^{-3}}{9.09 \times 10^{-6}}}$$

$$= 94.81$$

$$\delta_d = \delta_{max} = \delta_{st} \times I.f$$

$$= 9.09 \times 10^{-6} \times 94.81 = 0.861 \text{ mm}$$

Also,

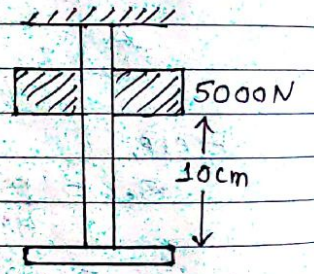
$$\sigma_{st} = \frac{M}{I} \times y = \frac{5 \times 10^3 \times 1 \times 50 \times 10^{-3}}{\frac{0.2 \times 0.1^3}{12}} = 1.5 \times 10^7 \text{ N/m}^2$$

$$\sigma_{inst} = \sigma_{st} \times If$$

$$= 1.42 \times 10^9 \text{ N/m}^2$$

5. A bar of 5m length & 6cm diameter hangs vertically & has a collar securely attached at the lower end. Find the maximum stress induced & instantaneous elongation when 5000N falls from height of 10cm on the collar. Take $E = 2\times10^5\ N/mm^2$

→ Here,



$W = 5000\ N$

$L = 5\ m$

$h = 10\times10^{-2}\ m = 0.1\ m$

$d = 0.06\ m$

$A = \frac{\pi d^2}{4}$

$= \frac{\pi \times 0.06^2}{4} = 2.827\times10^{-3}\ m^2$

$E = 2\times10^{11}\ N/m^2$

Then,

$\delta_{st} = \frac{WL}{AE} = \frac{5000\times5}{2.827\times10^{-3}\times2\times10^{11}} = 4.42\times10^{-5}\ m$

$IF = 1 + \sqrt{1+\frac{2h}{\delta_{st}}} = 1 + \sqrt{1+\frac{2\times0.1}{4.42\times10^{-5}}}$

$= 68.27$

$\delta = \delta_{st} \times IF$

$= 4.42\times10^{-5}\times68.27$

$= 3.017\ mm$

For stress,

$\sigma_{st} = W/A = 1.76\ N/mm^2$

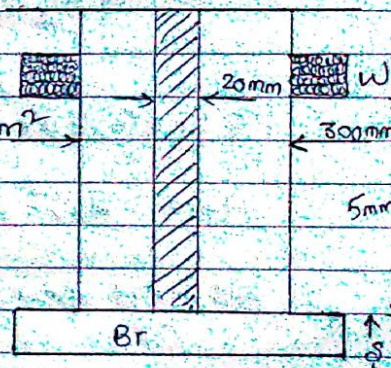
$\sigma_{max} = \sigma_{st} \times IF$

$= 120.15\ N/mm^2$

6. A steel bar of 20mm diameter is encased in a brass tube of 30mm outside diameter which is 1.5m long. The bar is fixed at the top end & collar is attached at the lower end. The mass of 100kg fall through a height of 5mm. Taking, $E_{brass} = 100\ GN/m^2$, $E_{steel} = 200\ GN/m^2$

Calculate maximum stress in brass & steel.

→ Here,



$A_S = \frac{\pi\times20^2}{4} = 314.16\ mm^2$

$A_B = \frac{\pi}{4}(30^2-20^2) = 70371.67\ mm^2$

$E_B = 100\times10^3\ N/mm^2$

$E_S = 200\times10^3\ N/mm^2$

Then,

$W(h+\delta) = U_B + U_S$

$W\left(h+\frac{\sigma_B\times L}{E_B}\right) = \frac{\sigma_B^2}{2E_B}\times A_B\times L + \frac{\sigma_S^2}{2E_S}\times A_S\times L$

$1000\times9.81\times10^{-3}\left(5+\frac{\sigma_B\times1500}{100\times10^3}\right) = \frac{\sigma_B^2}{2}\times1500 + \frac{\sigma_S^2}{2}\times1500$

$9.81\times10^6(5+0.015\sigma_B) = 750\sigma_B^2 + 750\sigma_S^2$ —①

From compability condⁿ,

$\epsilon_S = \epsilon_B$

$\Rightarrow \frac{\sigma_S}{E_S} = \frac{\sigma_B}{E_B} \Rightarrow \sigma_S = 2\sigma_B$

From ①

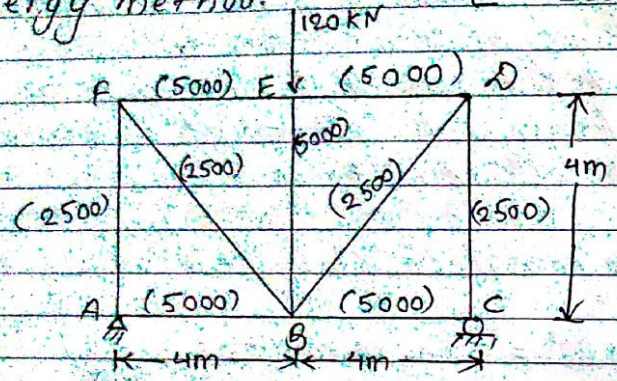
$49050000 + 147150\sigma_B = 750\sigma_B^2 + 750\times(2\sigma_B)^2$

$\Rightarrow 3750\sigma_B^2 - 147150\sigma_B - 49050000 = 0$

$\sigma_B = 135.65\ N/mm^2$

$\sigma_S = 271.3\ N/mm^2$

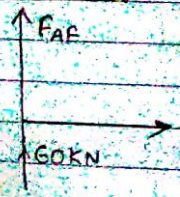
✻ Find the deflection of truss using strain Energy method. $E = 200\ KN/mm^2$



Strain Energy (S.E) = $\sum \frac{P^2 L}{2AE}$

Here, $R_{Ax} = 0$, $R_{Ay} = 60\ KN$, $R_{Cy} = 60\ KN$

At joint A:-

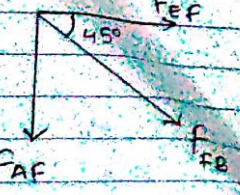


$(+\uparrow)\ F_{AF} + 60 = 0$

$F_{AF} = -60\ KN$

$(+\rightarrow)\ F_{AB} = 0$

At joint F



$(+\rightarrow)\ F_{EF} + F_{FB}\cos 45° = 0$ ← (1)

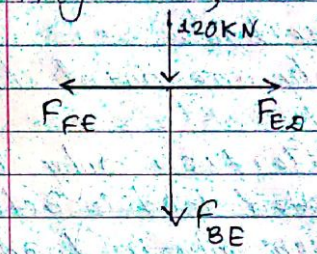
$(+\uparrow)\ -F_{AF} - F_{FB}\sin 45° = 0$

$F_{FB}\sin 45° = +60$

$F_{FB} = +84.85\ KN$

From (1) $F_{EF} = -84.85 \times \frac{1}{\sqrt{2}} = -60\ KN$

At joint E,



$(+\uparrow)\ \Sigma F_y = 0$

$-120 - F_{BE} = 0$

$F_{BE} = -120\ KN$

Member	Force	Nature
AB	0	
BC	0	
CD	60KN	T
DE	60 KN	C
EF	60KN	T
AF	60KN	C
BF	84.85 KN	T
BD	84.85 KN	T
BE	120 KN	C

$S.E = \sum \left(\frac{P^2 L}{2AE}\right)$

$= 2\times 0 + \frac{2\times 60^2 \times 4000}{2\times 2500\times 200} + \frac{2\times 60^2\times 4000}{2\times 5000\times 200} + \frac{2\times 84.85^2\times 4000}{2\times 2500\times 200}$

$+ \frac{120^2\times 4000}{2\times 5000\times 200}$

$= 153.45\ KJ$

$S\varepsilon = P/2 \times S$

$153.45 = \frac{120}{2} \times S \Rightarrow S = 2.55\ mm$

ch-3 Virtual Work Method:-

* Limitation of real work method:
→ It is applicable when only one point load is acting on structure.
→ We can calculate the deflection only at that point where concentrated load is acting.
→ We can find the deflection only in load acting direction.

Note:- Deflection by method of real work is similar to deflection by strain energy method.

** Principle of virtual Work:-
→ According to principle of virtual work, virtual work is the product of real force & imaginary displacement or the product of imaginary force & real displacement.

Virtual work = Real force x Imaginary displacement
or
= Imaginary force x Real displacement

→ Incase of rigid body, the distance betn two points is constant & virtual workdone by external force is zero. But in deformable bodies, the virtual workdone may be caused by real force acting through virtual displacement.

→ The principle of virtual work is used to find deflection of structures.
→ Virtual work method is also called unit load method or dummy load method.



virtual work = $\frac{1}{2} \times P \times \delta = \frac{1}{2} \times V \times \Delta$

Here, P = Real force
Δ = Real displacement
V = Virtual force
δ = virtual displacement.

we have,

$$\frac{1}{2} \times V \times \Delta = \int \frac{1}{2} \times \sigma' \times \epsilon \times dv$$

where, σ' = stress due to virtual load
ϵ = strain due to real load
Δ = Real deflection

For unit load i.e V=1,

$$\Delta = \int \sigma' \times \epsilon \times dv \quad \text{---------(1)}$$

From flexural eqn, $\frac{\sigma'}{y} = \frac{m}{I}$

$$\sigma' = \frac{my}{I}$$

where, m = moment in consider section due to unit load.

From (i)

$$\Delta = \int \frac{my}{I} \times e \times dv$$

$$= \int \frac{my}{I} \times \frac{\sigma}{E} \times dv$$

where, σ = stress due to real load

$$= \int \frac{my}{I} \times \frac{M}{I} \times y \times \frac{1}{E} \times dx \times dA$$

$$= \int_L \left(\frac{Mm}{EI^2} \int_A (y^2 dA) \right) dx$$

We know, $I = \int_A y^2 dA$

$$\Delta = \int_L \frac{Mm}{EI} \times dx$$

$$\therefore \Delta = \int_0^L \frac{Mm}{EI} dx \quad ---- (ii)$$

Where, M = moment due to external loading

m = Moment due to unit load.

Then, using eqn (ii), we can find the deflection incase of beam, frame, truss etc.

* Steps of solving problem / finding deflection using dummy load method or unit load method.

① Firstofall, find support rxn if necessary & draw FBD.

② consider a small section at a distance 'x' & calculate 'M' due to external loading.

③ Remove all external forces & apply unit load & similarly calculate all force & find 'm' due to unit load.

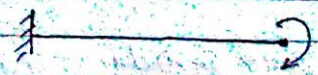
④ Then use formula, $\Delta = \int_0^L \frac{Mm}{EI} dx$

⑤ Draw following table,

SN	portion	origin	Limit	M	m	m_1	EI
eg 1	AB	A	0–100	0.5x+100	-3.5x	0.2x	EI
	- -	- -	- -	- -	- -	- -	- -

⑥ If we have to find slope, then draw column of m_1 but for deflection only column of m_1 is not necessary.

⑦ To find m_1, we have to apply unit moment at the point where we are going to find the deflection.



⑧ While calculating 'm' we have to apply unit load at the point where we are going to find deflection.



⑤ After calculating Δ & Θ,

If Δ +ve → deflection in dirⁿ of applied load

Δ -ve → deflection in opposite dirⁿ of applied load.

If Θ +ve → deflection occur in dirⁿ of applied Unit moment

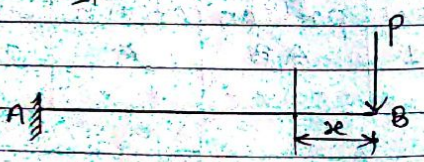
Θ -ve → deflection occur in opposite direction of applied Unit moment

※ Find the deflection & slope at point B of the cantilever beam using Unit load method:-

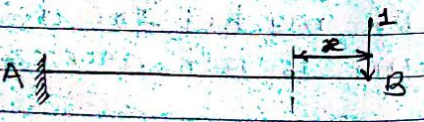


→

a) For deflection,



$M = -P \times x$



$m = -1 \times x$

Then,

$$\Delta = \int_0^L \frac{M m \, dx}{EI}$$

$$= \int_0^L \frac{-P x \times (-x) \, dx}{EI}$$

$$= \frac{P}{EI} \times \frac{L^3}{3}$$

$$= \frac{PL^3}{3EI}$$

b) For slope,

$$\Theta_B = \int_0^L \frac{M m_1 \, dx}{EI}$$

For m_1



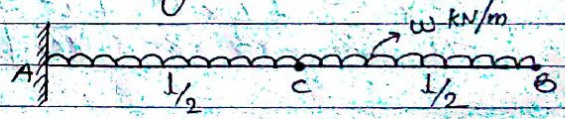
$m_1 = -1$ kNm

Then,

$$\Theta_B = \int_0^L \frac{-P x \times x \times (-1) \, dx}{EI} = \int_0^L \frac{P x \, dx}{EI}$$

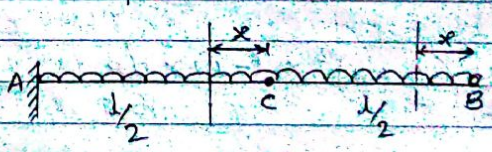
$$\Theta_B = \frac{P}{EI} \times \frac{L^2}{2} = \frac{PL^2}{2EI}$$

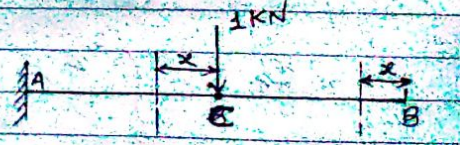
※ Find the deflection & slope of beam, shown in figure using virtual work method. ↳ point C

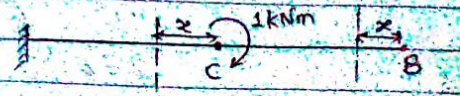


→

※ For slope & deflection:-







SN	portion	origin	limit	M	m	m_1	EI
1	AC	C	$0-L/2$	$-wx\left(\frac{L}{2}+x\right)\times\frac{\left(\frac{L}{2}+x\right)}{2}$	$-x$	-1	EI
2	CB	B	$0-L/2$	$-\frac{wx^2}{2}$	0	0	EI

$$\Delta_c = \int_0^{L/2} -\frac{w}{2}\left(\frac{L}{2}+x\right)^2 \times (-x) \times \frac{1}{EI}\,dx$$

$$= \frac{w}{2EI}\int_0^{L/2} x\left(\frac{L^2}{4}+Lx+x^2\right)dx$$

$$= \frac{w}{2EI}\int_0^{L/2}\left(\frac{L^2}{4}x+Lx^2+x^3\right)dx$$

$$= \frac{w}{2EI}\left[\frac{L^2}{4}\times\frac{L^2}{8}+\frac{L}{3}\times\frac{L^3}{8}+\frac{L^4}{64}\right]$$

$$= \frac{17\,wL^4}{384EI}$$

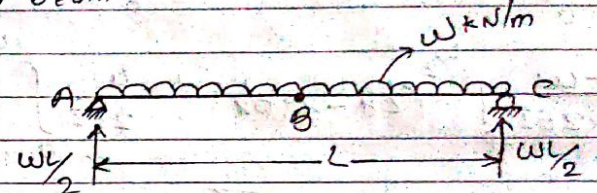
$$\theta_c = \int_0^{L/2} -\frac{w}{2}\left(\frac{L}{2}+x\right)^2 \times (-1) \times \frac{dx}{EI}$$

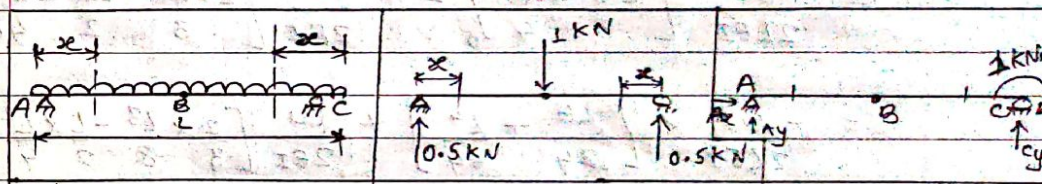
$$= \frac{w}{2EI}\int_0^{L/2}\left(\frac{L}{2}+x\right)^2 dx$$

$$= \frac{w}{2EI}\times\left[\frac{\left(\frac{L}{2}+x\right)^3}{3}\right]_0^{L/2}$$

$$= \frac{w}{2EI\times 3}\left[L^3-\frac{L^3}{8}\right] = \frac{7wL^3}{48EI}$$

** Find the deflection & slope of following simply supported beam.





M- diagram | m- diagram | m_1 - diagram.

calculation of rxn in m_1 diagram,

$+\circlearrowright \Sigma M_A = 0$

$-C_y \times L + 1 = 0 \Rightarrow C_y = 1/L$

$(+\uparrow)\ \Sigma f_y = 0$

$A_y + 1/L = 0 \Rightarrow A_y = -1/L$

$(+\rightarrow)\ \Sigma f_x = 0$

$A_x = 0$

SN	Portion	Origin	Limit	M	m	m_1	EI
1	AB	A	$0-L/2$	$\frac{wL}{2}x-\frac{wx^2}{2}$	$0.5x$	$-x/L$	EI
2	BC	C	$0-L/2$	$\frac{wL}{2}x-\frac{wx^2}{2}$	$0.5x$	$-1+\frac{1}{L}\times x$	EI

$$\Delta_B = \int_0^{L/2}\frac{0.5x}{EI}\left(\frac{wLx}{2}-\frac{wx^2}{2}\right)dx + \int_0^{L/2}\frac{0.5x}{EI}\times\left(\frac{wLx}{2}-\frac{wx^2}{2}\right)dx$$

$$= \frac{2\times 0.5\times w}{2EI}\int_0^{L/2}(Lx^2-x^3)\,dx$$

$$= \frac{w}{2EI}\times\left[\frac{L\times L^3}{24}-\frac{L^4}{64}\right] = \frac{5wL^4}{384EI}$$

$$\theta_C = \int_0^{L/2} \frac{\frac{w}{2}(Lx - x^2) \times \frac{-x}{L}\, dx}{EI} + \int_0^{L/2} \frac{\frac{w}{2}(Lx - x^2) \times \left(\frac{x}{L} - 1\right) dx}{EI}$$

$$= \frac{-w}{L \times 2EI} \int_0^{L/2} (Lx^2 - x^3)\, dx + \frac{w}{2EI} \int_0^{L/2} \left(x^2 - Lx - \frac{x^3}{L} + x^2\right) dx$$

$$= \frac{-w}{2LEI} \left[\frac{Lx^3}{3} - \frac{x^4}{4}\right]_0^{L/2} + \frac{w}{2EI} \left[\frac{2 \times x^3}{3} - \frac{Lx^2}{2} - \frac{x^4}{4L}\right]_0^{L/2}$$

$$= -\frac{w}{2LEI} \times \left[\frac{L^4}{24} - \frac{L^4}{64}\right] + \frac{w}{2EI} \left[\frac{2}{3} \times \frac{L^3}{8} - \frac{L}{2} \times \frac{L^2}{4} - \frac{L^4}{64L}\right]$$

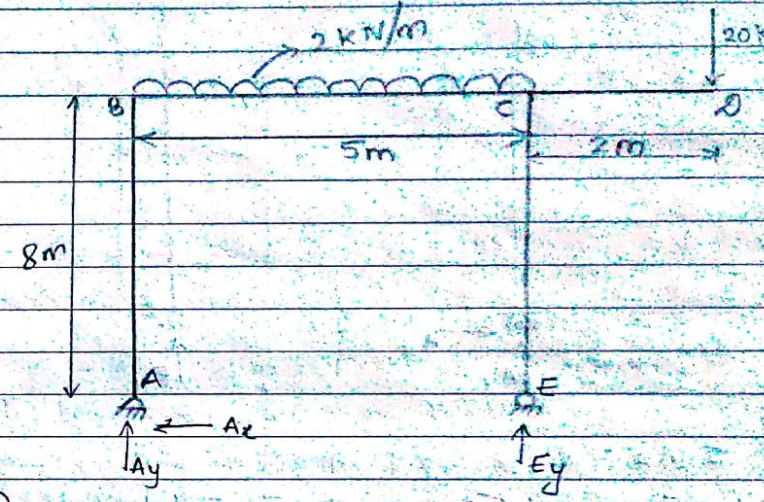
$$= \frac{-w}{2LEI} \times \frac{5L^4}{192} + \frac{w}{2EI} \times \left(\frac{-11}{192}\right) \times L^3$$

$$= -\frac{5wL^3}{384EI} - \frac{11wL^3}{384EI}$$

$$= -\frac{wL^3}{24EI}$$

* Find rotation at A & E. Also find $(\Delta C)_V$ & $(\Delta C)_H$.

EI - constant



$+\circlearrowright \Sigma M_A = 0$

$\Rightarrow 10 \times 2.5 + 20 \times 7 - E_y \times 5 = 0 \quad \Rightarrow E_y = 33\,kN$

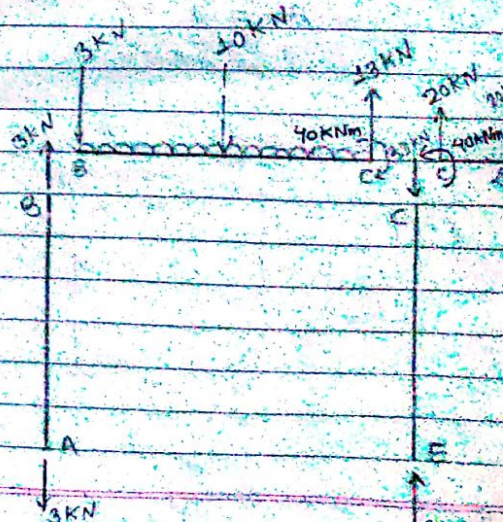
$(+\uparrow) \Sigma F_y = 0$

$\Rightarrow A_y - 10 - 20 + 33 = 0$

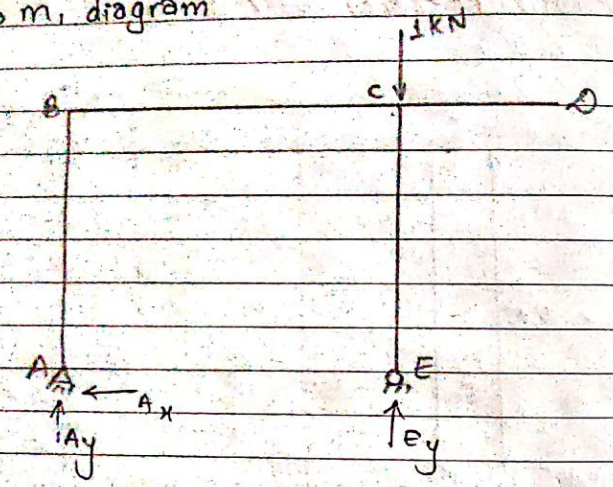
$\Rightarrow A_y = -3\,kN$

$(+\rightarrow) \Sigma F_x = 0 \quad \Rightarrow A_x = 0$

For M :-



For $(\Delta_C)_V \Rightarrow m_1$ diagram



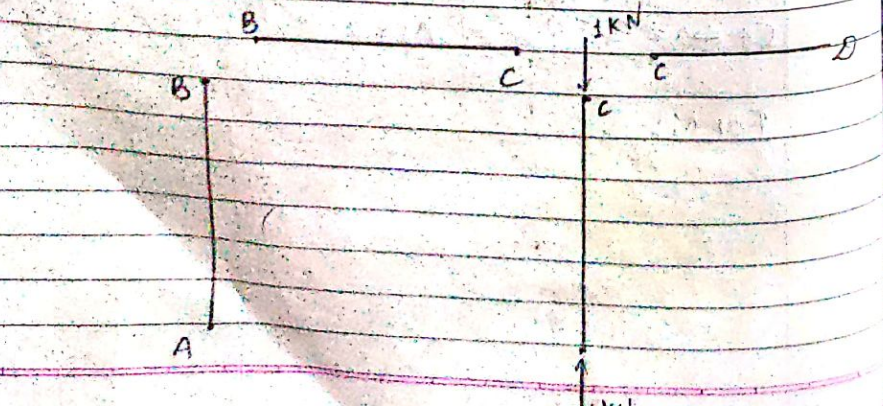
$+\circlearrowleft \Sigma M_A = 0$

$1 \times 5 - E_y \times 5 = 0 \Rightarrow E_y = 1\ kN$

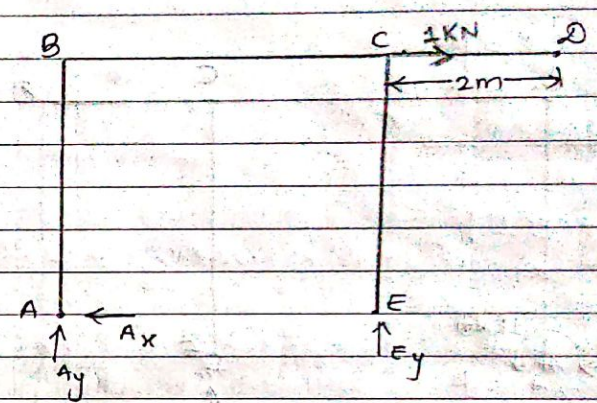
$(+\uparrow) \Sigma F_y = 0$

$A_y - 1 + E_y = 0 \Rightarrow A_y - 1 + 1 = 0$

$A_x = 0$



For $(\Delta_C)_{H_1} \Rightarrow m_2$ diagram

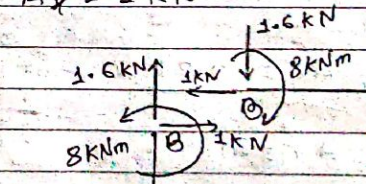


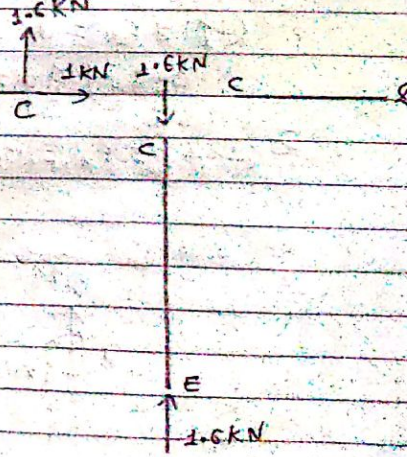
$+\circlearrowleft \Sigma M_A = 0$

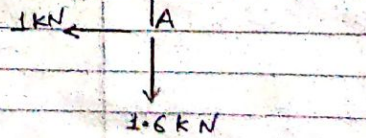
$-E_y \times 5 + 1 \times 8 = 0 \Rightarrow E_y = 8/5 = 1.6\ kN$

$(+\uparrow) \Sigma F_y = 0 \Rightarrow A_y + E_y = 0 \Rightarrow A_y = -1.6\ kN$

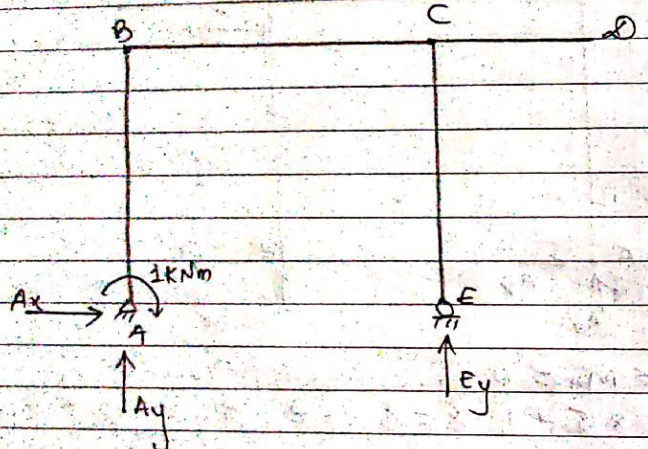
$A_x = 1\ kN$







For rotation at A ⇒ m_3 diagram.



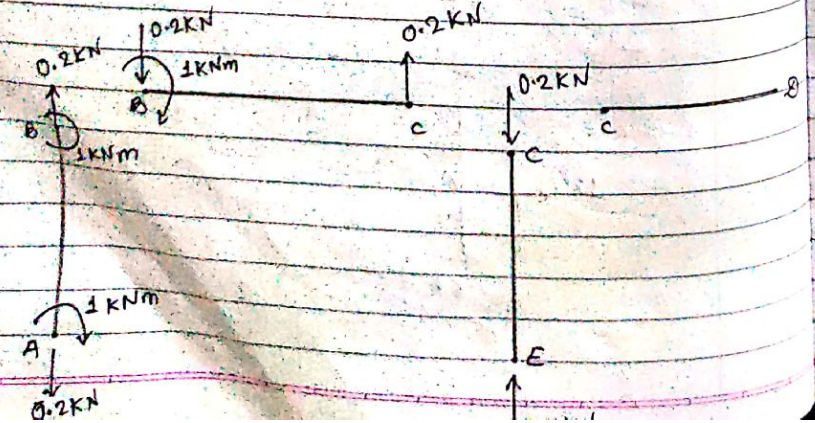
$+\circlearrowleft \Sigma M_A = 0$

$\Rightarrow 1 - E_y \times 5 = 0 \quad \Rightarrow E_y = 0.2kN$

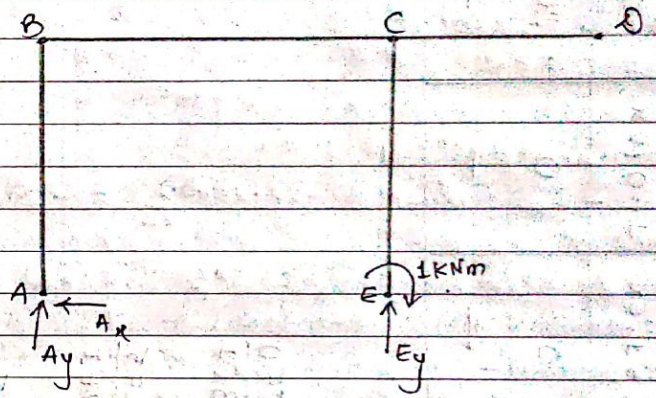
$(+\uparrow) \Sigma F_y = 0$

$\Rightarrow 0.2 + A_y = 0 \quad \Rightarrow A_y = -0.2kN$

$(+\rightarrow) \Sigma F_x = 0, \quad \Rightarrow A_x = 0$



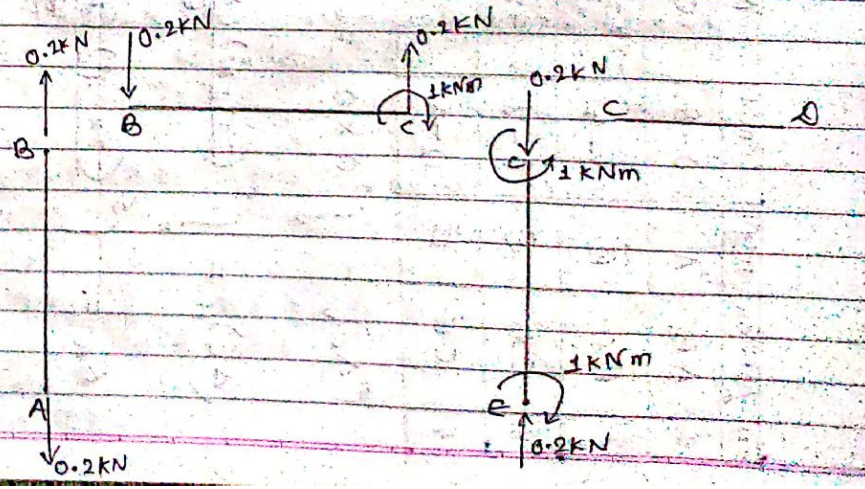
For rotation at E ⇒ m_4 diagram



$+\circlearrowleft \Sigma M_A = 0 \quad \Rightarrow 1 - E_y \times 5 = 0 \quad \Rightarrow E_y = 0.2kN$

$(+\uparrow) \Sigma F_y = 0 \quad \Rightarrow A_y + 0.2 = 0 \quad \Rightarrow A_y = -0.2kN$

$A_x = 0$



SN	Portion	Origin	Limit	M	m_1	m_2	m_3	m_4	EI
1	AB	A	0-8	0	—	—	—	—	EI
2	BC	B	0-5	$-3x-2x\frac{x}{2}$ $=-3x-x^2$	0	$8-1.6x$	$1-0.2x$	$-0.2x$	EI
3	CD	D	0-2	$-20x$	0	0	0	0	EI
4	CE	E	0-8	0	—	—	—	—	EI

$$(\Delta_C)_V = \int_0^5 0 \times 0 = 0$$

$$(\Delta_C)_H = \int_0^5 \frac{(-3x-x^2)(8-1.6x)dx}{EI} = -\frac{550}{3EI}$$

$$\theta_A = \int_0^5 \frac{(-3x-x^2)(1-0.2x)dx}{EI} = -\frac{275}{12EI}$$

$$\theta_E = \int_0^5 \frac{(-3x-x^2)(-0.2x)dx}{EI} = \frac{225}{4EI}$$

* Deflection of truss by unit load method

we know,

$$\Delta = \int \sigma' \epsilon dv \qquad [P' \times \Delta = P \times \delta]$$

(Imaginary force × real disp = Real force × Imag. disp)

$$= \Sigma \sigma' \times \epsilon \times A \times l$$

$$= \Sigma \left(\frac{P'}{A}\right) \times \frac{P}{AE} \times A \times l$$

$$\Delta = \Sigma \left(\frac{PP'L}{AE}\right)$$

* Find the vertical deflection of steel truss shown at end 'c'. Cross-sectional areas in mm^2 of all the members. Take $E = 200\ kN/mm^2$

Step 1: For calculation of member force 'P'

At joint C

$(+\uparrow)\Sigma F_y = 0$

$F_{BC}\sin 45° - 20 = 0$

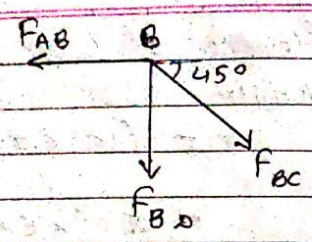
$F_{BC} = 28.28\ N$

$(+\leftarrow)\Sigma F_x = 0$

$F_{CD} + F_{BC}\cos 45° = 0$

$F_{CD} = -20\ kN$

At joint B



$(+\rightarrow) \Sigma F_x = 0$

$-F_{AB} + F_{BC}\cos 45° = 0$

$F_{AB} = 28.28 \cos 45°$

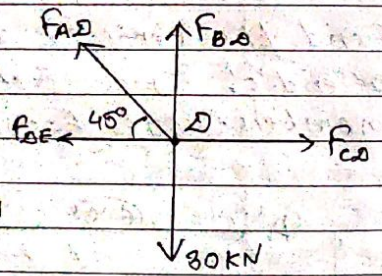
$= 20\,kN$

$(+\uparrow) \Sigma F_y = 0$

$-F_{BD} - F_{BC}\sin 45° = 0$

$F_{BD} = -F_{BC}\sin 45° = -28.28 \times \frac{1}{\sqrt{2}} = -20\,kN$

At joint D



$(+\uparrow) \Sigma F_y = 0$

$F_{BD} + F_{AD}\sin 45° - 80 = 0$

$-20 + F_{AD}\sin 45° - 80 = 0$

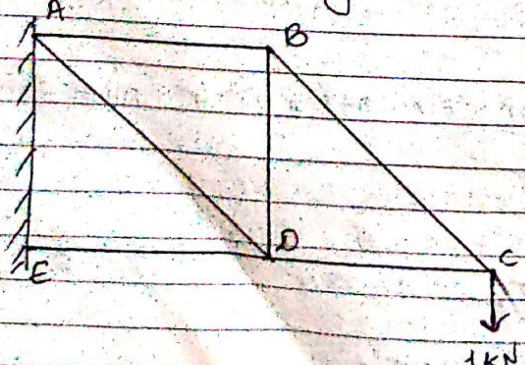
$F_{AD} = \frac{50}{\sin 45°} = 70.71\,kN$

$(+\leftarrow) \Sigma F_x = 0$

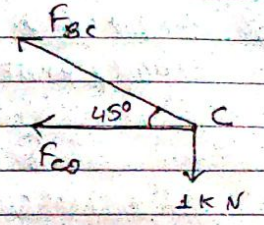
$F_{DE} + 70.71 \cos 45° + 20 = 0$

$F_{DE} = -70\,kN$

Step 2: For calculation of member force p' due due to unit loading at 'c' in vertical direction.



At joint C,



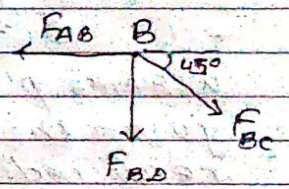
$F_{BC}\cos 45° = 1$

$F_{BC} = 1.41\,kN$

$-1.41 \cos 45° = F_{CD}$

$F_{CD} = -1\,kN$

At joint B:-

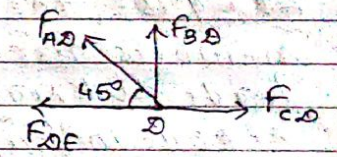


$-F_{AB} + 1.41\cos 45° = 0$

$F_{AB} = 1\,kN$

$F_{BC}\sin 45° + F_{BD} = 0 \Rightarrow F_{BD} = -1\,kN$

At joint D



$F_{AD}\sin 45° - 1 = 0$

$F_{AD} = 1.41\,kN$

Anjanth $F_{CD} - F_{DE} - F_{AD}\cos 45° = 0$

$-1 - F_{DE} - 1.41\cos 45° = 0$

$F_{DE} = -2\,kN$

$E = 200\,kN/mm^2$

Member	P	p'	L	A (mm^2)	(kN) AE	$(\Delta_c)_v = \Sigma \frac{PP'L}{AE}$	(mm)
AB	20 kN	1 kN	4m	600	120000	0.67	
BC	28.28kN	1.41kN	$\sqrt{32}$ m	600	120000	1.87	
CD	-20 kN	-1 kN	4m	1200	240000	0.33	
BD	-20kN	-1 kN	4m	600	120000	0.67	
AD	70.71kN	1.41 kN	$\sqrt{32}$ m	600	120000	4.69	
DE	-70kN	-2kN	4m	1200	240000	2.33	

$(\Delta C)_v = 10.56\,mm.$

Deflection due to lack of fit & temperature change

From principle of virtual work,

$$\delta = \sum \frac{pp'L}{AE}$$

$$\delta = \sum p' \times \left(\frac{pL}{AE}\right)$$

$$\delta = \sum p' \times \Delta L \quad \text{—①}$$

Where, p' is the unit load applied at the joint in the direction of which deflection is required.

In eqn①, the change in length (ΔL) of the members need not be only due to applied loads but may be due to any other cause. change in length may be due to selecting improper lengths while fabricating or may be due to temperature changes.

let,

$(\Delta L)_e \Rightarrow$ Change in length due to external loading

$(\Delta L)_T \Rightarrow$ Change in length due to temp^r

$(\Delta L)_M \Rightarrow$ Change in length due to misfit.

Then,

Total deflection $(\delta) = (\Delta L)_e + (\Delta L)_T + (\Delta L)_M$

$(\Delta L)_T \rightarrow$ +ve for rise in temp^r

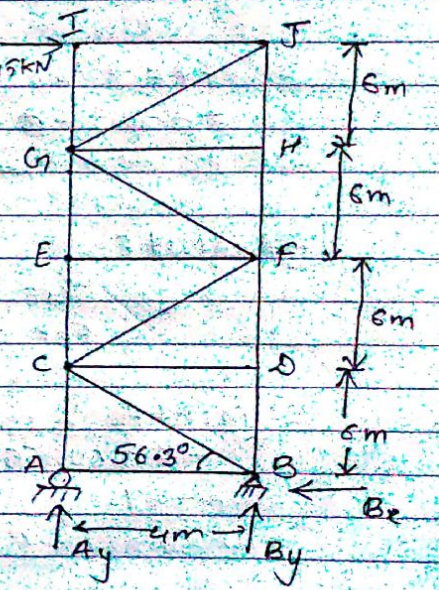
-ve for fall in temp^r

$(\Delta L)_M \rightarrow$ Member too short by ___ → -ve

Member too long by ___ → +ve

2014 Spring

2) A pin jointed frame is as shown in figure. Find horizontal displacement of point J using unit load method. Member CD is fabricated 2cm too long, temp^r of member GJ is increased by 30°c & GF is decreased by 30°c. The area of x-sectn of all members is 2000 mm^2. $E = 2\times10^5$ N/mm^2 & $\alpha = 12\times10^{-6}$/°c.



a) For calculation of member force due to external loading,

$+\circlearrowleft \sum M_A = 0$

$\Rightarrow 45\times24 - B_y\times4 = 0$

$\Rightarrow B_y = 270$ kN

$(+\uparrow)\sum F_y = 0$

$A_y + B_y = 0$

$A_y = -270$ kN

$(+\rightarrow)\ \sum F_x = 0, \quad B_x = 45$ kN

At joint A,

$f_{AB} = 0 \qquad f_{AC} - 270 = 0$

$f_{AC} = 270$ kN

At joint B,

$-f_{BC}\cos 56.3° + 0 - 45 = 0 \Rightarrow f_{BC} = -81.1$ kN

$270 + f_{BD} + (-81.1)\sin 56.3° = 0$

$f_{BD} = -202.52$ kN

At joint D

$F_{DF} - F_{BD} = 0$ $\quad F_{DC} = 0$

$F_{DF} = -202.52\ kN$

At joint C

$F_{BC}\cos 56.3° + F_{CF}\cos 56.3° = 0$

$F_{CF} = -F_{BC} = 81.1\ kN$

$F_{CE} - F_{AC} + 81.1\sin 56.3° + 81.1\sin 56.3° = 0$

$-270 + F_{CE} + 134.94 = 0$

$F_{CE} = 135.06\ kN$

At joint E,

$F_{GE} = 135.06\ kN$

$F_{EF} = 0$

At joint F,

$-F_{FC}\cos 56.3° - F_{GF}\cos 56.3° = 0$

$-81.1\cos 56.3° = F_{GF}\cos 56.3°$

$F_{GF} = -81.1\ kN$

At joint H, $F_{FH} + 202.52 - 81.1\sin 56.3° + (-81.1)\sin 56.3° = 0$

$F_{FH} = -67.57\ kN$

At joint H,

$F_{GH} = 0$

$F_{HJ} = -67.57\ kN$

At joint J, $-F_{IJ} - F_{GJ}\cos 56.3° = 0$ — (1)

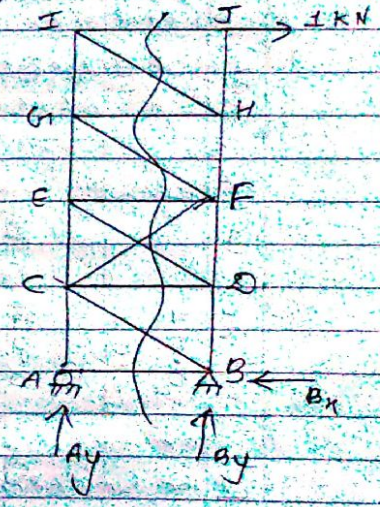
$-(-67.57) - F_{GJ}\sin 56.3° = 0$

$F_{GJ} = 81.22\ kN$

From (1)

$-F_{IJ} - 81.22\cos 56.3° = 0$ $\quad F_{IJ} = -45.06\ kN$

b) For calculation of member force due to unit loading at J.



$\circlearrowleft \sum M_A = 0$

$1 \times 24 - B_y \times 4 = 0$

$B_y = 6\ kN$

$A_y = -6\ kN$

$B_x = 1\ kN$

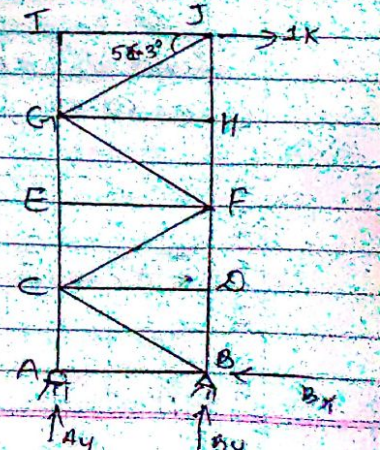
At joint J,

$F_{GJ}\cos 56.3° = 1$

$F_{GJ} = 1.802\ kN$

$1.802\sin 56.3° + F_{JH} = 0$

$F_{JH} = -1.5\ kN$



SN	Member	P	P'	L/Ae	$(\Delta L)_e$ (mm)	$(\Delta L)_t$	$P'(\Delta L)_t$ (mm)	$(\Delta L)_m$	$(\Delta L)_m P'$
1	AB	0	0	0.011	0	0	0	0	0
2	CD	0	0	0.011	0	0	0	0	0
3	EF	0	0	0.011	0	0	0	0	0
4	GH	0	0	0.011	0	0	0	0	0
5	IJ	-45.06kN	0	0.011	0	0	0	0	0
6	BC	-81.1kN	-1.802kN	0.018	2.63	0	0	20mm	-36.04
7	CF	81.1 kN	1.802kN	0.018	2.63	0	0	0	0
8	FG	-81.1 kN	-1.802kN	0.018	2.63	-2.59mm	4.66	0	0
9	GI	81.22 kN	1.802 kN	0.018	2.63	2.59 mm	4.66	0	0
10	AC	270 kN	6 kN	0.016	24.3	0	0	0	0
11	CE	135.06kN	3 kN	0.015	6.07	0	0	0	0
12	EG	135.06kN	3 kN	0.015	6.07	0	0	0	0
13	GI	~~-45.06kN~~ 0	0	0.015	0	0	0	0	0
14	BD	-202.52kN	-4.5 kN	0.016	13.67	0	0	0	0
15	DF	-202.52kN	+4.5kN	0.016	13.67	0	0	0	0
16	FH	+67.57kN	-1.5 kN	0.016	1.52	0	0	0	0
17	HJ	-67.57kN	-1.5 kN	0.016	1.52	0	0	0	0

$(\Delta)_m = -36.04$ mm

$(\Delta)_t = 9.33$ mm

$(\Delta)_e = 77.34$ mm

$\Delta = \Delta_e + \Delta_t + \Delta_m = 50.63$ mm

Ch-4

Displacement theorems:-

- Betti's law
- Maxwell Reciprocal theorem
- Castigliano's theorem & application for beams & plane frames.

Betti's law:-

For a linearly elastic structure, the workdone by external force P_1 acting through displacement δ_2 produced by another force P_2 is equal to the workdone by external force P_2, acting through displacement δ_1 produced by P_1.

$$\frac{1}{2} \times P_1 \times \delta_2 = \frac{1}{2} \times P_2 \times \delta_1$$

Maxwell Reciprocal Theorem:-

The deflection of point 'n' due to force 'p' at a point 'm' is numerically equal to the deflection of point 'm' due to force 'p' applied at point 'n'.

deflection of point 'n' due to force 'p' applied at m (δ_1) = deflection of point 'm' due to force 'p' applied at n (δ_2)

$$[\delta_1 = \delta_2]$$

※ Castigliano's theorem:-

↳ Castigliano published two important theorem in structural analysis.

↳ The first theorem helps in determining deflection & second one in determining redundant reaction component.

↳ We study only first theorem in this chapter.

* First theorem:-

In a linearly elastic structure, the deflection of point in dirn of applied load is equal to the partial derivative of strain energy wrt load.

$$\left.\begin{aligned} \Delta &= \frac{\partial U}{\partial P} \\ \theta &= \frac{\partial U}{\partial M} \end{aligned}\right\} \quad \text{—— (1)}$$

* Second theorem:-

The partial derivative of the strain energy with respect to deflection of a point in dirn of applied load is equal to the load.

$$\frac{\partial U}{\partial \Delta} = P$$

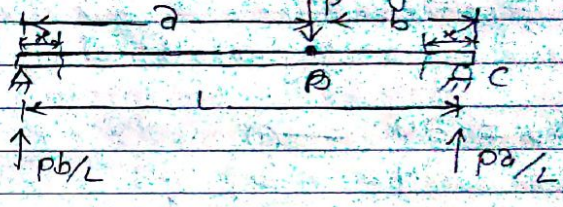
※ Application of first theorem of castigliano:-

↳ We use this theorem to find deflection.

↳ First we have to calculate strain energy & find partial derivative of 'U' wrt 'P' or M.

1) A simply supported beam of length 'L', carries a concentrated load 'P' at a distance 'a' from left hand side support shown in figure. Find deflection under load using castigliano's theorem.

→



$$U = \int \frac{M^2 dx}{2EI}$$

$$= \int_0^a \frac{\left(\frac{Pb}{L} \times x\right)^2 dx}{2EI} + \int_0^b \frac{\left(\frac{Pa}{L} \times x\right)^2 dx}{2EI}$$

$$= \frac{P^2 b^2}{2L^2 EI} \int_0^a x^2 dx + \frac{P^2 a^2}{2L^2 EI} \int_0^b x^2 dx$$

$$= \frac{P^2 b^2}{2L^2 EI} \times \frac{a^3}{3} + \frac{P^2 a^2}{2L^2 EI} \times \frac{b^3}{3}$$

$$= \frac{P^2 a^2 b^2 (a+b)}{6L^2 EI} = \frac{P^2 a^2 b^2 \times L}{6L^2 EI}$$

$$U = \frac{P^2 a^2 b^2}{6LEI}$$

By castigliano's first theorem,

$$\frac{\partial U}{\partial P} = \Delta$$

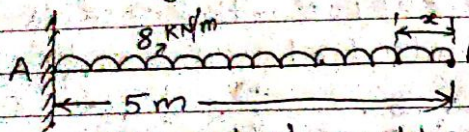
$$\Rightarrow \Delta_B = \frac{\partial}{\partial P}\left(\frac{P^2 a^2 b^2}{6LEI}\right)$$

$$\Rightarrow \Delta_B = \frac{P a^2 b^2}{3LEI}$$

when $a = b = L/2$,

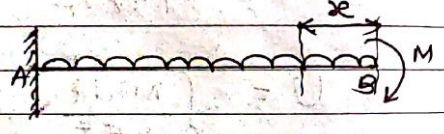
$$\Delta = \frac{PL^3}{48EI}$$

2) Using castiglano's theorem, determine the rotation at B for the given cantilever beam.



→ we know, By castiglano theorem,

$$\theta_B = \frac{\partial U}{\partial M} \quad \text{—①}$$



For strain energy,

$$U = \int_0^5 \frac{\left(-M - \frac{8x^2}{2}\right)^2 dx}{2EI}$$

From ①,

$$\theta_B = \frac{\partial}{\partial M} \int_0^5 \left(M + \frac{8x^2}{2}\right)^2 dx \times \frac{1}{2EI}$$

$$2EI\theta_B = \int_0^5 2\left(M + \frac{8x^2}{2}\right) \times 1 \times dx$$

$$EI\theta_B = \int_0^5 (M + 4x^2)\, dx$$

put M=0, [dummy moment]

$$\theta_B = \int_0^5 \frac{4x^2 dx}{EI}$$

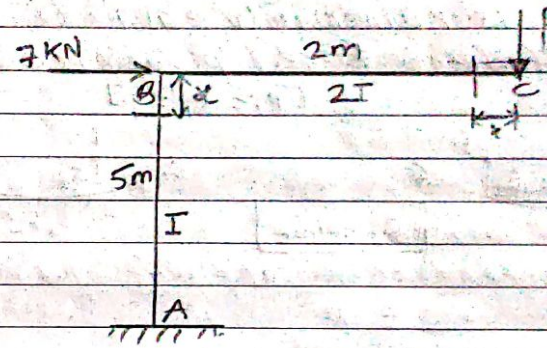
$$\theta_B = 4 \times \frac{5^3}{3} \times \frac{1}{EI}$$

$$\theta_B = \frac{500}{3EI}$$

$\theta = \frac{WL^3}{6EI}$

2011 Fall

3ⓐ Calculate the vertical deflection at point C in the following given frame using castiglano's theorem. Take $I = 160000 cm^4$ & $E = 210 GN/m^2$



We know, Strain energy $(U) = \int_0^l \frac{M^2 dx}{2EI}$ ——①

And by castiglano's theorem,

$$\Delta_C = \frac{\partial U}{\partial P} \quad \text{———②}$$

From ①,

$$U = \int_0^2 \frac{(-Px)^2 dx}{4EI} + \int_0^5 \frac{(-Px2 - 7x)^2 dx}{2EI}$$

$$= \int_0^2 \frac{P^2x^2}{4EI} dx + \int_0^5 \frac{(2P + 7x)^2}{2EI} dx$$

From ②

$$\Delta_C = \int_0^2 \frac{2Px^2}{4EI} dx + \int_0^5 \frac{2(2P + 7x)}{2EI} \times 2\, dx$$

put P = 0,

$$\Delta_C = \int_0^5 \frac{4}{2EI} \times 7x\, dx = 14 \times \frac{5^2}{2} \times \frac{1}{EI}$$

$$= 7 \times 25 \times \frac{1}{210 \times 10^6 \times 160000 \times 10^{-8}}$$

$$= 0.52 \text{ mm}$$

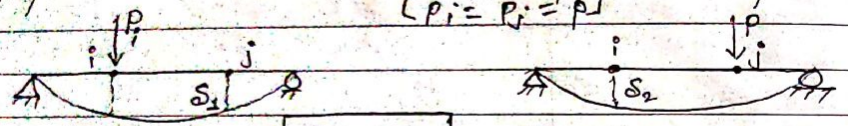
Proof

* Maxwell theorem of reciprocal deflection:-

Statement:-

The deflection of point 'j' due to force 'p' applied at point 'i' is numerically equal to the deflection of point 'i' due to force 'p' applied at point 'j'.

$[P_i = P_j = P]$



$$\boxed{S_1 = S_2}$$

Proof

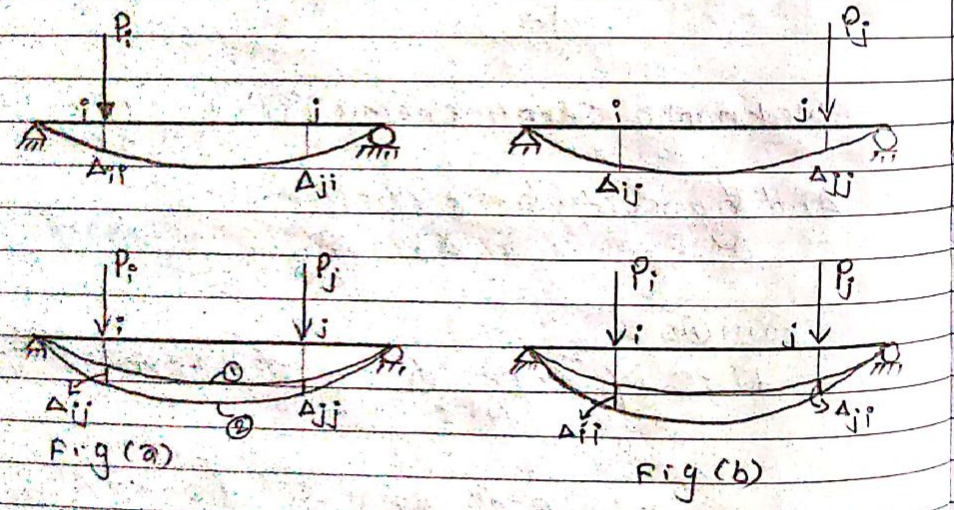


Fig (a) Fig (b)

In frg 'a', case(i) load P_i is acting at point 'i'.

Let Δ_{ii} = Deflection of point 'i' due to load at 'i'

Δ_{ji} = Deflection of point 'j' due to load at 'i'.

In case(ii) of frg(a), the above curve ① is deflection curve due to load P_i & further deflection takes place when load P_j is applied at j.

Let, Δ_{jj} = Add^n deflection at 'i' due to load at 'j'

P_{jj} = Add^n deflection at 'j' due to load at 'j'.

Page No.:
Date:

Workdone $= \frac{1}{2} \times P_i \times \Delta_{ii} + \frac{1}{2} \times P_i \times \Delta_{ij} + \frac{1}{2} \times P_j \times \Delta_{jj}$ —— (i)

Similarly, the explanation is similar to frg(a).

The workdone is,

Workdone $= \frac{1}{2} \times P_i \times \Delta_{ii} + \frac{1}{2} \times P_j \times \Delta_{jj} + \frac{1}{2} \times P_j \times \Delta_{ji}$ —— (ii)

Hence eqn (i) & (ii) are equal

$$\frac{1}{2} \times P_i \times \Delta_{ii} + \frac{1}{2} \times P_i \times \Delta_{ij} + \frac{1}{2} \times P_j \times \Delta_{jj} = \frac{1}{2} \times P_i \times \Delta_{ii} + \frac{1}{2} \times P_j \times \Delta_{ji} + \frac{1}{2} \times P_j \times \Delta_{jj}$$

$$\Rightarrow \boxed{\frac{1}{2} P_i \times \Delta_{ij} = \frac{1}{2} P_j \times \Delta_{ji}} \rightarrow \text{Proof of Bettil's theorem}$$

But, In Maxwell theorem of reciprocal deflection, load acting at 'i' & 'j' are same. so,

$$P_i = P_j$$

Then,

$$\Delta_{ij} = \Delta_{ji}$$

This means, deflection at point 'i' due to load 'p' at 'j' is equal to the deflection of point 'j' due to load at 'i'.

✱ Castigliano's theorem:-

First theorem:-

In a linearly elastic structure, the deflection of ~~structure~~ point in direction of applied load is equal to the partial derivative of s.e wrt load.

$$\Delta = \frac{\partial U}{\partial P}$$

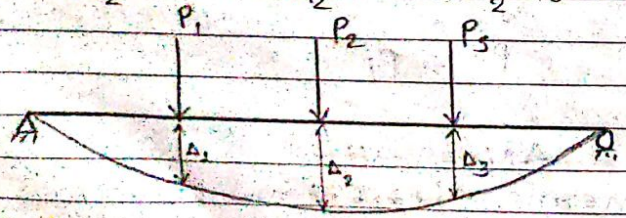
second theorem

The partial derivative of s.e wrt deflection of a point in direction of applied load is equal to load.

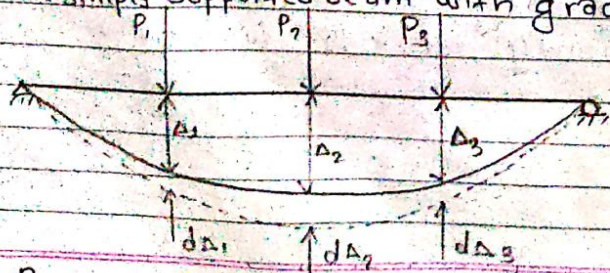
$$\frac{\partial U}{\partial \Delta} = P$$

Proof: To prove this theorem, consider a simply supported beam shown in fig. in which P_1, P_2 & P_3 are applied gradually. Let deflections under the loads P_1, P_2 & P_3 be Δ_1, Δ_2 & Δ_3 respectively.

$$\therefore U = \frac{1}{2} P_1 \Delta_1 + \frac{1}{2} P_2 \Delta_2 + \frac{1}{2} P_3 \Delta_3$$



Eg: A simply supported beam with gradually applied loads



Eg: Beam

Let, the additional load dp_1 be added after the loads P_1, P_2 & P_3 and applied & let the additional deflections be $d\Delta_1$, $d\Delta_2$, $d\Delta_3$. Then additional strain energy is,

$$dU = \frac{1}{2} dp_1 d\Delta_1 + P_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3 \quad \text{---(A)}$$

Total strain energy of the system is,

$$U + dU = \frac{1}{2} P_1 \Delta_1 + \frac{1}{2} P_2 \Delta_2 + \frac{1}{2} P_3 \Delta_3 + \frac{1}{2} dp_1 d\Delta_1 + P_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3 \quad \text{---(I)}$$

If $(P_1 + dp_1)$, P_2 & P_3 were to be applied simultaneously, strain energy stored is given by

$$= \frac{1}{2} (P_1 + dp_1)(\Delta_1 + d\Delta_1) + \frac{1}{2} P_2 \times (\Delta_2 + d\Delta_2) + \frac{1}{2} \times P_3 (\Delta_3 + d\Delta_3) \quad \text{---(II)}$$

Eqn (I) & (II) are same. So,

$$\frac{1}{2} P_1 d\Delta_1 + \frac{1}{2} P_2 d\Delta_2 + \frac{1}{2} P_3 d\Delta_3 = \frac{1}{2} dp_1 \Delta_1 \quad \text{---(III)}$$

From (A),

$$\frac{1}{2} \left(dU - \frac{1}{2} dp_1 d\Delta_1 \right) = \frac{1}{2} P_1 d\Delta_1 + \frac{1}{2} P_2 d\Delta_2 + \frac{1}{2} P_3 d\Delta_3$$

From (III),

$$\frac{1}{2} \left(dU - \frac{1}{2} dp_1 d\Delta_1 \right) = \frac{1}{2} dp_1 \Delta_1$$

$$\Rightarrow dU - \frac{1}{2} dp_1 d\Delta_1 = dp_1 \Delta_1$$

Neglecting small quantity,

$$dU = dp_1 \Delta_1$$

$$\therefore \Delta_1 = \frac{dU}{dp_1}$$

Similarly, if moments are considered, it may be shown that $\theta = \frac{dU}{dM}$

ch-5

slope & deflection of beam

↳ Moment Area Method:-

↳ Conjugate-beam method:-

※ Moment Area Method:-

This method is based on following two theorems.

Theorem 1:-

The change in the slope between two points on a straight member under flexure is equal to area of M/EI diagram betⁿ these two points.

i.e,

$$\theta_{CD} = \int_C^D \left(\frac{M}{EI}\right) dx$$

Proof Consider the beam AB. let C & D be any two points on this beam. The (M/EI) diagram is shown in fig (a). Fig (b) shows the elastic curve of the beam after loading.

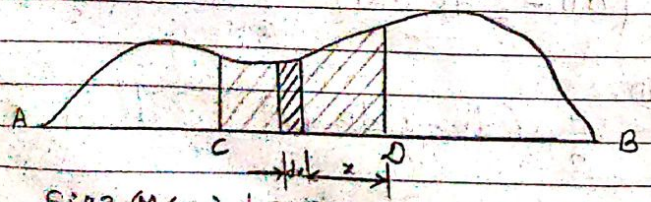


Fig a (M/EI) diagram



Fig b: Angle & deflection

Fig 'c' shows the elemental length 'dx' of fig 'a' to an enlarged scale.

let R be the radius of curvate.

Then,

$$d\theta = \frac{dx}{R}$$

$$\Rightarrow \frac{1}{R} = \frac{d\theta}{dx} \quad ---(i)$$

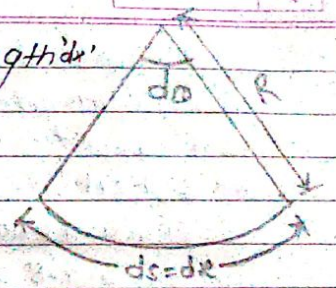


Fig c: Deformed shape of elemental length

we know, from flexural eqⁿ

$$\frac{M}{I} = \frac{E}{R}$$

$$\Rightarrow \frac{M}{EI} = \frac{d\theta}{dx}$$

$$\Rightarrow d\theta = \frac{M}{EI} x\, dx \quad ---(ii)$$

Then,

$$\theta_{CD} = \int_C^D \frac{M}{EI} dx$$

which proves theorem 1 of Moment Area Method.

Theorem 2:-

Deflection at a point in a beam in the direction perpendicular to its original straight line position measured from the tangent to the elastic curve at another point is given by the moment of (M/EI) diagram about the point where deflection is required.

$$\Delta = \int_C^D \left(\frac{M}{EI}\right) x\, dx$$

Proof: Fig 'a' 'b' & their explanation is similar. Now, consider figure 'd' in which portion CD of fig 'a' is shown in enlarged scale. Let the change of slope in elemental length 'dx' be 'dθ'. Distance of elemental length from D is 'x'. Then,

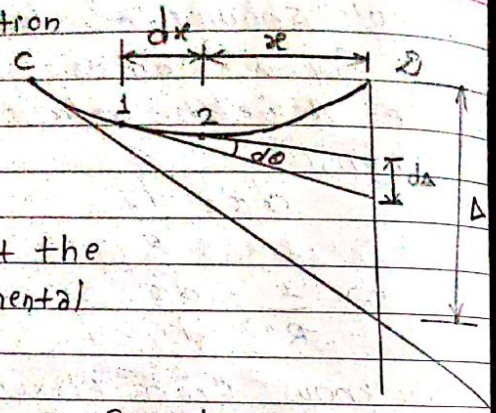


Fig: d: Enlarge view portion CD.

$$d\theta = \frac{d\Delta}{x}$$

$$d\Delta = x\,d\theta \quad ---(iii)$$

Using eqn (ii),

$$d\Delta = x \times \frac{M}{EI}\,dx$$

On integration,

$$\Delta = \int_C^D \left(\frac{M}{EI}\right) x\,dx \quad ---(iv)$$

Eqn (iv) proves the second theorem of moment area Method.

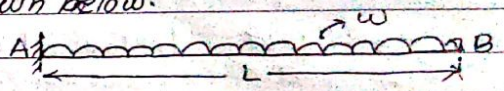
*Sign convention:-

1. Sagging moment area is positive, which means that, the tangent at 'D' makes an anticlockwise angle with tangent at 'C'.

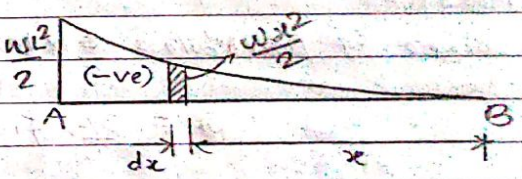
2. The moment of positive moment gives rise to positive deflection, which implies that the deflected position of point 'D' is above the tangent drawn at other point 'C'.

Examples:-

1. Determine the rotation & deflection at free end of cantilever beam shown below.



→ The bending moment diagram of above cantilever beam is,

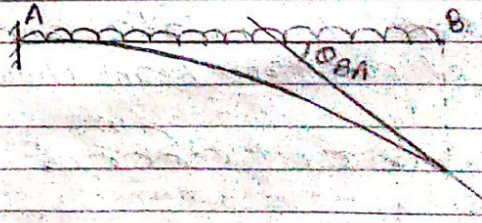


Now,

$\theta_{BA} = \theta_B - \theta_A$

$= \theta_B \quad [\theta_A = 0]$

Then from moment area theorem,



$$\theta_B = \int_0^L \left(\frac{M}{EI}\right) dx$$

$$= \int_0^L \frac{-wx^2}{2EI}\,dx$$

$$= \frac{-w}{2EI} \times \frac{L^3}{3} = \frac{-wL^3}{6EI}$$

$\therefore \theta_B = \frac{wL^3}{6EI}$; clockwise with tangent at A.

Δ_B = deflection of 'B' wrt tangent at A
Here, tangent at 'A' is horizontal.

$$\therefore \Delta_B = \int_0^L \left(\frac{M}{EI}\right) x\,dx$$

$$= \int_0^L \frac{-wx^3}{2EI}\,dx$$

$$= -\frac{w}{2EI} \times \frac{L^4}{4} = -\frac{wL^4}{8EI}$$

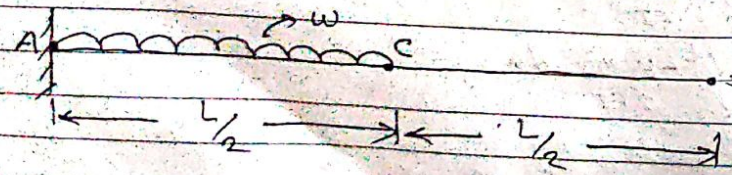
$\therefore \Delta_B = \frac{wL^4}{8EI}$ below the tangent at A.

Note:-

Area of parabola = $\frac{1}{3} \times L \times$ ordinate at end

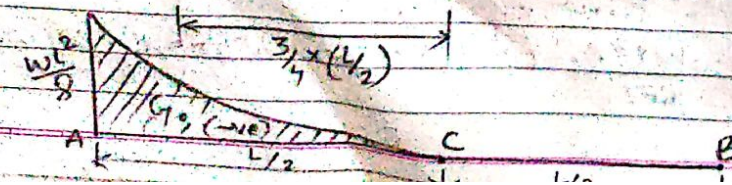
CG = $\frac{3L}{4}$ from the end where value is zero.

2) Find the rotation and deflection of free end of cantilever beam shown.



→

The BMD of above cantilever beam is,



Δ → below -ve
θ → 2 -ve

θ_B = Area of M/EI diagram

$$= \frac{1}{EI} - \left(\frac{1}{3} \times \frac{L}{2} \times \frac{wL^2}{8}\right)$$

$$= -\frac{wL^3}{48EI}$$

$\therefore \theta_B = \frac{wL^3}{48EI}$, clockwise with tangent at A.

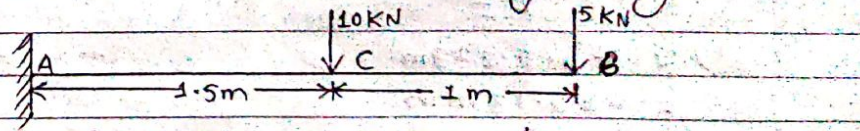
Δ_B = deflection at B wrt tangent at A
= Moment of area of M/EI diagram

$$= -\frac{wL^3}{48EI} \times \left(\frac{3L}{8} + \frac{L}{2}\right)$$

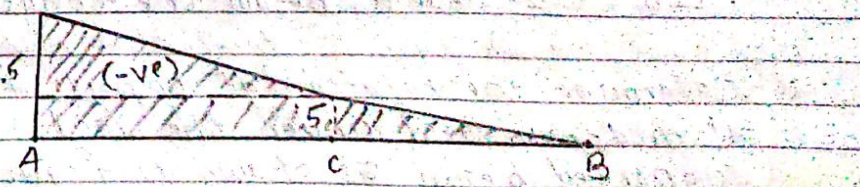
$$= -\frac{7wL^4}{384EI}$$

$\therefore \Delta_B = \frac{7wL^4}{384EI}$, below the tangent at A

2.3 Determine the slope and deflection at free end of a cantilever beam as shown in figure by MAM.



→ The BMD of the above cantilever beam is,



Given, $EI = 4000\ kNm^2$

θ_B = Area of bending moment diagram / EI

$= \frac{-1}{EI}\left[\frac{1}{2}\times 1\times 5 + 5\times 1.5 + \frac{1}{2}\times(27.5-5)\times 1.5\right]$

$= \frac{-26.875}{EI}$

$= \frac{-26.875}{4000}$

$= -6.71875\times 10^{-3}$ rad $= -0.38°$

$\therefore \theta_B = 0.38°$; clockwise with tangent at A.

Then,

Δ_B = moment of area of M/EI diagram

$= -\frac{1}{EI}\left[\left(\frac{1}{2}\times 1\times 5\right)\times\frac{2}{3} + (5\times 1.5)\times(1+0.75) + \left(\frac{1}{2}\times(27.5-5)\times 1.5\right)\times\left(1+\frac{2\times 1.5}{3}\right)\right]$

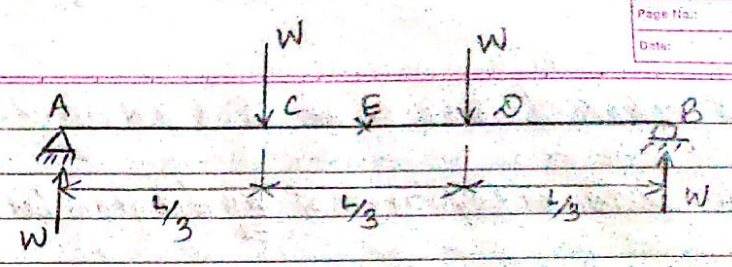
$= \frac{-48.542}{EI}$

$= -0.01213$ m

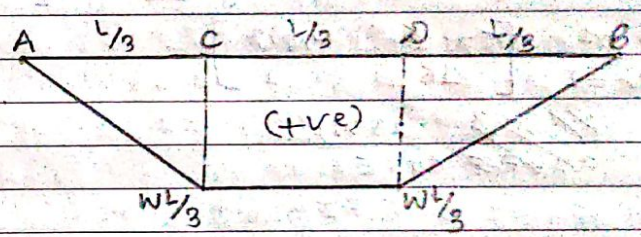
$= -12.13$ mm

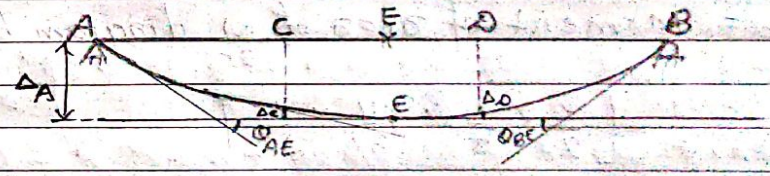
$\therefore \Delta_B = 12.13$ mm, below the tangent at A.

** Determine the rotation at supports & deflection at midspan and under the loads in simply supported beam as shown in figure.



→ The BMD of above simply supported beam is,





Due to symmetry, the slope at E is zero. In other words, the tangent at E is horizontal. Hence, it is convenient to workout rotations & deflection wrt tangent at E.

$\theta_A = \theta_{AE}$ = Area of M/EI diagram bet^n A & E.

$= \frac{1}{EI}\left[\frac{1}{2}\times\frac{L}{3}\times\frac{WL}{3} + \frac{1}{2}\times\left(\frac{L}{3}\times\frac{WL}{3}\right)\right]$

$= \frac{WL^2}{9EI}$

$\therefore \theta_A = \frac{WL^2}{9EI}$; anticlockwise with tangent at E.

Deflection of E w.r.t A

Δ_A = Moment of area of $\frac{M}{EI}$ diagram bet^n AE w.r.t A

= Deflection at E

$$= \frac{1}{EI}\left[\frac{1}{2}\times\frac{L}{3}\times\frac{WL}{3}\times\left(\frac{L}{3}+\frac{L}{12}\right) + \frac{1}{2}\times\frac{L}{3}\times\frac{WL}{3}\times\frac{2}{3}\times\frac{L}{8}\right]$$

$$= \frac{1}{EI}\left[\frac{5WL^3}{216} + \frac{WL^3}{81}\right]$$

$$= \frac{23WL^3}{648EI}$$

Deflection of 'C' above E

Δ_C = Moment of area of $\frac{M}{EI}$ diagram bet^n CE above C'

$$= \frac{1}{EI}\left[\frac{1}{2}\times\frac{L}{8}\times\frac{WL}{3}\times\frac{L}{12}\right]$$

$$= \frac{WL^3}{216EI}$$

Actual deflection of point C = $\frac{23WL^3}{648EI} - \frac{WL^3}{216EI}$

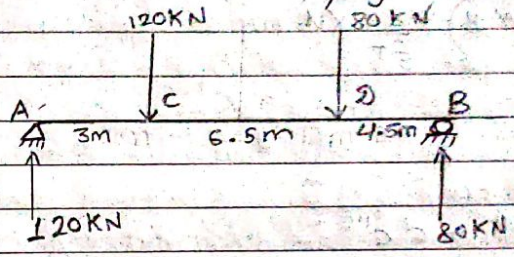
$$= \frac{5WL^3}{162EI}$$

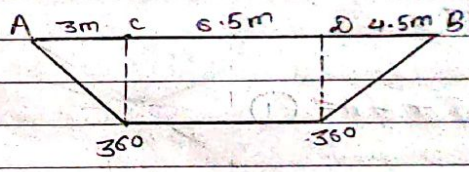
Due to symmetry, Deflection of D below B

$$= \frac{5WL^3}{162EI}$$

2.c

Determine the slope and deflection at the point 'C' and end of the beam shown in figure, EI is constant throughout





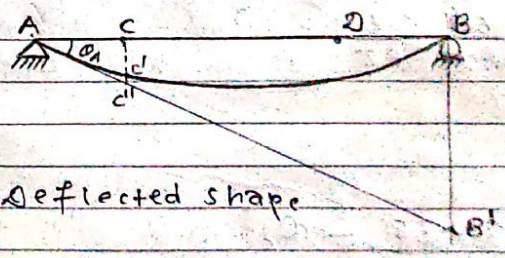
BMD

In similar Δ Acc'' & ABB'

$$\frac{Cc''}{AC} = \frac{BB'}{AB}$$

$$Cc'' = \frac{27045}{EI}\times\frac{3}{14}$$

$$= \frac{5795.36}{EI}$$



Deflected shape

BB' = Moment of M/EI diagram about B in bet^n A & B

$$= \frac{1}{EI}\left[\frac{1}{8}\times3\times360\times\left(\frac{11}{3}\times3\right) + (360\times6.5)\times\left(4.5+\frac{6.5}{2}\right) + \left(\frac{1}{2}\times360\times4.5\right)\times\left(\frac{2}{3}\times4.5\right)\right]$$

$$= \frac{27045}{EI}$$

$C'C''$ = Moment of $\frac{M}{EI}$ diagram in betⁿ A & C about C

$= \left(\frac{1}{2} \times 360 \times 3\right) \times \frac{1}{EI} \times 1$

$= \frac{540}{EI}$

Then,

$\Delta_C = CC' = CC'' - C'C''$

$= \frac{5795.36}{EI} - \frac{540}{EI}$

$= \frac{5255.36}{EI}$

Now,

$\Theta_{AC} = \Theta_A - \Theta_C$ ---(1)

Here,

$\Theta_A = \frac{BB'}{AB} = \frac{27045}{EI \times 14} = \frac{1931.78}{EI}$

Θ_{AC} = Area of $\frac{M}{EI}$ diagram betⁿ A & C

$= \frac{1}{2} \times 3 \times 360 \times \frac{1}{EI}$

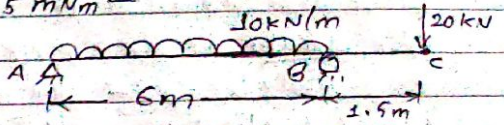
$= \frac{540}{EI}$

From (1)

$\frac{540}{EI} = \frac{1931.78}{EI} - \Theta_C$

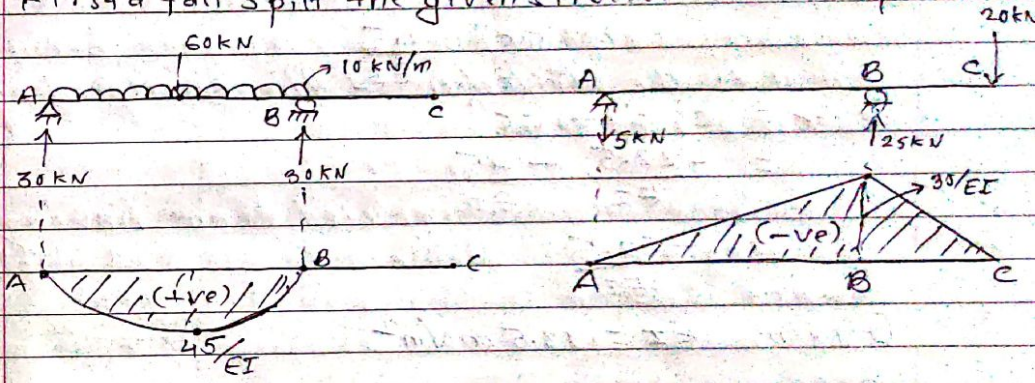
$\Theta_C = \frac{1931.78 - 540}{EI} = \frac{1391.78}{EI}$

* Find the deflection at free end 'C' by using moment Area Method. $EI = 13.5\ mNm^2$

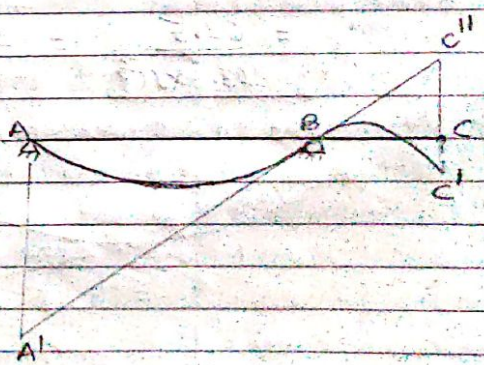


→

Firstly fall spilt the given structure into two parts.



The deflected shape is,



We have to find the value of CC'

i.e,

$\Delta_C = CC' = ?$

$C'C''$ = Moment of M/EI diagram betⁿ B & C about C

$= -\frac{1}{2} \times 1.5 \times \frac{30}{EI} \times \frac{2}{3} \times 1.5$

$= \frac{-22.5}{EI}$

AA' = Moment of $\frac{M}{EI}$ diagram betⁿ A & B about A

$$= \frac{2}{3} \times \frac{45}{EI} \times 6 \times 3 - \frac{1}{2} \times 6 \times \frac{30}{EI} \times \frac{2}{3} \times 6$$

$$= \frac{180}{EI}$$

In similar Δ, $\frac{180}{EI \times 6} = \frac{CC''}{1.5}$

$$\Rightarrow CC'' = \frac{45}{EI}$$

$$\therefore \Delta_C = C'C'' - CC''$$

$$= \frac{-22.5}{EI} - \frac{45}{EI}$$

$$= \frac{-62.5}{EI}$$

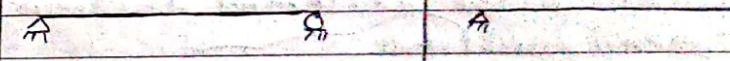
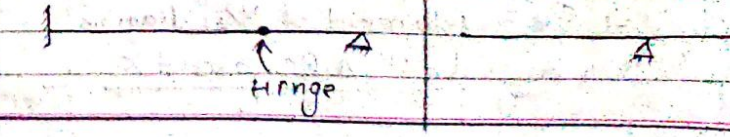
Given, $EI = 13.5\ mNm^2$

$$\therefore \Delta_C = \frac{62.5}{13.5 \times 10^3} = 4.63\ mm \downarrow \quad \text{Ans.}$$

Conjugate Beam:-

Conjugate beam is an imaginary beam of same span as the original beam loaded with (M/EI) diagram of original beam, such that the shear force & bending moment at a section will represent the rotation and deflection at that section in the original beam.

* Actual beam & corresponding conjugate beam.

Types of Beam	Actual Beam	Conjugate Beam
① Cantilever		
② Simply Supported		
③ Beam with Overhanging on one side.		
④ Beam with overhang on both sides.		
⑤ Beam with hinge.		
⑥ Beam with fixed end Hinge		

✻ Conjugate Beam method:-

The conjugate beam method is based on two theorems.

Theorem 1:
The shear at any point on conjugate beam is equal to the slope at the corresponding point on the real beam.

Theorem 2:
The bending moment at any point on conjugate beam is equal to the deflection of the corresponding point on real beam.

Proof: Consider a simply supported beam having M/EI diagram as shown in figure (a) & deflected shape of beam is shown in figure (b)

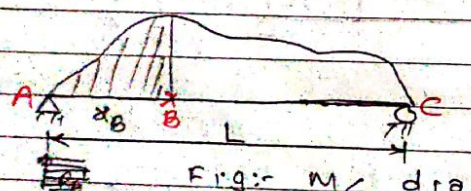
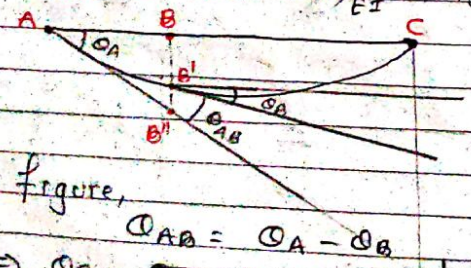


Fig:- M/EI diagram of real beam



From figure,

$\theta_{AB} = \theta_A - \theta_B$

$\Rightarrow \theta_B = \theta_A - \theta_{AB}$

$\Rightarrow \theta_B = \frac{CC'}{L}$ − Area of $\frac{M}{EI}$ diagram bet^n A & B

$\Rightarrow \theta_B = \frac{\text{Moment of } M/EI \text{ diagram bet}^n A \text{ \& } C \text{ about } C}{L}$ − Area of $\frac{M}{EI}$ diagram bet^n A & B.

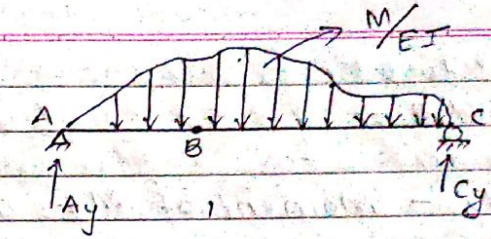


Fig: conjugate beam

$+\circlearrowright \Sigma M_C = 0$

$A_y \times l$ − Moment of M/EI diagram bet^n A & C about C $= 0$.

$A_y = \frac{\text{Moment of } M/EI \text{ diagram bet}^n A \text{ \& } C \text{ about } C}{L}$

Now,
shear force at B,

$V_B = A_y$ − Area of M/EI diagram bet^n A & B

$V_B = \frac{\text{Moment of } M/EI \text{ diagram bet}^n A \text{ \& } C \text{ about } C}{L}$ − Area of M/EI diagram bet^n A & B.

$\boxed{V_B = \theta_B}$ [using (1)]

This proves first theorem.

Also,

$\Delta_B = BB' = BB'' - B'B'' \quad \text{---(ii)}$

$\Rightarrow B'B''$ = Moment of M/EI diagram about B bet^n A & B

$BB'' = \frac{CC'}{AC} \times AB = \frac{\text{Moment of } M/EI \text{ diagram bet}^n A \text{ \& } C \text{ about } C}{L} \times x_B$

Then, from (ii)

$\Delta_B = \frac{\text{Moment of } M/EI \text{ diagram bet}^n A \text{ \& } C \text{ about } C}{L} \times x_B$ − Moment of M/EI diagram about B bet^n A & B ---(iii)

In fig 'c',
Moment about B is,

M_B = $A_y * x_B$ - Moment of M/EI diagram betn A & B about B.

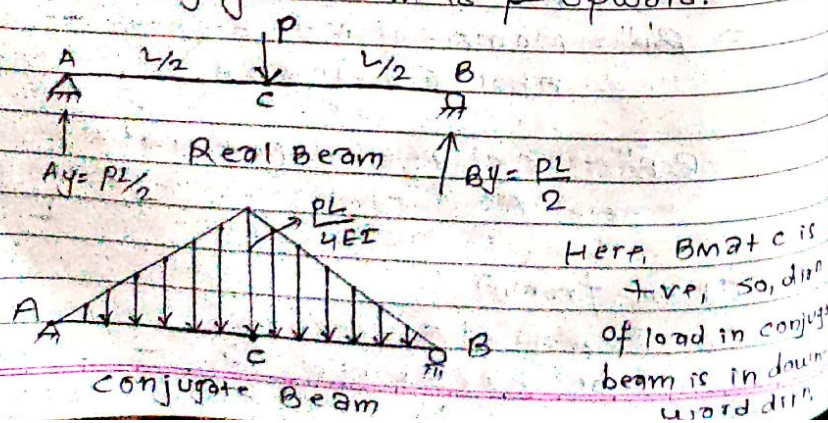
= Moment of M/EI diagram betn A&C about C $* \frac{L}{2}$ x_B − Moment of M/EI diagram betn A & B about B.

$M_B = \Delta_B$ [Using eqn (iii)]

∴ The moment about any point in conjugate beam gives the deflection at that point in real beam.
This proves the second theorem of conjugate beam method.

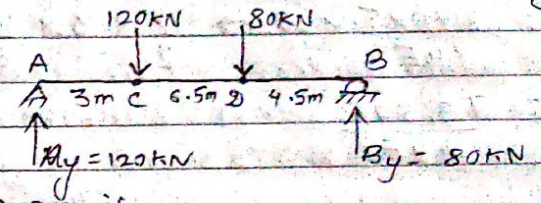
* Sign Convention:-
→ If the bending moment is positive, the direction of load in conjugate beam is downward.
→ If the bending moment is negative, the direction of load in conjugate beam is upward.

Eg

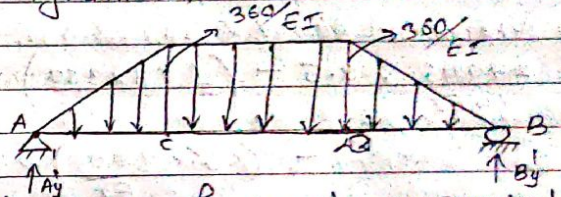


Here, BM at C is +ve, so, dir of load in conjug beam is in downward dir.

* Find slope at A & B, deflection at C & D using conjugate Beam method.



→ The M/EI diagram is,



The conjugate beam of simply supported beam is simply supported also, so,

$+\circlearrowleft \Sigma M_B = 0$

$A_y' \times 14 - \frac{1}{2} \times 3 \times \frac{360}{EI} \times (11+1) - \left(6.5 \times \frac{360}{EI}\right) \times \left(4.5 + \frac{6.5}{2}\right)$

$- \left(\frac{1}{2} \times 4.5 \times \frac{360}{EI}\right) \times \frac{2}{3} \times 4.5 = 0$

$\Rightarrow A_y' = \frac{27045}{14EI}$

$(+\uparrow)\ \Sigma F_y = 0$

$A_y' - \frac{1}{2} \times 3 \times \frac{360}{EI} - \frac{1}{2} \times 4.5 \times \frac{360}{EI} - 6.5 \times \frac{360}{EI} + B_y' = 0$

$\Rightarrow B_y' = \frac{24615}{14EI}$

Now,
θ_A = shear force at A
$= \frac{27045}{14EI}$

θ_B = Shear force at B $= \frac{24615}{14EI}$

Δ_C = Moment about C

$= \frac{27045}{14EI} \times 3 - \frac{1}{2} \times 3 \times \frac{360}{EI} \times \left(\frac{1}{3} \times 3\right)$

$= \frac{73575}{14EI}$

Δ_D = Moment about D

$= \frac{27045}{14EI} \times 9.5 - \left(\frac{1}{2} \times 3 \times \frac{360}{EI}\right) \times \left(6.5 + \frac{1}{3} \times 3\right)$

$- \left(6.5 \times \frac{360}{EI}\right) \times \frac{6.5}{2}$

$= \frac{187515}{28EI}$

Sign convention

a) Moment area method:-

Slope → +ve → Anticlockwise angle with horizontal

Slope → −ve → Clockwise angle with horizontal

deflection → +ve → above the tangent

deflection → −ve → below the tangent

b) Conjugate beam:-

positive shear → clockwise rotation

negative shear → anticlockwise rotation

positive moment → downward deflection

negative moment → upward deflection

Ch-6 Influence Line Diagram for simple structures:-

* Moving static loads:-

Structure which is used in bridges, gantry girders, crane beams etc are subjected to loads which change their position often such loads are called Moving static load.

Few standard loads are

i) Single concentrated load

ii) UDL greater than span

iii) UDL smaller than span

iv) Two concentrated load with specified distance betⁿ them

v) Multiple concentrated loads.

* Influence Line Diagram:-

→ The graphical presentation of various force parameter like reaction, shear force and bending moment at a section as a unit force moves from one side of the beam to the other side is called Influence Line Diagram.

→ ILD is always drawn for a concentrated load & it is independent of any type of system of loading.

→ ILD's are very important in bridge analysis & design of bridges which are subjected to rolling or moving loads from one side of bridge to other side.

→ Generally ILD are drawn for

- Reactions
- Shear force
- Bending Moment
- Radial shear & normal thrust.

* Difference betⁿ ILD & BMD or SFD :-

The ordinate of B.M.D or S.F.D gives the value of BM or SF at that section where the ordinate has been drawn whereas the ordinate of I.L.D at any point gives the value of BM or SF at the given section for which the ILD has been drawn & not at the point at which ordinate has been drawn.

* Influence Line Diagram

a) Simply Supported beam :-

1) ILD for R_A,

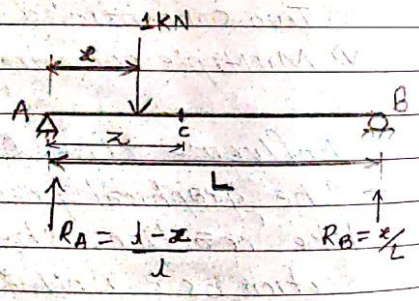
we have,

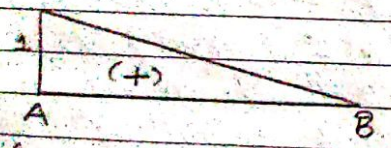
$$R_A = \frac{l-x}{l}$$

When $x = 0$, $R_A = 1$

When $x = L$, $R_A = 0$

[varies linearly]





2) ILD for R_B

we have,

$$R_B = x/L$$

When $x = 0$, $R_B = 0$

When $x = L$, $R_B = 1$



3) ILD for shear force at C (F_C) :-

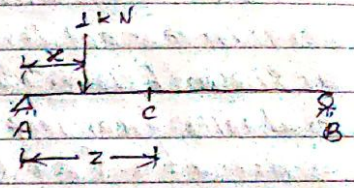
a) For portion AC,

$$(F)_C = R_A - 1 \text{ or } -R_B$$

$$= -x/L$$

When $x = 0$, $F_C = 0$

$x = z$, $F_C = -z/L$



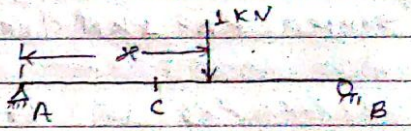
b) For portion CB

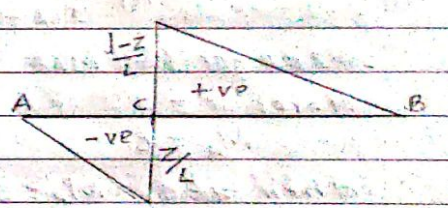
$$(F_C) = R_A$$

$$= \frac{l-x}{l}$$

When $x = z$, $F_C = \frac{l-z}{l}$

When $x = L$, $F_C = 0$.





ILD for F_C.

4) ILD for Moment at C (M_C) :-

a) For portion AC,

$$M_C = x/L \times (L-z)$$

When $x = 0$, $M_C = 0$ [A+ A]

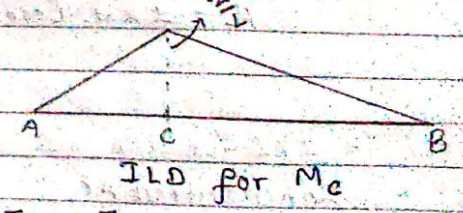
When $x = z$, $M_C = \frac{z}{L}(L-z)$ [A+ C]

b) For portion CB,

$$M_C = \frac{l-x}{l} \times z$$

When $x = z$, $M_C = \frac{z}{L}(L-z)$ [A+C]

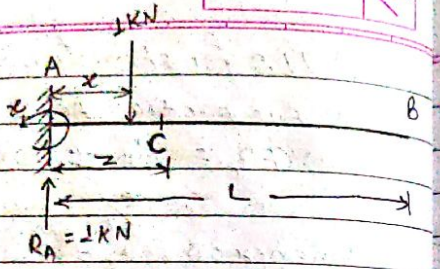
When $x = L$, $M_C = 0$.



ILD for M_C

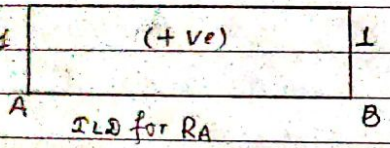
b) Cantilever Beam:-

Consider a cantilever beam as shown below. Let unit load rolls from left to right.



1) ILD for R_A,

Since, $R_A = 1$ KN [constant].



ILD for R_A

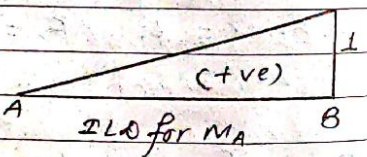
2) ILD for ~~BM~~ M_A:-

Here,

$M_A = x$

When $x = 0$, $M_A = 0$

$x = L$, $M_A = L$



ILD for M_A

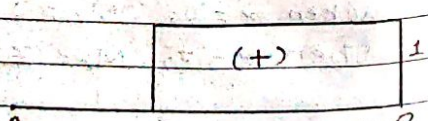
3) ILD for shear force at C, (F_C):-

For portion AC [i.e when 1 KN load moves from A to C]

$F_C = 0$

For portion CB,

$F_C = 1$ KN. [const]



ILD for F_C

4) ILD for M_C,

For portion AC,

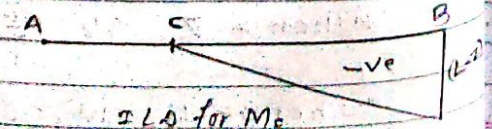
$M_C = 0$

For portion CB,

$M_C = -1 \times (x - z)$

When $x = z$, $M_C = 0$

$x = L$, $M_C = -(L - z)$



ILD for M_C

c) Overhanging beam:-

Consider an overhanging beam which is simply supported. First of all, we have to calculate the reactions R_A & R_B when load is in betⁿ (A & C), (A & B) & (B & D).

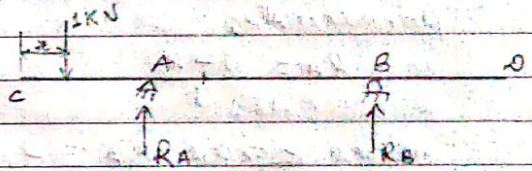


i) When 1KN load is in betⁿ C And A,

$+\circlearrowleft \Sigma M_A = 0$

$-1 \times (a - x) - R_B \times L = 0$

$R_B = \frac{x - a}{L}$



$(+\uparrow) \Sigma F_y = 0$,

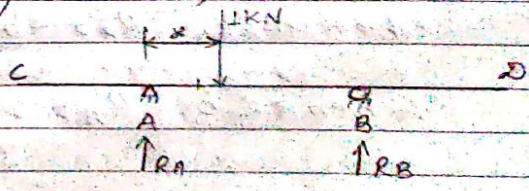
$R_A + R_B - 1 = 0$

$R_A = 1 - \frac{x - a}{L} = \frac{L - x + a}{L}$

ii) When 1 KN load is moving from A to B,

$R_A = \frac{1 - x}{L}$

$R_B = x/L$

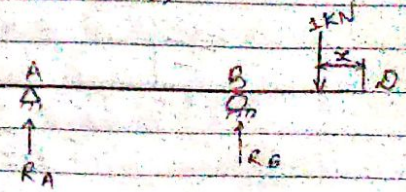


iii) When 1 KN load is moving from B to D,

$+\circlearrowleft \Sigma M_A = 0$

$\Rightarrow -R_B \times L + 1 \times (L + b - x) = 0$

$\Rightarrow R_B = \frac{L + b - x}{L}$



$(+\uparrow) \Sigma F_y = 0$

$\Rightarrow R_A = 1 - R_B = 1 - \frac{L + b - x}{L} = \frac{x - b}{L}$

i) ILD for R_A,

portion CA,

$$R_A = \frac{L - x + a}{L}$$

When $x = 0$ (A+C), $R_A = \frac{L+a}{L}$

When $x = a$ [A+A], $R_A = 1$

portion AB,

$$R_A = \frac{l - x}{l}$$

When $x = 0$, $R_A = 1$ [a+A]

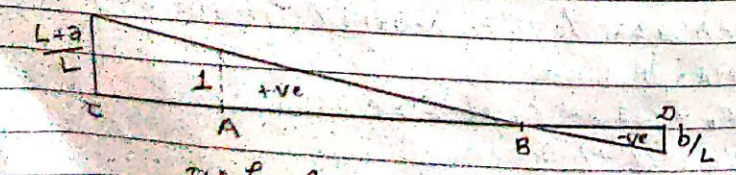
When $x = l$, $R_A = 0$ [a+B]

portion BD,

$$R_A = \frac{x - b}{L}$$

When $x = 0$, $R_A = -b/L$ [A+D]

When $x = b$, $R_A = 0$ [A+B].



ILD for R_A

ii) ILD for R_B

portion CA,

$$R_B = \frac{x - a}{L}$$; When $x = 0$, $R_B = -a/L$ [C]

When $x = a$, $R_B = 0$ [A]

portion AB,

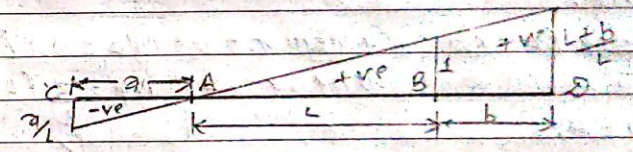
$$R_B = x/L$$; When $x = 0$, $R_B = 0$ [A]

When $x = L$, $R_B = 1$ [B]

portion BD,

$$R_B = \frac{L + b - x}{L}$$; When $x = 0$, $R_B = (L+b)/L$ [D]

When $x = b$, $R_B = 1$ [B]



iii) ILD for F_E (shear force at E)

portion CA

$$F_E = -R_B = \frac{a - x}{L}$$; When $x = 0$, $F_E = a/L$ [C]

When $x = a$, $F_E = 0$ [A]

portion AE

$$F_E = -x/L$$; When $x = 0$, $F_E = 0$ [A]

When $x = z$, $F_E = -z/L$ [E]

portion EB

$$F_E = \frac{l - x}{l}$$; When $x = z$, $F_E = \frac{l - z}{l}$ [A+E]

When $x = L$, $F_E = 0$ [A+B]

portion BD

$$F_E = \frac{x - b}{L}$$; When $x = 0$, $F_E = -b/L$ [A+D]

When $x = b$, $F_E = 0$ [A+B]

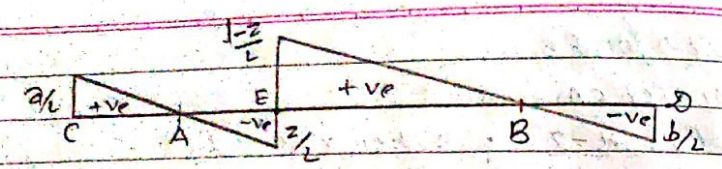


Fig: ILD for SF

iv) ILD for moment at E,

portion CA

$M_E = \left(\frac{x-a}{L}\right) \times (L-z)$; when $x=0$, $M_E = -\frac{a}{L}(L-z)$ [at C]

when $x=a$, $M_E = 0$ [at A]

portion AE,

$M_E = \frac{x}{L} \times (L-z)$; when $x=0$, $M_E = 0$ [at A]

when $x=z$, $M_E = \frac{z}{L}(L-z)$ [at E]

portion EB,

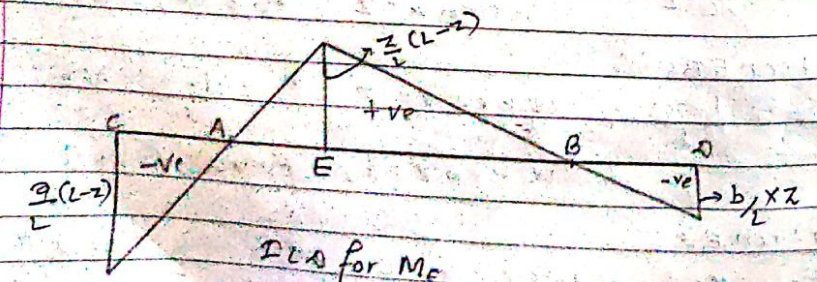
$M_E = \left(\frac{L-x}{L}\right) \times z$; when $x=z$, $M_E = \frac{z}{L}(L-z)$ [at E]

when $x=L$, $M_E = 0$ [at B]

portion BD,

$M_E = \left(\frac{x-b}{L}\right) \times z$; When $x=0$, $M_E = -\frac{b}{L} \times z$ [at D]

when $x=b$, $M_E = 0$ [at B]

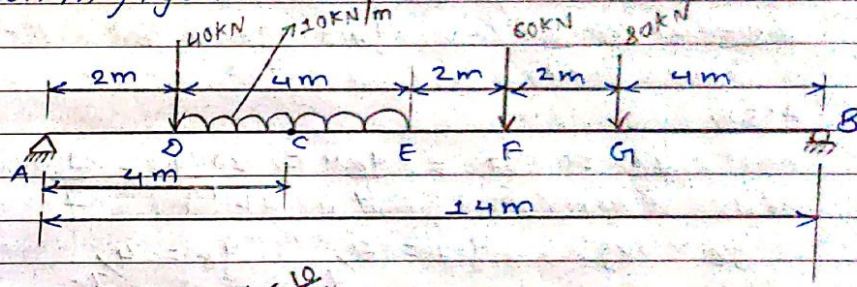


ILD for M_E

✱ Use of influence line diagram (ILD):-

Using influence diagrams for shear force & bending moment at a section, SF & BM at that section for any given loading can be found easily.

Q* Using ILD, determine the shear force & bending moment at section 'c' in the simply supported beam shown in figure.



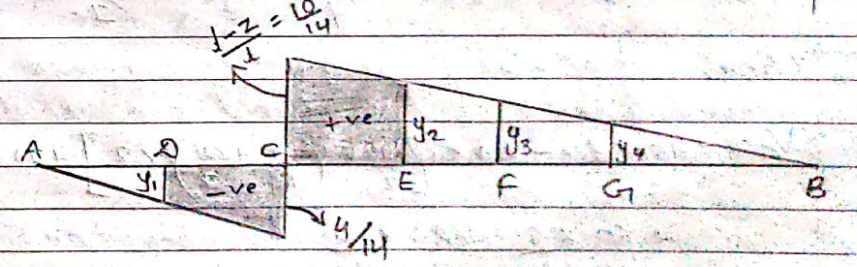


Fig: ILD for SF at c.

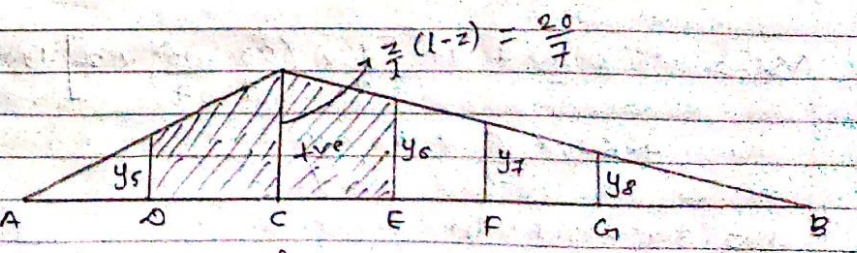


Fig: ILD for BM at c.

In similar Δs, [in ILD for SF at C]

$$\frac{y_1}{2} = \frac{4}{14\times 4} \Rightarrow y_1 = 1/7$$

Also, $\frac{y_2}{8} = \frac{y_3}{6} = \frac{y_4}{4} = \frac{10/14}{10} = \frac{1}{14}$

$$y_2 = \frac{8}{14},\ y_3 = \frac{6}{14},\ y_4 = \frac{4}{14}$$

Then, in ILD for BM at C,

$$\frac{y_5}{2} = \frac{20}{7\times 4} \Rightarrow y_5 = \frac{10}{7}$$

Also,

$$\frac{y_6}{8} = \frac{y_7}{6} = \frac{y_8}{4} = \frac{20/7}{10} = \frac{1}{7}$$

$$y_6 = 8/7,\ y_7 = 6/7,\ y_8 = 4/7$$

Then,

$$V_C = -40\times\frac{1}{7} - 10\times\left[\frac{1}{2}\times\left(\frac{4}{14}+\frac{1}{7}\right)\times 2\right] + 10\times\left[\frac{1}{2}\times\left(\frac{10}{14}+\frac{8}{14}\right)\times 2\right] + \frac{60\times 6}{14} + \frac{80\times 4}{14}$$

$$= 51.43\ \text{KN}$$

$$M_C = 40\times\frac{10}{7} + 10\left[\frac{1}{2}\times\left(\frac{20}{7}+\frac{10}{7}\right)\times 2\right] + 10\left[\frac{1}{2}\times\left(\frac{20}{7}+\frac{8}{7}\right)\times 2\right] + \frac{60\times 6}{7} + \frac{80\times 4}{7}$$

$$= 345.71\ \text{KN-m}$$

* Maximum S.F & B.M values due to Moving loads.

A designer finds the position of moving loads for which shear force & bending moment values are maximum, Influence line diagram are used for this purpose.

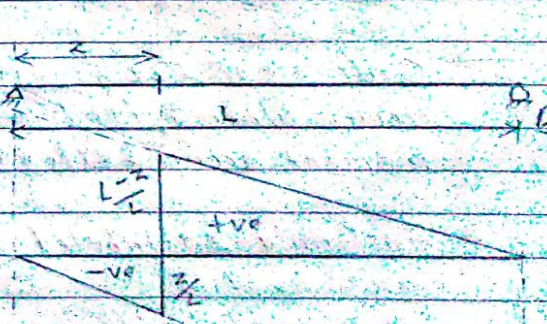
Cases:-

1) Single point load
2) Uniformly distributed load longer than span
3) Uniformly distributed load shorter than span
4) A train of point load.

1) Single Point load:-

a) Maximum S.F and B.M at a given point:-

Let 'p' be the moving load & the values of maximum S.F & B.M required be at 'c', which is at a distance 'z' from A.

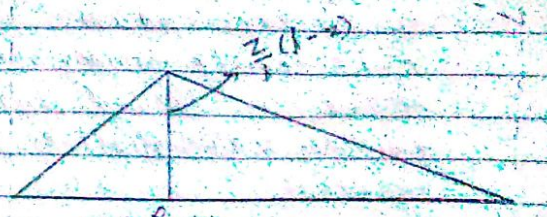


From figure, it is clear that maximum negative S.F occurs when the load is just to the left of section 'c' & its value is,

$$= P\times\left(\frac{z}{L}\right)$$

Similarly, maximum positive S.F occurs when the load is just to right of section 'c' & its value is

$$= P\times\left(\frac{L-z}{L}\right)$$



Also, from ILD for M_c, it is clear that Maximum bending moment will occur when the load is on the section itself & its value is $P * \left[\frac{z}{L}(L-z)\right]$

b) Absolute Maximum Values anywhere in the beam:

→ For achieving this, ordinate of ILD should be Maximum. Negative S.F ordinate is Maximum when $z = L$ & is equal to 1.

Therefore, absolute maximum negative S.F = P and it occurs when the load is at $z = L$ (i.e at B)

→ For positive S.F, ILD ordinate has maximum value of 1, when ($z = 0$, i.e at A).

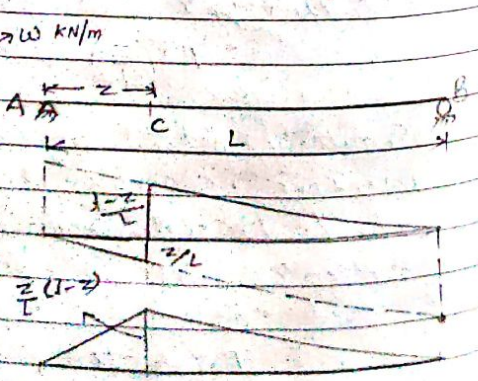
Absolute Maximum S.F = P.

→ Ordinate of ILD for Moment is Maximum when $z = L/2$

Hence, when a load is at mid span, absolute Maximum moment occurs & its value is $\frac{PL}{4}$.

3) Uniformly Distributed Load Longer than Span:-

Let a Uniformly distributed load of intensity 'w' move from left to move.



we already known load intensity times area of ILD over loaded length gives the value of stress resultant.

Negative S.F is maximum, when the load covers portion AC only

Maximum negative F_c = w * Area of ILD for F_c in length AC

$$= w * \left[\frac{1}{2} \times z \times \frac{z}{1}\right] = \frac{wz^2}{2L}$$

Positive S.F is maximum when the uniformly distributed load occupies the portion CB only and

Maximum positive F_c = w * Area of ILD for F_c in length CB

$$= w * \left[\frac{1}{2} \times (L-z) \times \left(\frac{L-z}{L}\right)\right]$$

$$= \frac{w * (L-z)^2}{2L}$$

From figure, it is clear that Maximum moment at C will be, when the UDL covers entire span.

$$M_{c,max} = w * \text{Area of ILD for } M_c$$

$$= w * \frac{1}{2} * L * \frac{z}{L}(L-z) = \frac{wz(L-z)}{2}$$

* Absolute Maximum values anywhere

Negative S.F is Maximum when $z = L$, i.e at B when the load occupies entire span AB.

Absolute Maximum S.F = $w \times \frac{1}{2} \times 1 \times L = \frac{wL}{2}$

Similarly, Maximum positive S.F occurs when $z = 0$; i.e at A when the load occupies entire span AB.

Maximum positive S.F = $w \times \frac{1}{2} \times 1 \times L = \frac{wL}{2}$

Max^m moment at any section = $\frac{1}{2} \times \frac{wz(1-z)}{L} = \frac{1}{2} \times wz \times (L-z)$

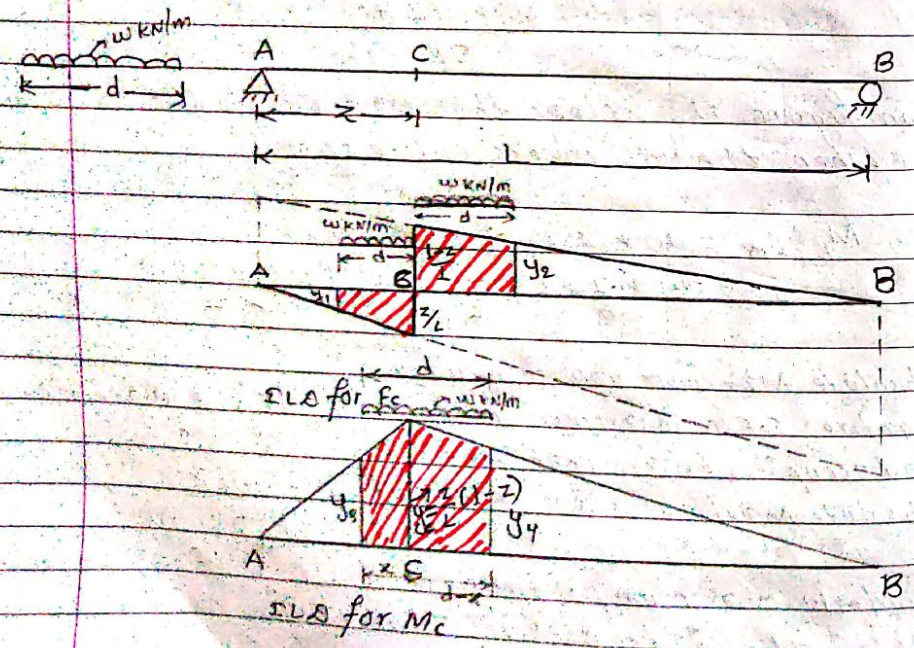
This is maximum when $z = L/2$ i.e at mid span

∴ Absolute Max^m moment $= \frac{1}{2} \times w \times \frac{L}{2} \times \left(L - \frac{L}{2}\right)$

$= \frac{wL^2}{8}$ at mid span

3) Uniformly distributed load smaller than span:

Let 'd' be the length of uniformly distributed load w/unit length. Let it move from left to right over beam AB of span 'L'. Here, $d < L$.



From ILD for shear force at 'c', it is clear that maximum shear force will develop when the head of the load reaches the section.

For maximum positive shear force, the tail of the UDL should reach the section.

Maximum bending moment will develop at c when the load is partly to the left of the section & partly to the right of section.

$$M_c = w * \left[\frac{1}{2}(y_3 + y_c) \times x\right] + w * \left[\frac{1}{2}(y_c + y_4) * (d - x)\right]$$

For M_c to be maximum

$$\frac{dM_c}{dx} = 0$$

$$\Rightarrow \frac{w}{2}(y_3 + y_c) + \frac{w}{2}(y_c + y_4) * (0) - \frac{w}{2}(y_c + y_4) \times 1 = 0$$

$$\Rightarrow y_3 + y_c - y_c - y_4 = 0$$

$$\Rightarrow \boxed{y_3 = y_4}$$

Thus, Moment at 'c' will be maximum when the ordinates of ILD for M_c at head & tail of the UDL are equal.

Since,

$$y_3 = y_4$$

$$\Rightarrow \left(\frac{z - x}{z}\right) y_c = \frac{(L - z) - (d - x)}{L - z} y_c$$

On solving, $\frac{x}{d} = \frac{z}{L}$

* position for absolute maximum moment:

For this, y_c should be Max^m.

$$y_c = \frac{z}{L}(L - z)$$

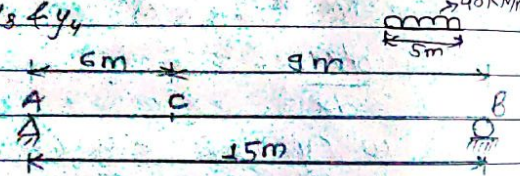
$$\frac{dy_c}{dz} = 0 \quad \Rightarrow L - 2z = 0$$

$$\Rightarrow z = L/2$$

This means absolute Max^m moment occurs at mid span.

* A simply supported beam has a span of 15m. Uniformly distributed load of 40kN/m & 5m long crosses the girder from left to right. Draw the influence line diagram for shear force & bending moment at a section 6m from left end. Use these diagrams to calculate the maximum shear force & bending moment at this section.

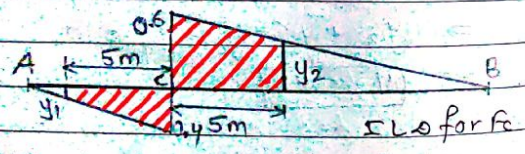
→ To find ordinate y_1, y_2, y_3 & y_4



$\frac{0.4}{6} = \frac{y_1}{1} \Rightarrow y_1 = 1/15$

$\frac{0.6}{9} = \frac{y_2}{4} \Rightarrow y_2 = 4/15$

For Max^m bending moment,

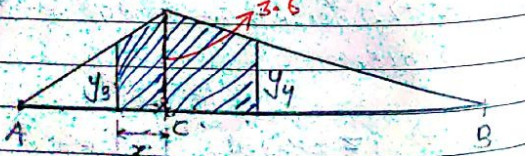


I.L.D for F_c

$\frac{x}{d} = \frac{z}{L}$

$\Rightarrow x = \frac{6}{15} \times 5 = 2m$

Then,



$\frac{3.6}{6} = \frac{y_3}{6-2} \Rightarrow y_3 = 2.4$

$\frac{3.6}{9} = \frac{y_4}{6} \Rightarrow y_4 = 2.4$

I.L.D for M_c

Now,

Maximum positive shear force $= w * \left[\frac{1}{2}(0.6 + 4/15) * 5\right]$

$= 40 \times \left[\left(\frac{1}{2}(0.6 + 4/15) * 5\right]\right.$

$= 86.67\ kN$

Maximum Negative shear force; $= 40 \times \left[\frac{1}{2} \times (0.4 + 1/15) \times 5\right] = 46.67\ kN$

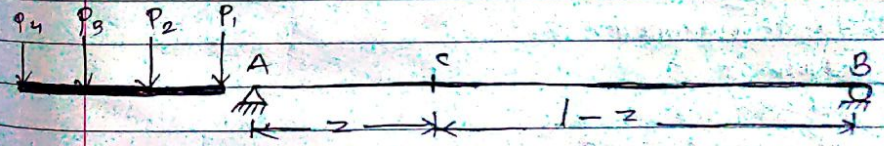
Maximum Moment $= w * \left[\frac{1}{2} \times (3.6+2.4) \times 2 + \frac{1}{2}(3.6+2.4) \times 3\right]$

$= 600\ kNm.$

Train of Concentrated load:-

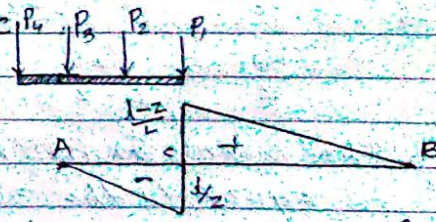
A train of concentrated loads moving over a simply supported beam from left to right is shown in figure. In this condition, we have to calculate

a) Maximum shear force at C
b) Maximum bending moment at C
c) Absolute Max^m shear force in beam
d) Absolute Max^m bending moment in beam.

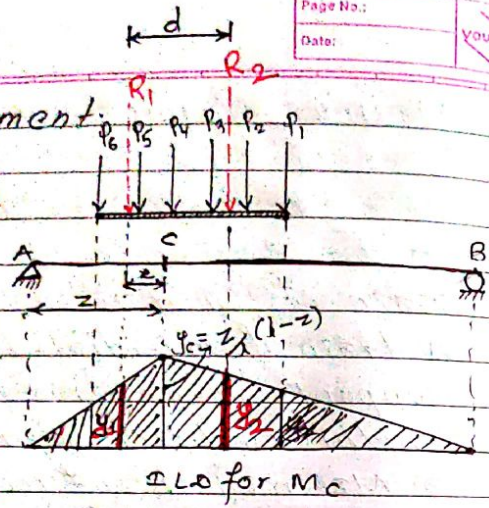


a) Maximum shear force at C

The negative shear force becomes maximum, when head of train of concentrated load just reached at C (or P_1 lies at C). Similarly, the positive shear force becomes maximum when tail of concentrated load just at C. Sometimes, if loads are varying, then we can find the maximum shear force by changing the position of loads. By two or three trial values it is possible to get maximum negative & positive shear force.



b) Maximum bending moment:



ILD for M_C

Let,
R_1 be the resultant of loads on the left of the section & R_2 be resultant of loads on the right of the section distance bet^n R_1 & R_2 be d' & R_1 be at a distance x from 'c'.

$$M_C = R_1 * y_1 + R_2 * y_2$$
$$= R_1 \left(\frac{z-x}{z}\right) y_c + R_2 * \frac{(L-z)-(d-x)}{L-z} \times y_c$$

For M_c to be maximum,

$$\frac{dM_c}{dx} = 0$$

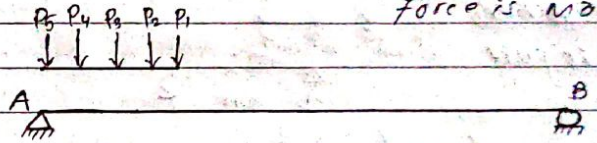
$$\Rightarrow -\frac{R_1 y_c}{z} + R_2 \left(\frac{y_c}{L-z}\right) = 0$$

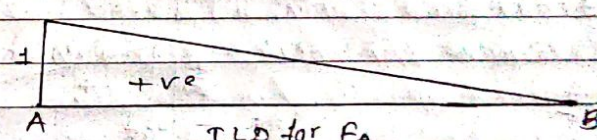
$$\Rightarrow \frac{R_1}{z} = \frac{R_2}{L-z}$$

That means, average load on left side portion of beam is equal to the average load on right side portion for Maximum bending Moment.
For this, we check the weight on left & right of 'c' & try to balance & draw this table.

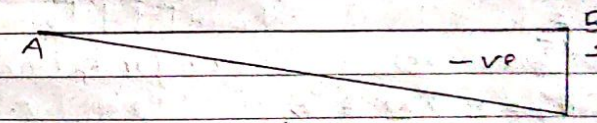
Sn	Load Crossing	Average weight W_{AC}	Average weight W_{BC}	Remarks:
1	P_1	- -	- -	$W_{AC} > W_{BC}$
2	P_2			$W_{AC} > W_{BC}$
3	P_3			$W_{AC} < W_{BC}$

c) Absolute Maximum shear force:-
At any section, influence line ordinate for negative shear is z/L and for positive shear is $(L-z)/L$.
i) When $z=0$, [i.e at A], ILD ordinate for positive shear force is Maximum (=1)
ii) When $z=L$, [i.e at B], ILD ordinate for negative shear force is Maximum (=1)





ILD for f_A



ILD for f_B

d) Absolute Maximum bending Moment under a load:-
→ Bending Moment will be Maximum near the centre of the span when heavier loads are near to the centre. Since, the Maximum moment always occurs under a wheel load, it can be concluded that absolute Maximum moment occur under one of the loads when the resultant of all the loads & the load under consideration are equidistance from the centre of span.

5.3 Four point loads 8, 15, 15 & 10 kN have centre to centre spacing of 2m betⁿ consecutive loads & they traverse a girder of 30m span from left to right with 10 kN load leading. Calculate the maximum bending moment & shear force at 8m from left support.

→ The +ve shear force will be maxᵐ when head of load is just at 'C'

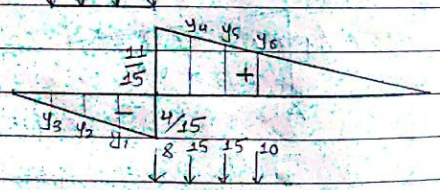


From figure,

$y_1 = \frac{4}{15} \times \frac{6}{8} = \frac{1}{5}$

$y_2 = \frac{4}{15} \times \frac{4}{8} = \frac{2}{15}$

$y_3 = \frac{4}{15} \times \frac{2}{8} = \frac{1}{15}$



ILD for F_C

Similarly, $y_4 = \frac{11}{15} \times \frac{20}{22} = \frac{2}{3}$

$y_5 = \frac{11}{15} \times \frac{18}{22} = \frac{3}{5}$

$y_6 = \frac{11}{15} \times \frac{16}{22} = \frac{8}{15}$

$-ve\ (V_C)_{max} = \frac{10 \times 4}{15} + \frac{15 \times 3}{5} + \frac{15 \times 2}{15} + \frac{8 \times 1}{15} = 8.2\ kN$

$+ve\ (V_C)_{max} = \frac{11}{15} \times 8 + \frac{2}{3} \times 15 + \frac{3}{5} \times 15 + \frac{8}{15} \times 10 = 30.2\ kN.$

For maximum bending moment, we have to find position of train of concentrated load. For this, draw table as follows.

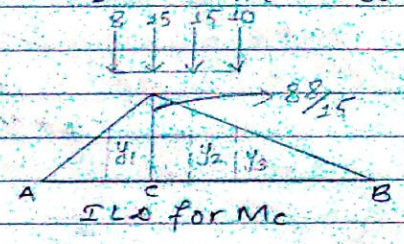
Sn	load Crossing	W_{AC}	W_{BC}	Remarks
1	10 kN	38/8	10/22	$W_{AC} > W_{BC}$
2	15 kN	23/8	25/22	$W_{AC} > W_{BC}$
3	15 kN	8/8	40/22	$W_{AC} < W_{BC}$

From figure,

$y_1 = \frac{88}{15} \times \frac{6}{8} = \frac{22}{5}$

$y_2 = \frac{88}{15} \times \frac{20}{22} = \frac{16}{3}$

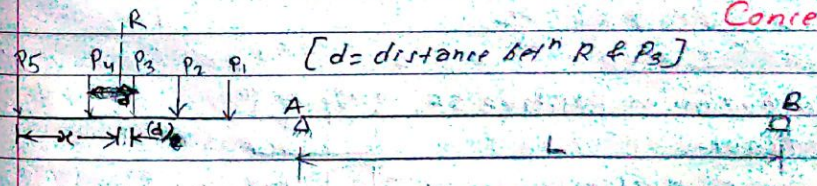
$y_3 = \frac{88}{15} \times \frac{18}{22} = \frac{24}{5}$



ILD for M_C

$(M_C)_{max} = \frac{8 \times 22}{5} + \frac{15 \times 88}{15} + \frac{16}{3} \times 15 + \frac{10 \times 24}{5} = 251.2\ kNm$

✻ Maximum Moment Under a load [In case of train of Concentrated load].

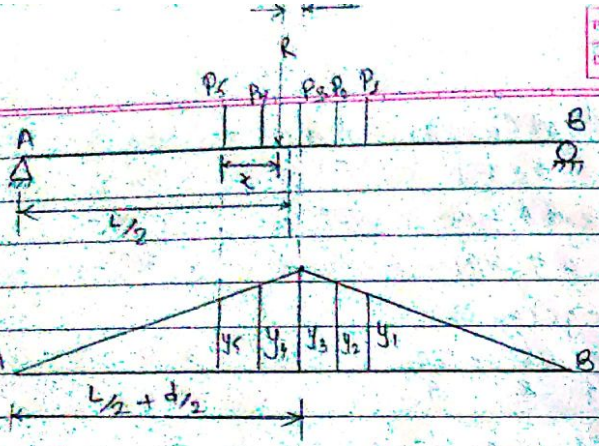


[d = distance betⁿ R & P_3]

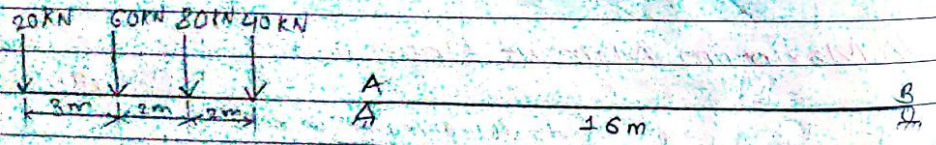
Let a train of concentrated loads $P_1, P_2, P_3 \ldots$ move on a simply supported beam AB from left to right as shown in figure. Let R be resultant of all loads.

a) If $P_3 > P_4$, placed the load over beam in such a way that R & P_3 bisect the centre of span equally. Then draw ILD & calculate moment.

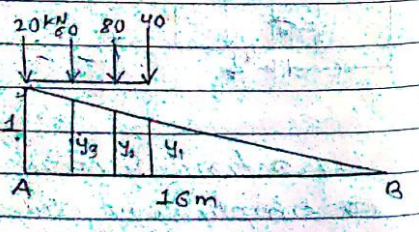
b) If $P_3 < P_4$, placed the load over beam in such a way that P_4 & R bisect the centre of span equally. Then draw ILD & calculate moment. The maximum value of moment obtained from above two cases give absolute value.



2 A train of concentrated loads shown in figure moves from left to right on a simply supported girder of span 16m. Determine the absolute maximum shear force & bending moment developed in the beam.



→ For maximum positive SF,



IL⊙ for f_A

From figure,

$y_1 = 9/16$, $y_2 = 11/16$,

$y_3 = 13/16$

1) When 20KN is just at A,

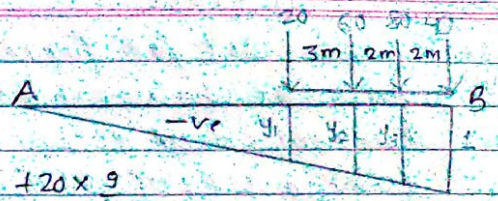
$+ve\ SF = 1 \times 20 + 60 \times \frac{13}{16} + 80 \times \frac{11}{16} + 40 \times \frac{9}{16} = 146.25\ kN$

ii) Since 20KN is lighter, so, 60kN is just at A & remove 20 kN,

$+ve\ SF = 1 \times 60 + 80 \times \frac{14}{16} + 40 \times \frac{12}{16} = 160\ kN$

$\therefore Max^m +ve\ SF = 160\ kN$

For Max^m negative SF:-



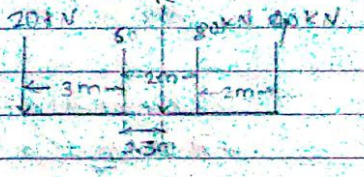
Maximum negative SF

$= 40 \times 1 + 80 \times \frac{14}{16} + 60 \times \frac{12}{16} + 20 \times \frac{9}{16}$

$= 166.25\ kN$

$\therefore$ Absolute Maximum shear force $= 166.25\ kN$.

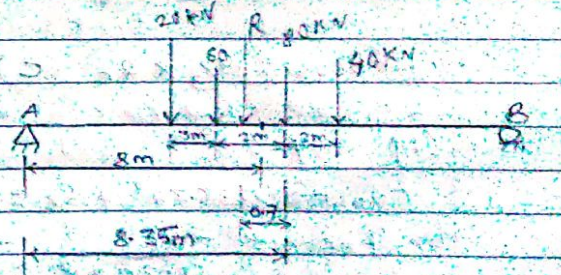
For absolute maximum moment



for position of resultant force,

$x = \frac{20 \times 0 + 60 \times 3 + 80 \times 5 + 40 \times 7}{20 + 60 + 80 + 20}$

$= 4.3m$



$y_c = \frac{z}{L}(L - z)$

$= \frac{8.35}{16}(16 - 8.35)$

$= 3.992$

$y_3 = 2.9483$

$y_2 = 3.035$

$y_1 = 1.601$



Then,

Absolute Max^m bending moment is

$= 1.601 \times 20 + 3.035 \times 60 + 2.9483 \times 80 + 3.992 \times 80$

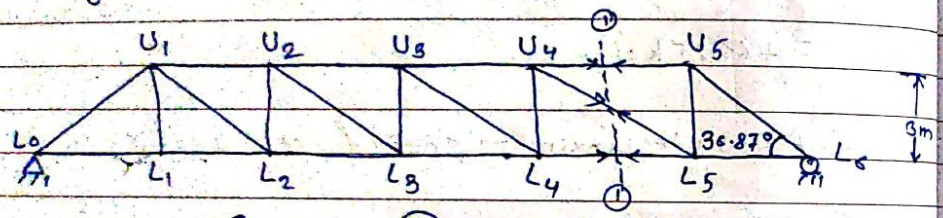
$= 651.412\ kNm$

(Since, heaviest load is near to R. so, not necessary to check for 60 kN).

ILD for truss

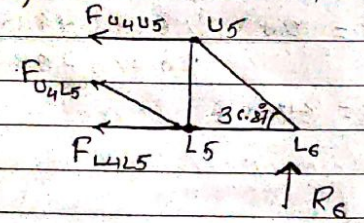
2012 Fall

Draw ILD for forces of Member U_4U_5, U_4L_5 & L_4L_5 as shown in figure. Unit load moves in bottom chord to L_6. Verify member forces with taking UDL of 8kN/m throughout the span.



6 panels @ 4m = 24m.

→ When unit load moves from L_0 to L_4



$\curvearrowright \Sigma M_{L_5} = 0$

$\Rightarrow -F_{U_4U_5} \times 3 - R_6 \times 4 = 0$

$\Rightarrow F_{U_4U_5} = -\frac{4}{3} \times R_6$

$\therefore F_{U_4U_5} = 4/3 \times R_6$ (C)

$(+\uparrow) \Sigma F_y = 0$

$F_{U_4L_5} \sin 36.87^\circ + R_6 = 0$

$\Rightarrow F_{U_4L_5} = \frac{-R_6}{\sin 36.87^\circ} = -\frac{5}{3} R_6$

$\therefore F_{U_4L_5} = 5/3 R_6$ (C)

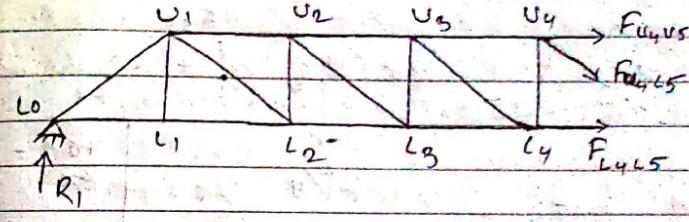
$(+\rightarrow) \Sigma F_x = 0$

$-F_{U_4U_5} - F_{U_4L_5} \cos 36.87^\circ - F_{L_4U_5} = 0$

$\frac{4}{3} R_6 + \frac{5}{3} R_6 \times \cos 36.87^\circ = F_{L_4U_5}$

$F_{L_4U_5} = \frac{8}{3} R_6$ (T)

When unit load moves from L_5 to L_6



$+\circlearrowleft \Sigma M_{U_4} = 0$

$R_1 \times 16 - F_{L_4L_5} \times 3 = 0 \Rightarrow F_{L_4L_5} = \frac{16}{3} R_1$ (T)

$(+\uparrow) \Sigma F_y = 0$

$R_1 - F_{U_4L_5} \sin 36.87^\circ = 0 \Rightarrow F_{U_4L_5} = \frac{5}{3} R_1$ (T)

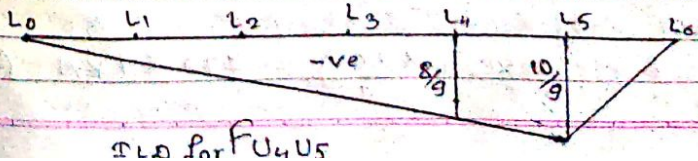
$(+\rightarrow) \Sigma F_x = 0$

$F_{U_4U_5} + F_{U_4L_5} \cos 36.87^\circ + F_{L_4L_5} = 0$

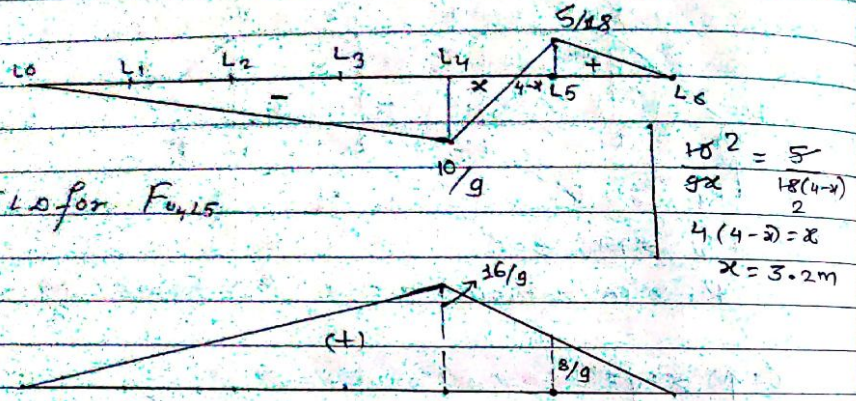
$F_{U_4U_5} + \frac{5}{3} \times R_1 \times \cos 36.87^\circ + \frac{16}{3} R_1 = 0$

$F_{U_4U_5} = \frac{-20 R_1}{3} \Rightarrow F_{U_4U_5} = \frac{20}{3} R_1$ (C)

Position → / Member ↓	$x = 0$ ($R_6 = 0$)	$x = 16$ ($R_6 = \frac{16}{24} = \frac{2}{3}$)	$x = 20$ ($R_1 = \frac{4}{24} = \frac{1}{6}$)	$x = 24$ ($R_1 = 0$)
U_4U_5	0	8/9 (C)	10/9 (C)	0
U_4L_5	0	10/9 (C)	5/18 (T)	0
L_4L_5	0	16/9 (T)	8/9 (T)	0



ILD for $F_{U_4U_5}$



$$\frac{10}{9x} = \frac{5}{18(4-x)/2}$$

$$4(4-x) = x$$

$$x = 3.2\,m$$

I.L.D for $F_{U_4L_5}$

I.L.D for $F_{L_4L_5}$

When unit load of 8 kN/m is throughout the span,

1) For $F_{U_4U_5}$

$$F_{U_4U_5} = \left(\frac{1}{2} \times 24 \times \frac{10}{9}\right) * 8 = 106.67\ kN\ (C)$$

2) For $F_{U_4L_5}$

$$-ve\ max^m = 8 * \left(\frac{1}{2} \times 19.2 \times \frac{10}{9}\right) = 85.33\ kN$$

$$+ve\ max^m = 8 * \left(\frac{1}{2} \times 4.8 \times \frac{5}{18}\right) = 5.33\ kN$$

$$F_{U_4L_5} = -85.33 + 5.33 = -53\ kN$$

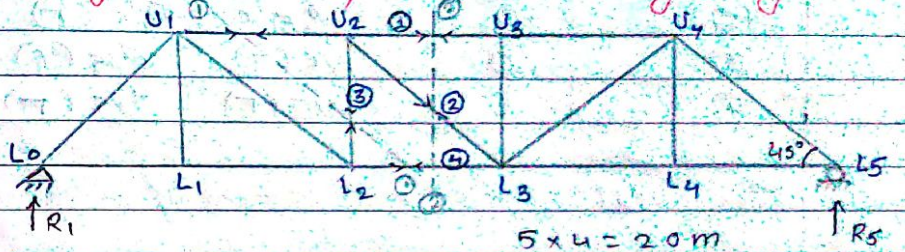
$$\therefore F_{U_4L_5} = 53\ kN\ (C)$$

3) For $F_{L_4L_5}$

$$F_{L_4L_5} = 8 * \left(\frac{1}{2} \times 24 \times \frac{16}{9}\right) = 170.67\ kN\ (T)$$

Q6

6.2 Determine the maximum forces in Members 1, 2, 3 & 4 of the truss shown in figure 6.12 when uniformly distributed load of 30 kN/m longer than the span traverses along the girder.



a) Considering section ① - ①

1) When unit load moves from L_0 to L_2

$$(+\uparrow) \Sigma F_y = 0 \Rightarrow -F_3 + R_5 = 0 \Rightarrow F_3 = R_5\ (T)$$

2) When unit load moves from L_3 to L_5

$$(+\uparrow) \Sigma F_y = 0 \Rightarrow F_3 + R_1 = 0 \Rightarrow F_3 = -R_1 \Rightarrow F_3 = R_1\ (C)$$

b) Considering section ② - ②

1) When unit load moves from L_0 to L_2

$$+\circlearrowright \Sigma M_{L_3} = 0 \Rightarrow -R_5 \times 8 - F_1 \times 4 = 0$$

$$\Rightarrow F_1 = -2R_5 \quad \therefore F_1 = 2R_5\ (C)$$

$$(+\uparrow) \Sigma F_y = 0 \Rightarrow F_2 \sin 45° + R_5 = 0$$

$$\Rightarrow F_2 = -\sqrt{2} R_5 \quad \therefore F_2 = \sqrt{2} R_5\ (C)$$

$$(+\rightarrow) \Sigma F_x = 0 \Rightarrow -F_1 - F_2 \cos 45° - F_4 = 0$$

$$\Rightarrow F_4 = 2R_5 + \sqrt{2} R_5 \times \frac{1}{\sqrt{2}} = 3R_5 \quad \therefore F_4 = 3R_5\ (T)$$

2) When unit load moves from L_3 to L_5

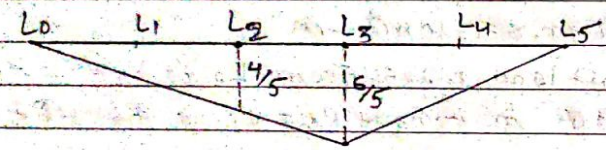
$$+\circlearrowright \Sigma M_{U_2} = 0 \Rightarrow R_1 \times 8 - F_4 \times 4 = 0 \Rightarrow F_4 = 2R_1\ (T)$$

$$(+\uparrow) \Sigma F_y = 0 \Rightarrow R_1 - F_2 \sin 45° = 0 \Rightarrow F_2 = \sqrt{2} R_1\ (T)$$

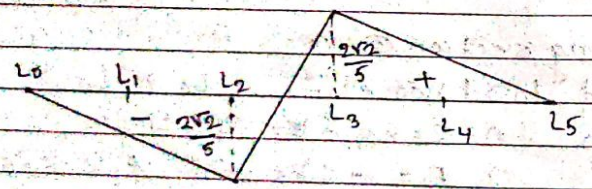
$$(+\rightarrow) \Sigma F_x = 0 \Rightarrow F_1 + F_2 \cos 45° + F_4 = 0$$

$$\Rightarrow F_1 = -3R_1 \Rightarrow F_1 = 3R_1\ (C)$$

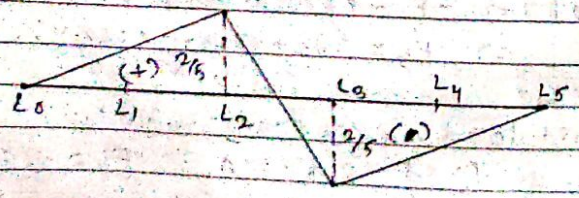
Position → / Member ↓	$x = 0$	$x = 8$	$x = 12$	$x = 20$
	$R_5 = 0$	$R_5 = 8/20 = 2/5$	$R_1 = 8/20 = 2/5$	$R_1 = 0$
①	0	4/5 (C)	6/5 (C)	0
②	0	$2\sqrt{2}/5$ (C)	$2\sqrt{2}/5$ (T)	0
③	0	2/5 (T)	2/5 (C)	0
④	0	6/5 (T)	4/5 (T)	0



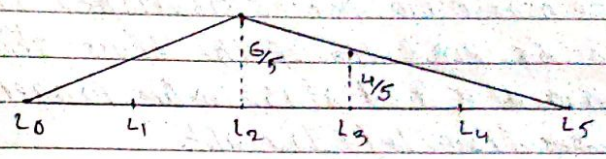
ILD for F_1



ILD for F_2

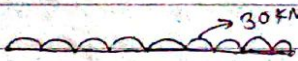


ILD for F_3



ILD for F_4

When UDL of 30 kN/m longer than span is moved, we have to find out maximum forces in member 1, 2, 3 & 4.



1) Force in Member 1,

$$F_1 = 30 * \left(\frac{1}{2} * 20 * \frac{6}{5}\right) = 360\,kN\,(C)$$

2) Force in Member 2,

$$(+ve)\ Maximum = 30 * \left(\frac{1}{2} \times 10 \times \frac{2\sqrt{2}}{5}\right) = 84.853\,kN$$

$$(-ve)\ Maximum = 30 * \left(\frac{1}{2} \times 10 \times \frac{2\sqrt{2}}{5}\right) = 84.853\,kN$$

3) Force in member 3,

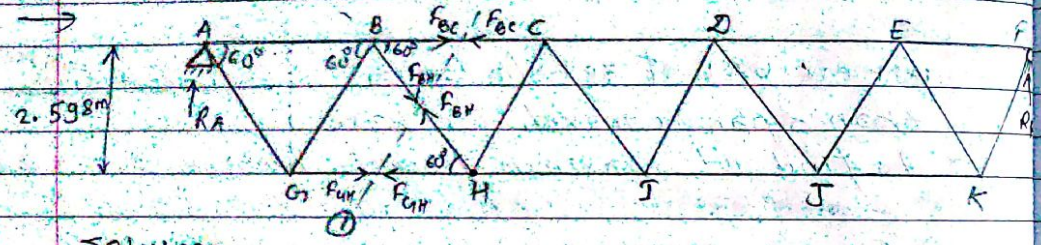
$$(+ve)\ Max^m = 30 * \left(\frac{1}{2} \times 10 \times \frac{2}{5}\right) = 60\,kN$$

$$(-ve)\ Max^m = 30 * \left(\frac{1}{2} \times 10 \times \frac{2}{5}\right) = 60\,kN$$

4) Force in Member 4,

$$F_4 = 30 * \left(\frac{1}{2} \times 20 \times \frac{6}{5}\right) = 360\,kN\ (T)$$

Q.3 Find the maximum forces developed in the members BC, BH & HG of the truss shown in figure. When a load of 30 KN/m longer than the span moves from left to right.



Solution

① When unit load moves from A to B,

$\rightarrow F_{BC} \times 2.598 + R_F \times 10.5 = 0$

$F_{BC} = -4.041 R_F$ $\quad \boxed{F_{BC} = 4.041 R_F \text{ (C)}}$

$\rightarrow F_{BH} \sin 60° + R_F = 0$ $\quad \boxed{F_{BH} = 1.154 R_F \text{ (T)}}$

$\rightarrow F_{BC} + F_{BH} \cos 60° + F_{GH} = 0$

or, $F_{GH} = 4.041 R_F + 1.154 \times \cos 60° \times R_F$

or, $F_{GH} = 4.618 R_F$ $\quad \boxed{F_{GH} = 4.618 R_F \text{ (T)}}$

② When unit load moves from C to F,

$\rightarrow F_{BC} \times 2.598 + R_A \times 4.5 = 0$

$F_{BC} = -1.732 R_A$ $\quad \boxed{F_{BC} = 1.732 R_A \text{ (C)}}$

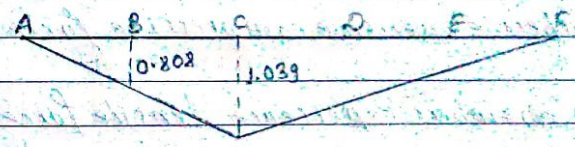
$\rightarrow R_A - F_{BH} \sin 60° = 0$ $\quad \boxed{F_{BH} = 1.154 R_A \text{ (T)}}$

$\rightarrow F_{BC} + F_{GH} + F_{BH} \cos 60° = 0$

or, $F_{GH} = 1.732 R_A - 1.154 R_A$

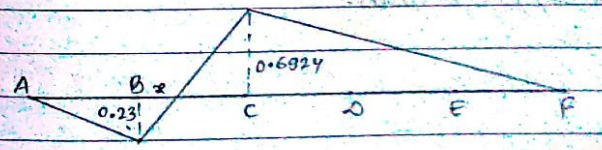
or, $F_{GH} = 0.578 R_A$ $\quad \boxed{F_{GH} = 0.578 R_A \text{ (T)}}$

position → / Member ↓	x = 0	x = 3 $R_F = 1$	x = 6 $R_A = 3$	x = 15
BC	0	0.808 (C)	~~0.6924~~ 1.039 (C)	0
BH	0	0.2308 (C)	0.6924 (T)	0
HG	0	0.923 (T)	0.3468 (T)	0

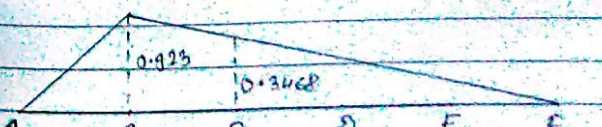


ILD for BC

$\frac{0.23}{x} = \frac{0.6924}{3-x} \quad \Rightarrow 3 - x = 3.01x \quad \Rightarrow x = 0.748m$



ILD for BH



ILD for HG

① $F_{BC} = \left(\frac{1}{2} \times 15 \times 1.039\right) \times 30 = 233.775 \text{ KN (C)}$

② F_{BH} (-ve) $= \left(\frac{1}{2} \times 0.23 \times 3.748\right) \times 30 = 12.93 \text{ KN (C)}$

F_{BH} (+ve) $= \left(\frac{1}{2} \times 0.6924 \times 11.252\right) \times 30 = 116.85 \text{ KN (T)}$

③ $F_{HG} = \left(\frac{1}{2} \times 15 \times 0.923\right) \times 30 = 207.67 \text{ KN (T)}$

Note

→ While drawing ILD in truss

a) For vertical & inclined member,

$$\Sigma F_y = 0$$

b) For horizontal Member,

$$\Sigma M_x = 0$$

Where, x is the point of intersection of remaining two member.

→ All top members experience compressive force & ILD is negative.

→ All bottom members experience tensile force & ILD is positive.

→ The vertical & inclined member experience tensile, compressive and both types of force depending on position of applied loads.

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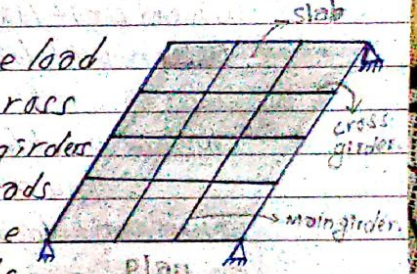
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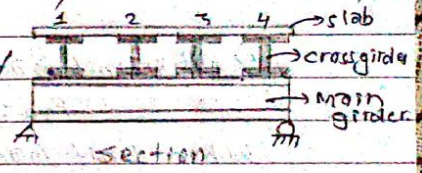
ILD for girders:-

→ It is the case of indirect loading / panel loading

→ Girder is a support beam, which supports smaller beam. Girder is mainly used in bridges.

→ In Girder, roof slab transmits the load to the cross beams & through cross beams to the girders. Hence, the girders are subjected to concentrated loads transmitted by cross beams. The point at which the girder supports the cross beam are referred as panel points 1, 2, 3 & 4 are panel points.





→ The end reactions are not affected by the presence of cross beam. Hence, the reactions are same as that of simply supported beam.

→ The ILD for SF at any point within the panel is same

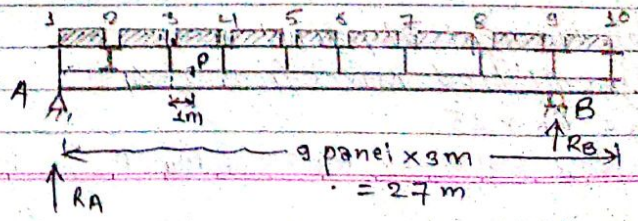
→ The ILD for BM at any point is identical to the ILD for simple beam.

→ But, for a section in beth panel points, the ILD differs only in that panel.

2011 Fall

Q Construct ILD for shear force in pannel 3-4, Moment at panel point 4 & Moment at 'p' for parallel beam as shown in figure.

→



1) ILD for shear force in pannel 3-4

a) When unit load moves from 1-3,

$F_{34} = -R_B$

When unit load is at 1, $R_B = 0$, so, $F_{34} = 0$

When unit load is at 3 $R_B = \frac{6}{24}$, $F_{34} = \frac{-6}{24} = -0.25$

b) When unit load moves from 4-9.

$F_{34} = R_A$

When unit load is at 4, $R_A = \frac{15}{24}$ $\therefore F_{34} = \frac{15}{24} = 0.625$

When unit load is at 9, $R_A = 0$ $\therefore F_{34} = 0$

c) When unit load moves from 9 to 10,

$F_{34} = R_A$

When unit load is at 9, $R_A = 0$, $\therefore F_{34} = 0$

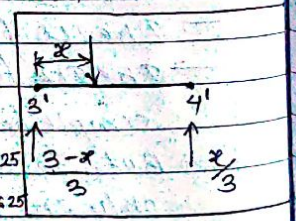
When unit load is at 10, $R_A = -\frac{1 \times 3}{24}$ $\therefore F_{34} = -0.125$

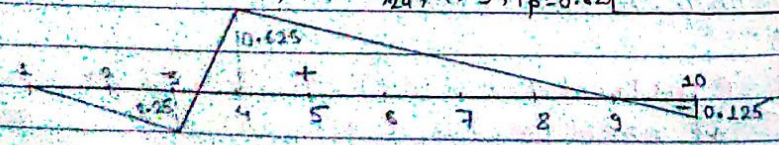
d) When unit load moves from 3-4,

$F_p = R_A - R_{3'} = R_A - \left(\frac{3-x}{3}\right)$

When unit load is at 3', $R_A = 18/24$, $x = 0$, $F_p = -0.25$

When unit load is at 4', $R_A = 15/24$, $x = 3$, $F_p = 0.625$





ILD for F_{34}

2) ILD for Moment at pannel 4:-

a) When unit load moves from 1-24

$M_4 = R_B * 15$

When unit load at 1, $R_B = 0$, $M_4 = 0$

When unit load at 4, $R_B = \frac{9}{24}$ $\therefore M_4 = 5.625$

b) When unit load moves from 4 to 9,

$M_4 = R_A * 9$

When unit load at 4, $R_A = \frac{15}{24}$ $\therefore M_4 = 5.625$

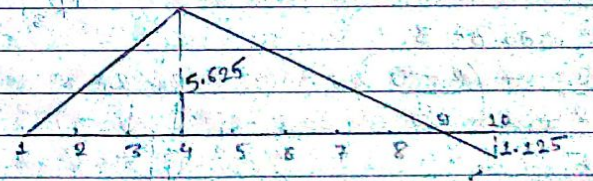
When unit load at 9, $R_A = 0$ $\therefore M_4 = 0$

c) When unit load moves from 9 to 10,

$M_4 = R_A * 9$

When unit load at 9, $R_A = 0$, $M_4 = 0$

When unit load at 10, $R_A = \frac{-1 \times 9}{24}$, $M_4 = -1.125$



ILD for M_4

3) ILD for Moment at 'p'.

a) When unit load moves from 1-3:-

$M_p = R_B \times 17$

When unit load is at 1, $R_B = 0$, $\therefore M_p = 0$

When unit load is at 3, $R_B = 6/24$, $\therefore M_p = \frac{6}{24} \times 17 = 4.25$

b) When unit load moves from 4-9

$M_p = R_A \times 7$

When unit load is at 4, $R_A = \frac{15}{24}$, $M_p = 4.375$

When unit load is at 9, $R_A = 0$ $\therefore M_p = 0$

c) When unit load moves from 9 to 10

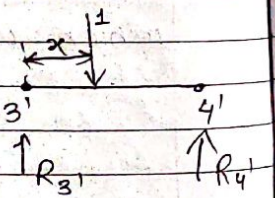
$M_p = R_A * 7$

When unit load is at 9, $R_A = 0$, $M_p = 0$

When unit load is at 10, $R_A = \frac{-1 \times 3}{24} = -0.125$

$\therefore M_p = -0.875$

d) When unit load moves from 3-4,



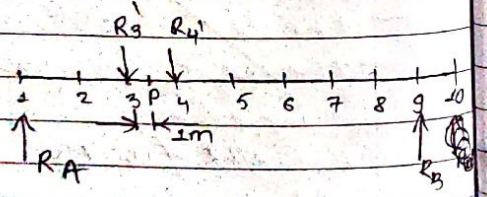
$M_p = R_A \times 7 - R_3' \times 1$

$= 7R_A - \left(\frac{3-x}{3}\right)$

When unit load at 3,

$M_p = 7 * \frac{18}{24} - \left(\frac{3-0}{3}\right)$

$= 4.25$

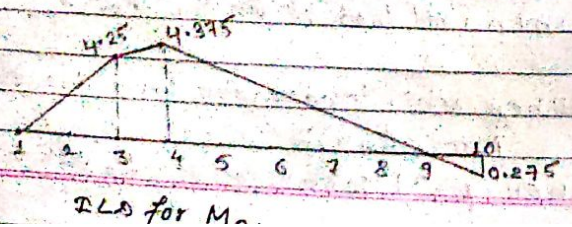


When unit load at 4,

$M_p = 7 * \frac{15}{24} - \left(\frac{3-3}{3}\right)$

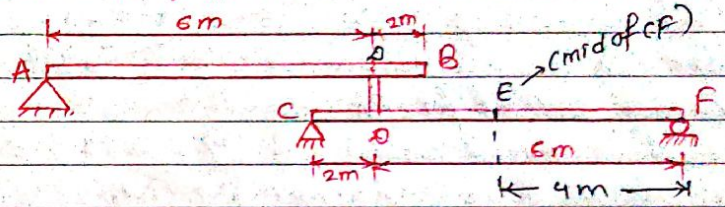
$= 4.375$

$R_3' = \frac{3-x}{3}$ $R_4' = \frac{x}{3}$



ILD for M_E

Q Draw the ILD for reaction at A & C and for B.M at E, the midpoint of lower beam CF of the ~~the~~ simply supported beam shown in figure.



Solution

→ we have to draw ILD for R_A, R_C & M_E.

1) When unit load moves from A to D,

Let unit load is at a distance 'x' from A,

$R_D = \frac{x}{6}$ & $R_A = 1 - \frac{x}{6}$

$R_C = \frac{6}{8} * \frac{x}{6} = \frac{x}{8}$ $R_F = \frac{x}{6} - \frac{x}{8} = \frac{x}{24}$

When $x = 0$ [i.e unit load at A]

$R_A = 1$, $R_D = 0$, $R_C = 0$, $R_F = 0$

When $x = 6$, $R_A = 0$, $R_D = 1$, $R_C = 0.75$, $R_F = 0.25$

2) When unit load moves from D to B,

Let unit load is at a distance 'x' from D,

$R_D = \frac{6+x}{6}$, $R_A = -\frac{x}{6}$, $R_C = \frac{6}{8} \times \left(\frac{6+x}{6}\right)$ $R_F = \frac{6+x}{24}$

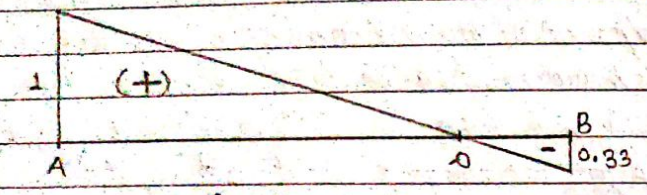
$= (6+x)/8$

When $x = 0$, [i.e unit load at D]

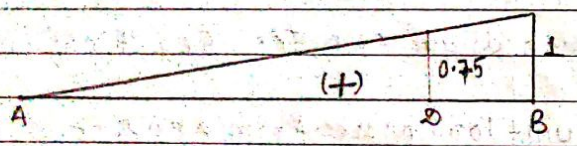
$R_D = 1$, $R_A = 0$, $R_C = 0.75$, $R_F = 0.25$

When $x = 2$, [i.e unit load at B]

$R_D = 1.33$, $R_A = -0.33$, $R_C = 1$, $R_F = 0.33$



ILD for R_A



ILD for R_C

* ILD for M_E

a) When unit load moves from A to D,

$M_E = R_F \times 4$

when unit load at A, $R_F = 0$, $M_E = 0$

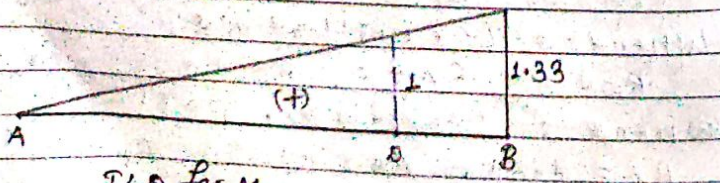
When unit load at D, $R_F = 0.25$, $M_E = 1$

b) When unit load moves from D to B,

$M_E = R_F \times 4$

When unit load at D, $R_F =$ ~~0.25~~ 0.25, $M_E = 1$

When unit load at B, $R_F = 0.33$, $M_E = 1.33$



ILD for M_E.

Ch-7 Statically Determinate Arch:-

Arch:-

→ Arch is a curved structure having convexity upwards & supported at ends. It may be subjected to vertical, horizontal or even incline loads. They transfer load to the abutment by axial compression & bending.

→ Beams transfer the applied load to the end supports by bending & shear action. For larger spans, beam are uneconomical & many a time the self weight of the beam contribute to the stress in such large portions that is difficult to design & requires larger section. So, there is requirement of arch for larger spans like bridges.

Types of Arches:-

1) On the basis of material used to built
   i) Steel arch
   ii) Reinforced concrete arch

2) On the basis of shape
   i) parabolic
   ii) Circular
   iii) Elliptical

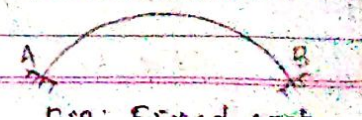


Fig: Three hinged arch

3) On the basis of structural behaviour
   i) Three hinged arch
   ii) Two hinged arch
   iii) Hingedless / Fixed arch.

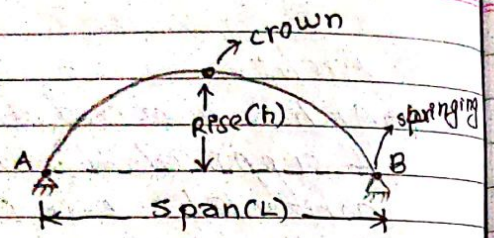


Fig: Two hinged arch



* Parts of three hinged arch:-

The topmost point of three hinged arch is called crown which has a hinge.



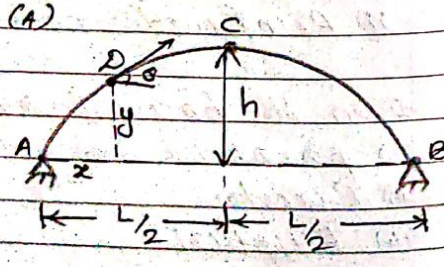
The height of the crown above support level is called rise (h). Arch transfer the loads to the ~~spanning~~ abutments at springing points. The length betn two springing is called span(L). The part of almost arches is same which is described above.

* Only three hinged arch is structurally determinate whereas two hinged arch & fixed arch is structurally indeterminate.

* Three Hinged Arch with Support at Same level:-

1) parabolic Arch:-

Taking springing point (A) as origin, the eqn of parabola is given by.



$$y = \frac{4hx}{L^2}(L-x)$$

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{L^2}(L-2x)$$

Taking crown as origin, the eqn of parabola is given by

$$\frac{x^2}{y} = a$$

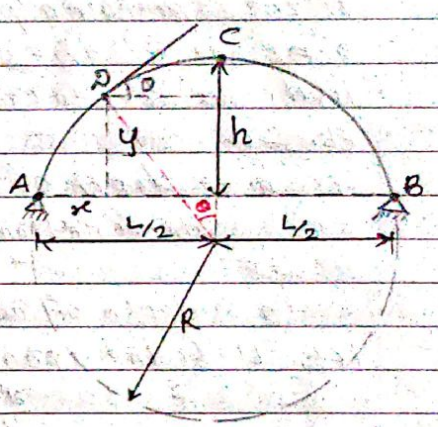
At $x = L/2$, $y = h$

So, $a = \frac{(L/2)^2}{h} = \frac{L^2}{4h}$

Then,

$$y = \frac{4h}{L^2} * x^2$$

2) Circular Arch:-



Let,

R = Radius of circle.

'D' be a point on arch having coordinate (x, y).

From property of circle,

$$ab = cd$$

$$\Rightarrow \frac{L}{2} \times \frac{L}{2} = h * (2R - h)$$

$$\Rightarrow \frac{L^2}{4h} = 2R - h$$

$$\Rightarrow R = \frac{h}{2} + \frac{L^2}{8h}$$



Taking origin at support A,

$$\frac{L}{2} = x + R\sin\theta$$

$$\sin\theta = \frac{(L/2 - x)}{R} \quad ----(i)$$

$$y = R\cos\theta - (R-h)$$

$$y = R\cos\theta - R + h$$

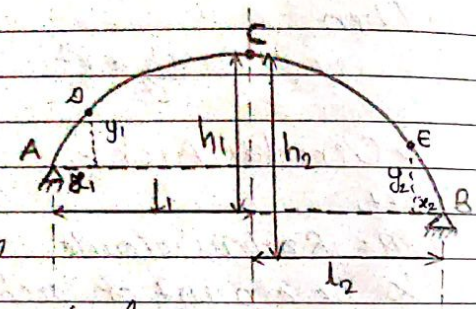
$$y = h + R(\cos\theta - 1) \quad ----(ii)$$

Eqn (i) gives the angle θ & using it calculate value of 'y' in eqn (ii).

※ Three Hinged Arch with support at different levels

1) parabolic:-

If the springing points are not at the same level, we have to calculate L_1 & L_2.



Let, h_1 → depth of abutment 'A' from crown

h_2 ⇒ depth of abutment 'B' from crown.

Let, 'L' be the span. Then,

$$\frac{x^2}{y} = \text{constant}$$

$$\frac{x}{\sqrt{y}} = \text{constant}$$

i.e., $\frac{L_1}{\sqrt{h_1}} = \text{const}$ & $\frac{L_2}{\sqrt{h_2}} = \text{const}$

This means,

$$\frac{L_1}{\sqrt{h_1}} = \frac{L_2}{\sqrt{h_2}} = \frac{L_1+L_2}{\sqrt{h_1}+\sqrt{h_2}} \qquad \left[\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}\right]$$

$$\Rightarrow \frac{L_1}{\sqrt{h_1}} = \frac{L_2}{\sqrt{h_2}} = \frac{L}{\sqrt{h_1}+\sqrt{h_2}}$$

$$\Rightarrow L_1 = \frac{\sqrt{h_1}\times L}{\sqrt{h_1}+\sqrt{h_2}} \quad \& \quad L_2 = \frac{\sqrt{h_2}\times L}{\sqrt{h_1}+\sqrt{h_2}}$$

Also,

$$y_1 = \frac{4h_1x_1(2L_1-x_1)}{(2L_1)^2} \qquad [L \to 2L_1]$$

$$y_2 = \frac{4h_2x_2(2L_2-x_2)}{(2L_2)^2} \qquad [L \to 2L_2]$$

* Analysis of three Hinged Arch:-

i) Determination of support reactions:-

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_A = 0$ or $\Sigma M_B = 0$ [All loads]

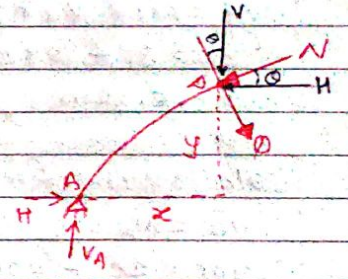
$\Sigma M_C = 0$ [considering only left or only right part]

ii) Determination of radial shear 'Φ' & Normal thrust (N) & bending Moment (M):-

Here, θ is the angle made by normal thrust with horizontal.

Normal Thrust (N) = $H\cos\theta + V\sin\theta$

Radial shear (Φ) = $V\cos\theta - H\sin\theta$.



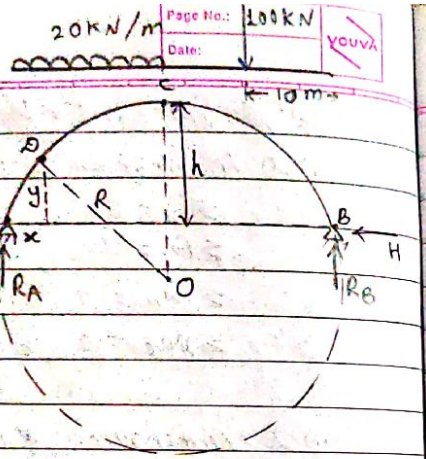
Q A three-hinged circular arch hinged at the springing & crown points has a span of 40m & central rise of 8m. It carries a UDL of 20kN/m over the left half of the span together with a concentrated load of 100kN at the right quarter span point. Find the reactions at the supports, normal thrust & shear at a 10m from left support.

→ Given,

span (L) = 80m

Central rise (h) = 8m

x = 10m



The radius of circle is,

$$R = \frac{h}{2} + \frac{l^2}{8h}$$

$$= \frac{8}{2} + \frac{80^2}{8\times 8}$$

$$= 29\,m$$

$$\sin\phi = \frac{l/2 - x}{R}$$

$$\phi = \sin^{-1}\left(\frac{20-10}{29}\right) = 20.17^\circ$$

$$y = h + R(\cos\phi - 1)$$

$$= 8 + 29(\cos 20.17^\circ - 1)$$

$$= 6.22\,m$$

Calculation of reaction force:

$$+\circlearrowright \Sigma M_A = 0$$

$$\Rightarrow 400\times 20 + 100\times 30 - R_B \times 40 = 0$$

$$\Rightarrow R_B = 175\,KN$$

$$(+\uparrow)\ \Sigma F_y = 0\ ;\quad R_A - (400+100) + 175 = 0$$

$$R_A = 325\,KN$$

$+\circlearrowright \Sigma M_C = 0$ [Taking left part only]

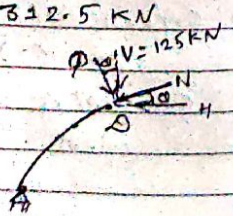
$$R_A \times 20 - H\times 8 - 400\times 10 = 0$$

$$\frac{325\times 20 - 400\times 10}{8} = H \quad \Rightarrow H = 312.5\,KN$$

Calculation of N & ϕ,

$$N = H\cos\phi + V\sin\phi = 312.5\cos 20.17^\circ + 125\sin 20.17^\circ$$

$$= 336.43\,KN$$

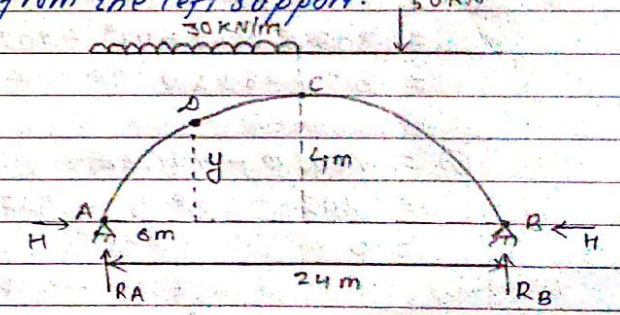


$$\phi = V\cos\phi - H\sin\phi = 125\cos 20.17^\circ - 312.5\sin 20.17^\circ$$

$$= 9.582\,KN$$

$$M_D = 325\times 10 - 312.5\times 6.22 - 200\times 5 = 306.25\,KNm.$$

Q. A three hinged parabolic arch hinged at the supports and at the crown has a span of 24m and a central rise of 4m. It carries a concentrated load of 50KN at 18m from left support & a uniformly distributed load of 30KN/m over the left half portion. Determine the moment, thrust & radial shear at a section 6m from the left support.



→ Given,

$L = 24m$

$h = 4m$

$$+\circlearrowright \Sigma M_A = 0$$

$$\Rightarrow 360\times 6 + 50\times 18 - R_B\times 24 = 0$$

$$\Rightarrow R_B = 127.5\,KN.$$

$$(+\uparrow)\ \Sigma F_y = 0$$

$$\Rightarrow 127.5 + R_A - 360 - 50 = 0$$

$$\Rightarrow R_A = 282.5\,KN$$

$$+\circlearrowright \Sigma M_C = 0\ ;$$

$$\Rightarrow 282.5\times 12 - H\times 4 - 360\times 6 = 0$$

$$\Rightarrow H = 307.5\,KN$$

$$y = \frac{4hx}{L^2}(L-x) = \frac{4\times 4\times 6}{24^2}(24-6) = 3m$$

$$M_D = 282.5\times 6 - 307.5\times 3 - 180\times 3 = 232.5\,KNm$$

Calculation of Radial shear & Normal thrust

We know,

$$\tan\theta = \frac{d}{dx}\left[\frac{4hx}{L^2}(L-x)\right]$$

$$\Rightarrow \tan\theta = \frac{4h}{L^2}(L-2x)$$

$$\Rightarrow \tan\theta = \frac{4\times4}{24^2}\times(24-2\times6)$$

$$\Rightarrow \theta = 18.43^\circ$$

$V = R_A - 180$

$= 102.5\ KN$

$N = H\cos\theta + V\sin\theta$

$= 307.5\cos18.43^\circ + 102.5\sin18.43^\circ$

$= 324.133\ KN$

$Q = V\cos\theta - H\sin\theta$

$= 102.5\cos18.43^\circ - 307.5\sin18.43^\circ$

$= 0$

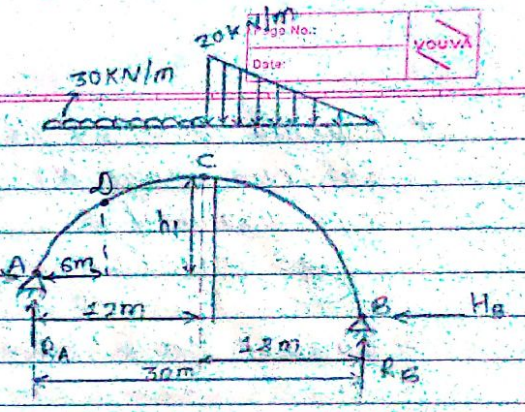
Q. A three hinged parabolic arch of span 30m has its supports A & B at different levels. A being higher level than B. The supports 'A' is 4m below the crown hinge 'C' and 12m horizontally from A. The arch carries UDL of 30KN/m in AC & UVL of 20kN/m in CB. The UVL has maximum intensity at crown. Determine vertical reaction at supports, horizontal thrust & maximum moment in the arch.

→ Given,

$L = 30m$

$h_1 = 4m$

$L_1 = 12m$



Calculation of reaction forces,

$$\frac{L_1}{\sqrt{h_1}} = \frac{L_2}{\sqrt{h_2}}$$

$$\Rightarrow \frac{12}{\sqrt{4}} = \frac{18}{\sqrt{h_2}}$$

$$\Rightarrow \sqrt{h_2} = \frac{18\times2}{12}$$

$$\Rightarrow h_2 = 9m$$

Now,

$$+\circlearrowright \Sigma M_A = 0$$

$$\Rightarrow (30\times12)\times6 + \frac{1}{2}\times18\times20\times\left(12+\frac{1}{3}\times18\right) + H_B\times(9-4) - R_B\times30 = 0$$

$$\Rightarrow 5H_B - 30R_B = -5400 \quad ---(1)$$

$$+\circlearrowright \Sigma M_C = 0 \quad \text{[Taking right part only]}$$

$$\Rightarrow \frac{1}{2}\times18\times20\times\left(\frac{1}{3}\times18\right) + H_B\times9 - R_B\times18 = 0$$

$$\Rightarrow 9H_B - 18R_B = -1080 \quad ---(11)$$

On solving,

$H_B = 360\ KN$ & $R_B = 240\ KN$

$$(+\uparrow)\Sigma F_y = 0$$

$$\Rightarrow R_A + R_B - 30\times12 - \frac{1}{2}\times18\times20 = 0$$

$$\Rightarrow R_A + 240 - 360 - 180 = 0$$

$$\Rightarrow R_A = 300\ KN$$

$$(+\rightarrow)\Sigma F_x = 0$$

$$\Rightarrow H_A - H_B = 0$$

$$\Rightarrow H_A = 360\ KN$$

* Normal thrust & Radial shear at a distance of 6m from hinge 'A'.

To find value of ϕ,

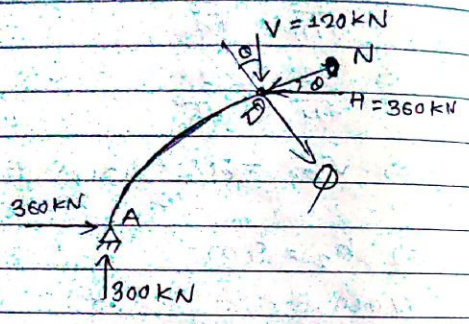
$$y = \frac{4hx}{L^2}(L-x)$$

$$\Rightarrow \tan\phi = \frac{4h}{L^2}(L-2x)$$

$$\Rightarrow \tan\phi = \frac{4 \times h_1}{(2L_1)^2}(2L_1 - 2x_1)$$

$$\Rightarrow \tan\phi = \frac{4 \times 4 \times (2\times 12 - 2*6)}{(2\times 12)^2}$$

$$\Rightarrow \phi = 18.43^\circ$$



Also, $y_1 = \frac{4 * h_1 * x_1}{(2L_1)^2}(2L_1 - x_1)$

$$= \frac{4*4*6}{(2\times 12)^2} \times (2*12 - 6)$$

$$= 3m$$

Now,

$N = H\cos\phi + V\sin\phi$

$= 360\cos 18.43^\circ + 120\sin 18.43^\circ$

$= 379.47$ kN

$Q = V\cos\phi - H\sin\phi$

$= 120\cos 18.43^\circ - 360 * \sin 18.43^\circ$

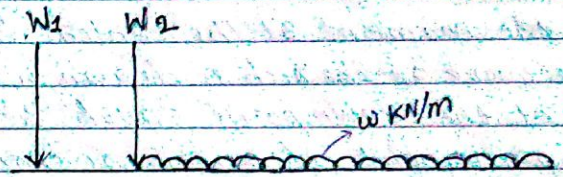
$= 0$

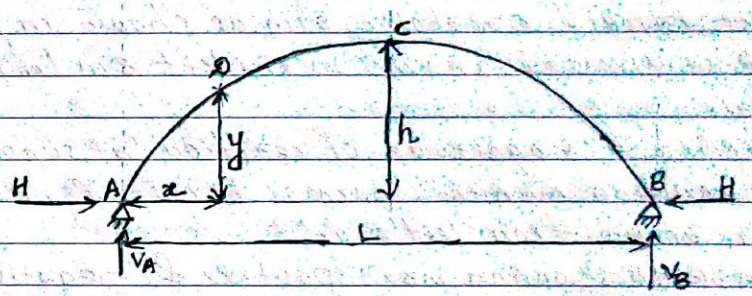
$M_D = R_A \times 6 - H_A \times 3 - (30\times 6)\times 3$

$= 300\times 6 - 360\times 3 - (30\times 6)\times 3$

$= 180$ kNm

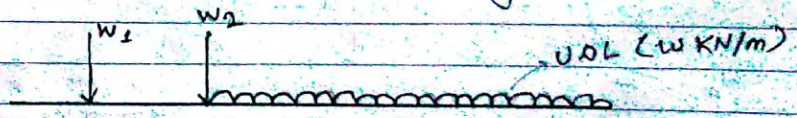
* Bending Moment Diagrams:-



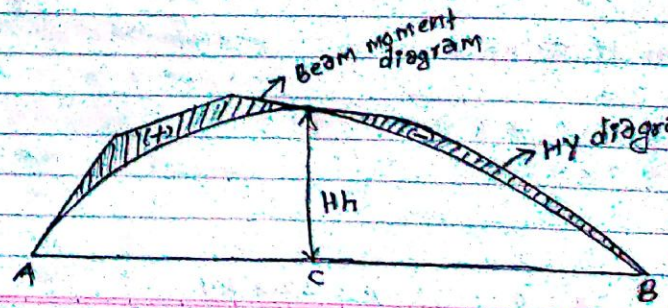


In an arch, at any section D(x,y), the bending moment may be looked as a sum of the moment in an equivalent beam minus the ordinate time the horizontal thrust. Thus

BMD = Beam moment - Hy.



Equivalent beam

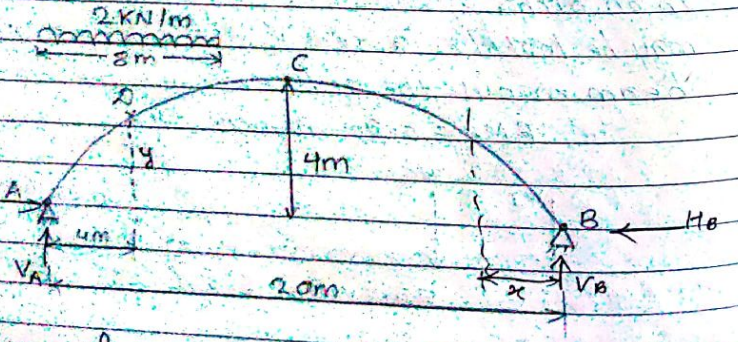


Bending Moment diagram

Hence, first bending moment diagram for the equivalent beam be drawn. Then, to subtract My moment, we note that the moment at the central hinge is zero. Hence, the curve of the arch is drawn with the ordinate at the crown point equal to the ordinate of bending moment diagram as shown in figure.

* A three hinged parabolic arch as shown in figure is loaded with UDL 2 KN/m on the left 8m length. Calculate
i) direction & magnitude of reaction at supports.
ii) the bending moment, normal thrust & radial shear at 4m from left end.
iii) Draw BMD showing maxm positive & negative value.

→



Calculation of support rxn:

$+\circlearrowright \Sigma M_A = 0$

$\Rightarrow -V_B \times 20 + (2*8)*4 = 0$

$\Rightarrow V_B = 3.2\ KN$

$(+\uparrow) \Sigma F_y = 0$

$\Rightarrow -16 + 3.2 + V_A = 0 \quad \Rightarrow V_A = 12.8\ KN$

$+\circlearrowright \Sigma M_C = 0$ [Taking right part only]

$\Rightarrow H_B * 4 - V_B * 10 = 0$

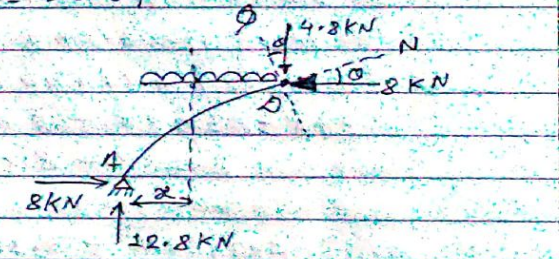
$\Rightarrow H_B = \frac{3.2 \times 10}{4} = 8\ KN$

$\therefore H_A = H_B = 8\ KN.$

For parabolic arch,

$$y = \frac{4hx}{L^2}(L-x) = \frac{4 \times 4 \times 4 (20-4)}{20^2} = 2.56m$$

$$\theta = \tan^{-1}\left[\frac{4h}{L^2}(L-2x)\right] = 25.64°$$



Normal thrust,

$N = 8\cos\theta + 4.8\sin\theta$

$= 9.29\ KN \swarrow 25.64°$

Radial shear,

$\Phi = 4.8\cos\theta - 8\sin\theta = 0.86KN \nwarrow \theta = 25.64°.$

$BM\ at\ D = 12.8 \times 4 - 8 \times 2.56 - 8 \times 2$

$= 14.72\ KNm$

* Maximum positive BM:-

Let maxm positive B.M occurs at a section x m from left end.

Then,

$$M = 12.8x - 2x \times \frac{x}{2} - 8y$$

$$= 12.8x - x^2 - 8\left[\frac{4 \times 4 \times x}{20^2}(20-x)\right]$$

$$= 12.8x - x^2 - 0.32x(20-x)$$

$$= 6.4x - 0.68x^2$$

For 'M' to be maximum,

$$\frac{dM}{dx} = 0$$

$\Rightarrow 6.4 - 0.68 \times 2x = 0$

$\Rightarrow x = 4.71\,m$

Then,

Max^m +ve BM $= 6.4 \times 4.71 - 0.68 \times 4.71^2 = 15\,kNm$

* Maximum negative BM:

If Max^m negative BM occurs at a distance x m from right support then,

$$M = 3.2x - 8 * y$$

$$= 8.2x - 8 \times \left[\frac{4 \times 4 \times x}{20^2}(20 - x)\right]$$

$$= 0.32x^2 - 3.2x$$

For 'M' to be maximum,

$$\frac{dM}{dx} = 0$$

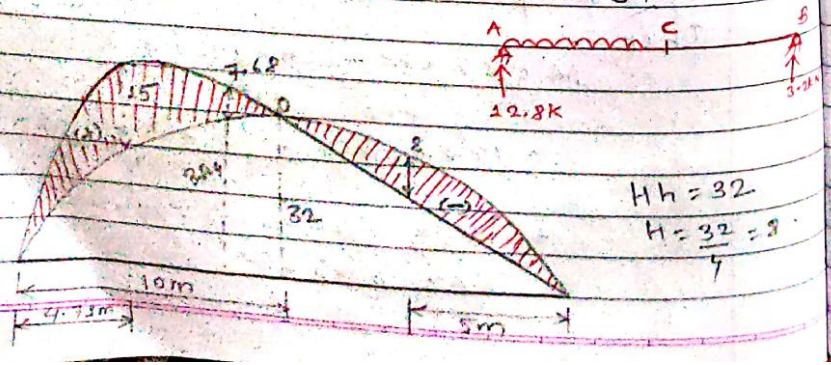
$\Rightarrow 0.32 \times 2x - 3.2 = 0$

$\Rightarrow x = 5\,m.$

Then,

Max^m -ve Moment $= 0.32 \times 5^2 - 3.2 \times 5 = -8\,kNm$

$M_B = 12.8 \times 8 - 8 \times 3.24 - \frac{1}{2} \times 4 \times 4 = 7.68\,kNm$



Ch-9 Cable Structures:-

A cable is a rope used for construction purpose as well as other different purpose. Mainly, from point of view of civil engineering, cable are used in suspension bridges.

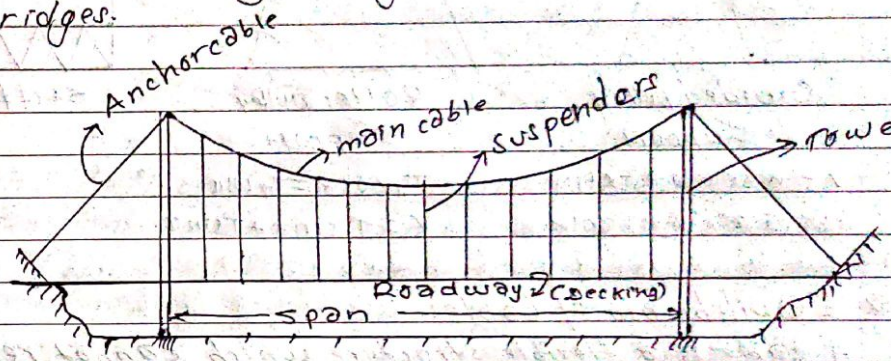


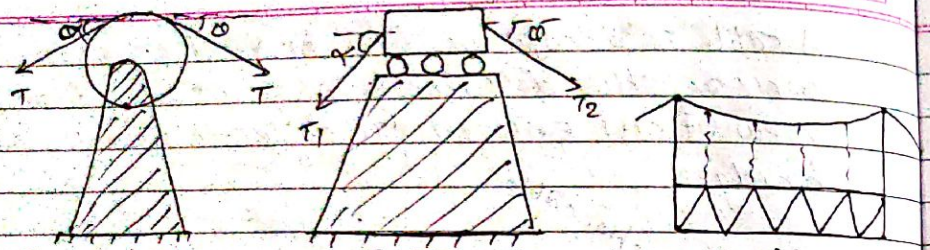
Fig:- Suspension bridge

→ Suspension bridges are used for highways where the span of bridge is more than 200m. Essentially a suspension bridge consists of following elements.

i) Main cable
ii) Suspender
iii) Decking / Roadway including stiffening girder
iv) Tower
v) Anchor cable.

→ The traffic load of decking is transferred to main cable through suspenders. Since, the cable is main load bearing member. The stiffened girder transfers a uniformly distributed or equal load to the suspender, irrespective of the load position on the decking.

→ The cable transfer load through tension & B.moment at any point on cable is zero.

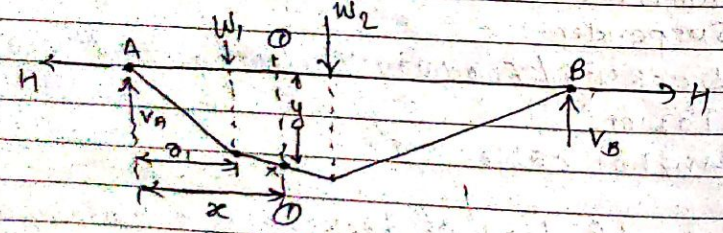


$V = T\sin\alpha + T\sin\alpha$ $\quad$ $T_1\cos\alpha = T_0\cos\alpha$

$H = T\cos\alpha - T\cos\alpha$ $\quad$ $V = T_1\sin\alpha + T\sin\alpha$.

※ Equilibrium of Cable:-

A cable is a flexible structure which cannot resist bending moment. It deflects so that the bending moment is zero at any point which is achieved by developing horizontal thrust at the support & thus developing appropriate deflection.



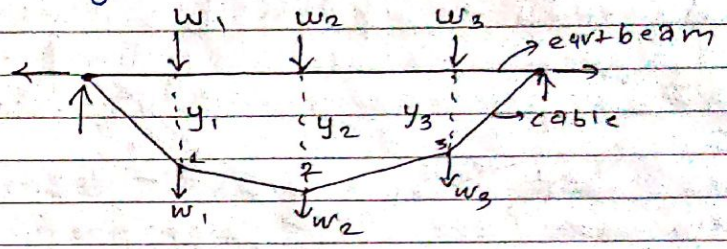
$M_x = V_A * x - W_1 * (x - a_1) - Hy$

Since, in cable $M_x = 0$,

$\therefore Hy = V_A * x - W_1(x - a_1)$

Hy = Beam moment.

※ Cable subjected to concentrated loads:-



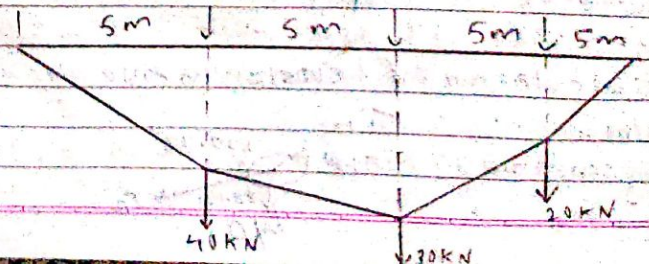
The deflected shape is similar to beam moment diagram. If M_1, M_2 & M_3 are beam moments at load points 1, 2 & 3, the deflections y_1, y_2 & y_3 are given by

$$y_1 = \frac{M_1}{H}, \quad y_2 = \frac{M_2}{H}, \quad y_3 = \frac{M_3}{H}$$

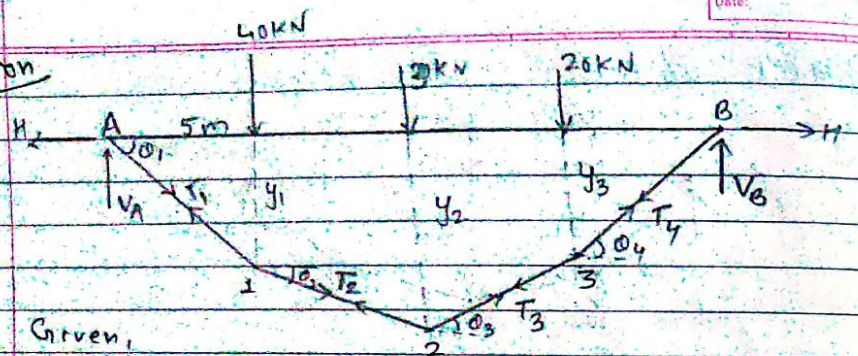
If horizontal thrust is known or position of cable is known, the deflection at all points can be calculated & shape is found. The actual length of cable is sum of length of each segment.

After finding deflection, the tension in each cable can be calculated.

Q. A light cable is supported at two points 20m apart which are at same level. The cable supports three concentrated loads as shown in figure. The deflection at first point was found to be 0.8m. Determine the tension in diffn segments & total length of the cable.



Solution



Given,

$y_1 = 0.8m$

$+\circlearrowleft \Sigma M_A = 0$

$40 \times 5 + 30 \times 10 + 20 \times 15 - V_B \times 20 = 0$

$V_B = 40\ KN$

$(+\uparrow) \Sigma F_y = 0$; $V_A = 50\ KN$.

Now,

$M_1 = H * y_1$

$\Rightarrow V_A \times 5 = H \times y_1 \Rightarrow H = \frac{50 \times 5}{0.8} = 312.5\ KN$

Similarly,

$y_2 = \frac{M_2}{H} = \frac{V_A \times 10 - 40 \times 5}{312.5} = 0.96m$

$y_3 = \frac{M_3}{H} = \frac{V_A \times 15 - 40 \times 10 - 30 \times 5}{312.5} = \frac{V_B \times 5}{312.5} = 0.64m$

From figure,

$\theta_1 = 9.1°$

$\theta_2 = 1.83°$

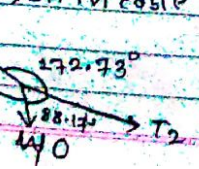
$\theta_3 = 3.66°$

$\theta_4 = 7.29°$

For calculation of tension in cable

At joint 1

Using sine law,



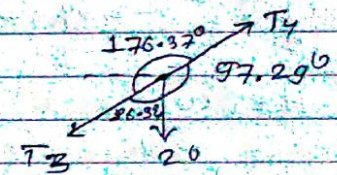
$$\frac{T_1}{\sin 88.17°} = \frac{T_2}{\sin 99.1°} = \frac{40}{\sin(172.73°)}$$

$T_1 = 316.47\ KN$

$T_2 = 312.63\ KN$

At joint 3

Using sine law,

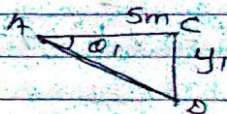


$$\frac{T_3}{\sin(97.29°)} = \frac{T_4}{\sin(86.34°)} = \frac{20}{\sin 176.37°}$$

$T_3 = 313.337\ KN$

$T_4 = 315.24\ KN$

For length of cable,



$\cos\theta_1 = \frac{5}{AD}$

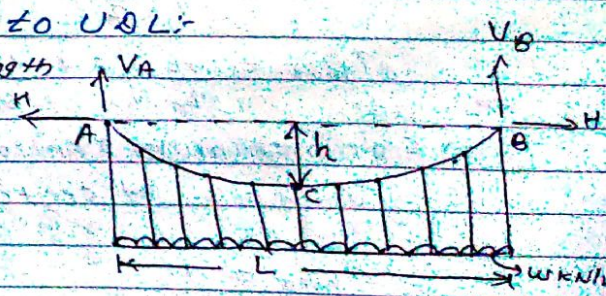
$AD = \frac{5}{\cos\theta_1}$

$L = 5\left[\frac{1}{\cos\theta_1} + \frac{1}{\cos\theta_2} + \frac{1}{\cos\theta_3}\right]$

$= 20.117m$

✻ Cable subjected to UDL:-

Consider a cable of length 'L' is subjected to udl of intensity w KN/m.



Now,

$+\circlearrowleft \Sigma M_A = 0$

$\Rightarrow -V_B \times L + w * L * \frac{L}{2} = 0$

$\Rightarrow V_B = \frac{wL}{2}$

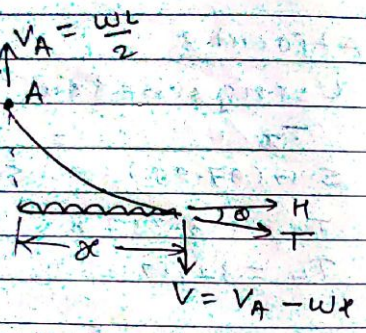
$(+\uparrow) \Sigma F_y = 0$; $V_A = \frac{wL}{2} = V_B$.

$+\circlearrowright \sum M_C = 0$ [Taking left part only]

$$V_A \times \frac{l}{2} - H \times h - \omega x \left(\frac{l}{2}\right) \times \frac{l}{4} = 0$$

$$\Rightarrow \frac{\omega l}{2} \times \frac{l}{2} - H \times h - \frac{\omega l^2}{8} = 0$$

$$\Rightarrow H = \frac{\omega l^2}{8h}$$



The tension in any point in cable is equal to resultant of V & H.

$$\therefore T = \sqrt{V^2 + H^2}$$

i) For Maximum Tension in cable,

$V_{max} = \frac{\omega l}{2}$ at support,

$$T_{max} = \sqrt{\left(\frac{\omega l}{2}\right)^2 + \left(\frac{\omega l^2}{8h}\right)^2}$$

$$T_{max} = \frac{\omega l}{2}\sqrt{1 + \frac{l^2}{16h^2}}$$

ii) For minimum Tension in cable,

$V_{min} = 0$ at centre,

$$T_{min} = H.$$

Also,

From figure, $T\cos\theta = H$ ---(i)

$T\sin\theta = V_A - \omega x$ ---(ii)

eqn (ii) / eqn (i)

$$\tan\theta = \frac{V_A - \omega x}{H}$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{\omega l}{2} - \omega x\right) \times \frac{1}{H}$$

On integration,

$$y = \left(\frac{\omega l}{2} x - \frac{\omega x^2}{2}\right) \times \frac{1}{H}$$

$$y = \frac{\omega x}{2H}(l - x)$$

Substituting value of H as, $H = \omega l^2/8h$

$$y = \frac{\omega x \times 8h}{2 \times \omega l^2}(l - x)$$

$$y = \frac{4hx}{l^2}(l - x) \quad ---(iii)$$

which is equation of parabola. Thus, the shape of cable is parabolic.

To find the length of cable in any curve,

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\simeq 1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2$$

$$= 1 + \frac{1}{2}\left[\left(\frac{\omega l}{2} - \omega x\right) \times \frac{1}{H}\right]^2$$

$$= 1 + \frac{1}{2}\left[\left(\frac{\omega l}{2} - \omega x\right) \times \frac{1 \times 8h}{\omega l^2}\right]^2$$

$$= 1 + \frac{1}{2}\left[\frac{4h}{l^2}(l - 2x)\right]^2$$

The length of cable is given by,

$$L = \int_0^l ds = \int_0^l \left\{1 + \frac{1}{2}\left[\frac{4h}{l^2}(l - 2x)\right]^2\right\} dx$$

$$= \int_0^l \left[1 + \frac{1}{2} \times \left(\frac{4h}{l^2}\right)^2 \left(l^2 - 4lx + 4x^2\right)\right] dx$$

$$= \int_0^l \left[1 + \frac{8h^2}{l^4} \left(l^2 - 4lx + 4x^2\right)\right] dx$$

$$= \left\{x + \frac{8h^2}{l^4}\left[l^2 x - 4l\frac{x^2}{2} + \frac{4x^3}{3}\right]\right\}_0^l$$

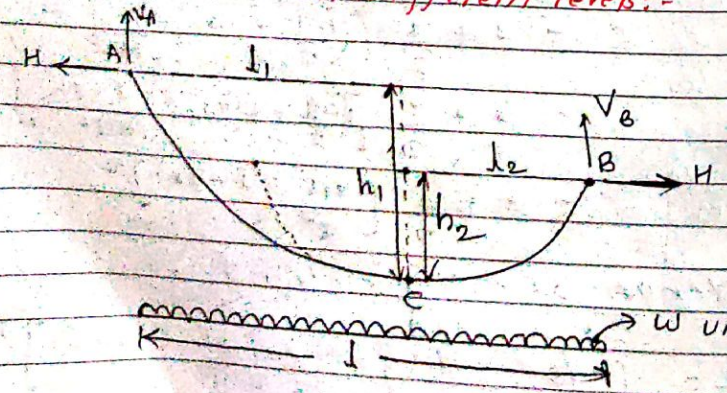
$$= l + \frac{8h^2}{l^4}\left[l^3 - \frac{4l}{2} \times l^2 + \frac{4l^3}{3}\right]$$

$$= l + \frac{8h^2}{l^4}\left[l^3 - 2l^3 + 4/3\, l^3\right]$$

$$\boxed{L = l + \frac{8h^2}{3l}}$$

where, l = length of span
L = length of cable.

Cable with ends at different levels:-



$$l_1 = \left(\frac{\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}\right) \times l \quad \& \quad l_2 = \left(\frac{\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}\right) \times l$$

When supports at same level,

length of cable, $(L) = l + \frac{8h^2}{3l}$

length of $AC = \frac{1}{2}\left[2l_1 + \frac{8h_1^2}{3 \times 2l_1}\right] = l_1 + \frac{8h_1^2}{3 \times 4l_1}$

$$= l_1 + \frac{2h_1^2}{3l_1}$$

length of $BC = \frac{1}{2}\left[2l_2 + \frac{8h_2^2}{3 \times 2l_2}\right] = l_2 + \frac{2h_2^2}{3l_2}$

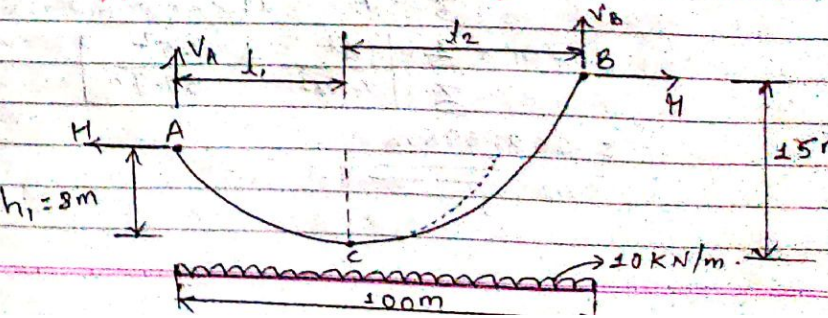
Then,

total length of cable $(L) = l_1 + l_2 + \frac{2h_1^2}{3l_1} + \frac{2h_2^2}{3l_2}$

$$= l + \frac{2}{3}\left[\frac{h_1^2}{l_1} + \frac{h_2^2}{l_2}\right]$$

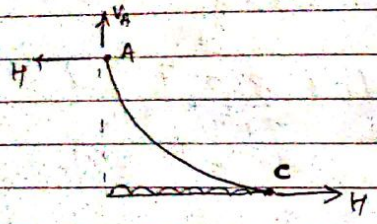
Q. A cable of span 100m has its ends at heights 8m & 15m above the lowest point of the cable. It carries a UDL of 10KN/m per unit horizontal run of the span. Determine the horizontal & vertical reactions at the support. What is the length of cable.

→



$$l_1 = \left(\frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right) l = \frac{\sqrt{8}}{\sqrt{8}+\sqrt{15}} * 100 = 42.21\,m$$

$l_2 = 100 - l_1 = 57.79\,m$



Shear force at 'C' $= V_A - 10 * 42.21$

$0 = V_A - 422.1$

$V_A = 422.1\,KN$

$(+\uparrow) \Sigma F_y = 0$

$\Rightarrow V_A + V_B - 100 \times 10 = 0$

$\Rightarrow 422.1 + V_B - 1000 = 0$

$\Rightarrow V_B = 577.9\,KN$

$\circlearrowleft \Sigma M_c = 0$ [Taking left]

$\Rightarrow V_A * 42.21 - H * 8 - 10 * 42.21 * \frac{42.21}{2} = 0$

$\Rightarrow H = 1113.55\,KN$

Length of cable,

$$L = l + \frac{8}{3}\left[\frac{h_1^2}{l_1} + \frac{h_2^2}{l_2}\right]$$

$$= 100 + \frac{8}{3}\left[\frac{8^2}{42.21} + \frac{15^2}{57.79}\right]$$

$= 103.606\,m$

$$T_{max} = \sqrt{V_A^2 + H^2}$$

$$= \sqrt{422.1^2 + 1113.55^2}$$

$= 1190.86\,KN$

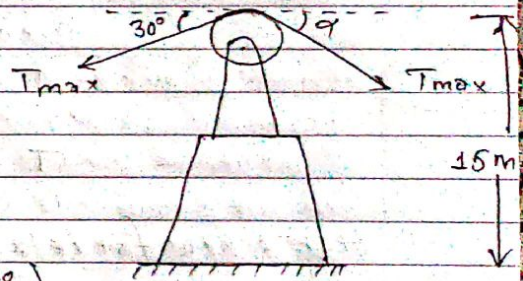
$T_{min} = \sqrt{0^2 + H^2} = H = 1113.55\,KN$

i) In case of Guided pulley support.

$$\tan\alpha = \frac{V_A}{H}$$

$$\alpha = \tan^{-1}\left(\frac{422.1}{1113.55}\right)$$

$= 20.76°$



Horizontal force on tower

$= T_{max}(\cos 20.76° - \cos 30°)$

$= 82.226\,KN\ (\longrightarrow)$

Vertical force on tower

$= T_{max}\sin\alpha + T_{max}\sin 30°$

$= T_{max}(\sin\alpha + \sin 30°)$

$= 1190.86(\sin 20.76° + \sin 30°)$

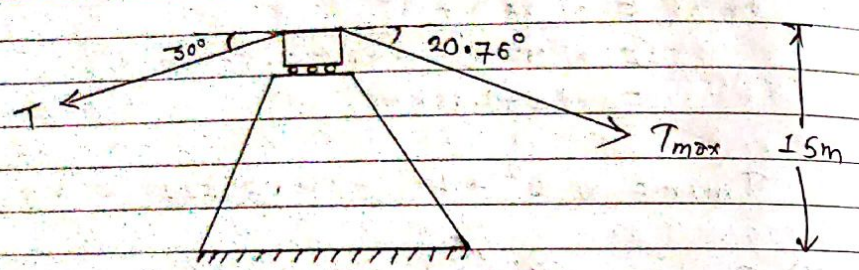
$= 1017.535\,KN\ (\downarrow)$

Maximum bending moment on tower

$= 82.226 \times 15$

$= 1233.39\,KNm$ ↺

ii) In case of Roller support, (saddle)



∴ In case of saddle,

$T \cos 30° = T_{max} \cos 20.76°$

$T = 1190.86 * \frac{\cos 20.76°}{\cos 30°}$

$T = 1285.807\ m$

Horizontal force = 0

Vertical force = $T \sin 30° + T_{max} \sin 20.76°$

$= 1285.807 \sin 30° + 1190.86 \sin 20.76°$

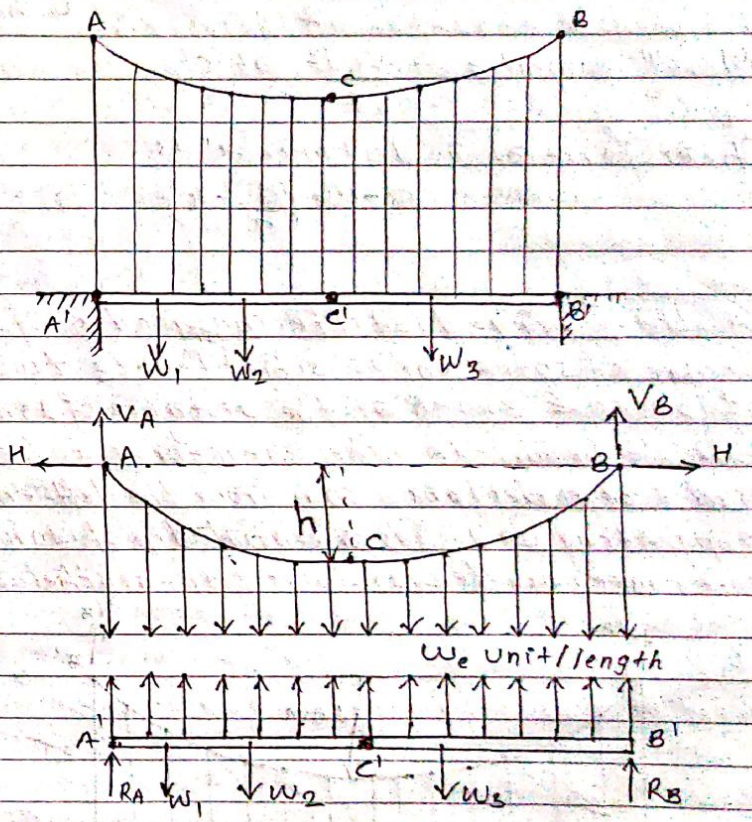
$= 1065\ KN\ (\downarrow)$

Moment at bottom of cable = H * 15

= 0

#

* Suspension bridge with three hinged stiffening Girder.



The girder can be a heavy beam or a truss which has three hinges two at the ends & one at the centre. The cable & girder are connected by a number of hangers or suspenders. Since, the number of suspenders are very large, the load on cable & girder due to forces in the suspenders, may be taken as uniformly distributed load. Let, it be We per unit horizontal length.

Bending moment at a distance 'x' is,

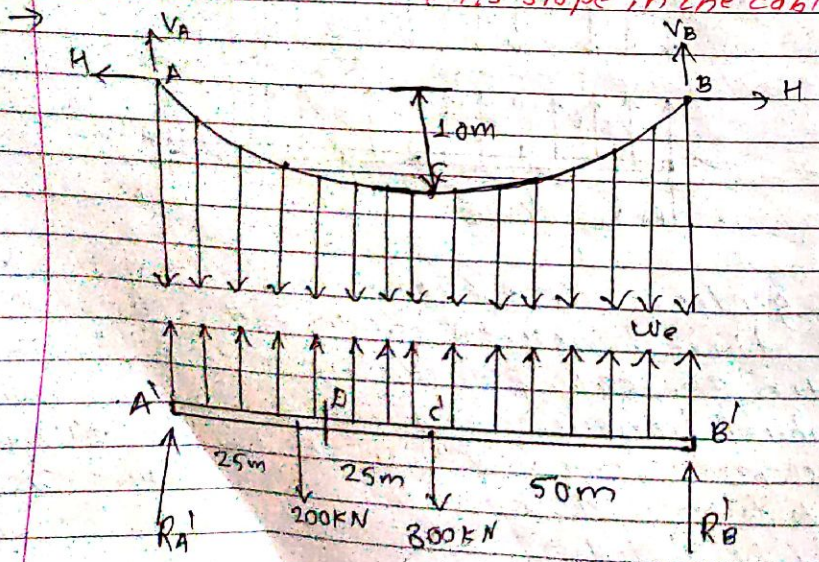
$$= -\frac{We\,x}{2}(l-x)$$ [-ve for hogging moment]

Max^m Moment $= -\frac{We\,l^2}{8}$ at 'C'.

Shear force at a distance 'x' is,

$$= -We\left(\frac{l}{2} - x\right)$$

Q A three hinged stiffening girder of a suspension bridge of span 100m is subjected to two point loads of 200KN & 300KN at the distance of 25m & 50m from the left end. Find the shear force & bending for the girder at a distance 30m from the left end. The supporting cable has a central dip of 10m. Find also the maximum tension & its slope in the cable.



$+\circlearrowleft \Sigma M_A = 0$

$200\times25 + 300\times50 - R_B' \times 100 = 0 \quad \Rightarrow R_B' = 200\,KN$

$R_A' = 300\,KN$

$+\circlearrowleft \Sigma M_C = 0$

$\Rightarrow -\frac{We\,x}{2}(l-x) + R_A' \times 50 - 200\times25 = 0$

$\Rightarrow -\frac{We\times50}{2}[100-50] + R_A'\times50 - 200\times25 = 0, \quad We = 8\,KN/m$

Shear force at D,

$$V_D = -We\left(\frac{l}{2} - x\right) + R_A' - 200$$

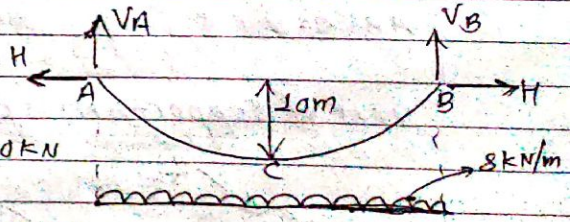
$$= -8(50-30) + 300 - 200$$

$$= -60\,KN$$

BM at D,

$$(BM)_D = -\frac{We\,x}{2}(l-x) + R_A' \times 30 - 200\times5$$

$$= -400\,KNm$$



Here,

$$V_A = \frac{wL}{2} = \frac{8\times100}{2} = 400\,KN$$

$$H = \frac{wL^2}{8h} = \frac{8\times100^2}{8\times10} = 1000\,KN$$

$$T_{max} = \sqrt{V_A^2 + H^2} = \sqrt{400^2 + 1000^2}$$

$$= 1077.032\,KN$$

$$\theta = \tan^{-1}\left(\frac{V_A}{H}\right) = 21.80^\circ$$

Ch-8

Space Truss:-

A space truss is one whose members do not lie in the same plane. When several members are joined together at their extremities to form a three dimensional configuration, we obtain a space truss.

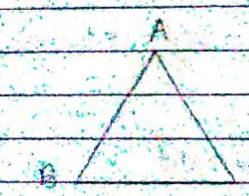


Fig:- 2D truss

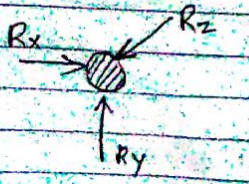


Fig:- 3D truss

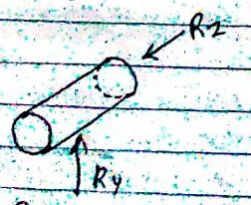
→ An elementary 2D truss consists of three members joined at their extremities to form the sides of triangle.

→ An elementary 3D space truss consists of six members joined at their extremities to form edge of tetrahedron ABCD.

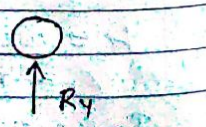
Types of Supports in space truss:-



Ball & Socket support.



Roller support



Ball support

The ball & socket support prevents the movement of support in each of the three directions while roller & ball supports prevent it in two & one dirns respectively.

* Analysis of space truss:-

The equilibrium of an entire space truss or its section is given by the following six eqns.

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$

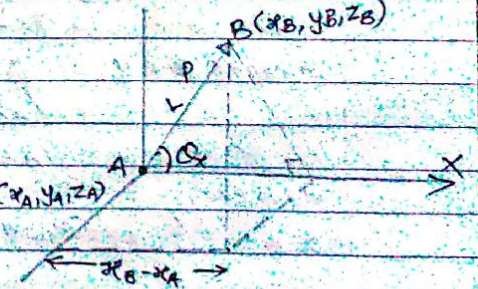
$\Sigma M_x = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$

In simple space truss, the analysis can be started at a joint where only three members are connected.

Th: At a joint having only three members not in are plane & where external forces are zero, the forces in all member meeting at that joint must be zero.

* Tension coefficient method:-

It is the method of finding member force in space truss. Let, $A(x_A, y_A, z_A)$ be the origin, then, the force 'p' is acting along AB which can be resolved into x, y & z dirns.



$$P_x = p\cos\theta_x = \frac{P}{L}(x_B - x_A) = t_{AB}(x_B - x_A)$$

$$\left[\cos\theta_x = \frac{x_B - x_A}{L},\ \text{where } L \text{ is length of line AB}\right]$$

$$P_y = p\cos\theta_y = \frac{P}{L}(y_B - y_A) = t_{AB}(y_B - y_A)$$

$$P_z = p\cos\theta_z = \frac{P}{L}(z_B - z_A) = t_{AB}(z_B - z_A)$$

Similarly,

$P_x = t_{AC}(x_C - x_A)$, $P_y = t_{AC}(y_C - y_A)$ & $P_z = t_{AC}(z_C - z_A)$

$R_x = t_{AD}(x_D - x_A)$, $R_y = t_{AD}(y_D - y_A)$, $R_z = t_{AD}(z_D - z_A)$

Then,

$\Sigma F_x = 0$

$\Rightarrow t_{AB}(x_B - x_A) + t_{AC}(x_C - x_A) + t_{AD}(x_D - x_A) + F_x = 0$ -- (I)

$\Sigma F_y = 0$

$\Rightarrow t_{AB}(y_B - y_A) + t_{AC}(y_C - y_A) + t_{AD}(y_D - y_A) + F_y = 0$ -- (II)

$\Sigma F_z = 0$

$\Rightarrow t_{AB}(z_B - z_A) + t_{AC}(z_C - z_A) + t_{AD}(z_D - z_A) + F_z = 0$ -- (III)

On solving eqn (I), (II) & (III), we get value of, t_{AB}, t_{AC} & t_{AD}.

Then,

$$t_{AB} = \frac{T_{AB}}{L_{AB}} \Rightarrow T_{AB} = t_{AB} \times L_{AB}$$

Similarly,

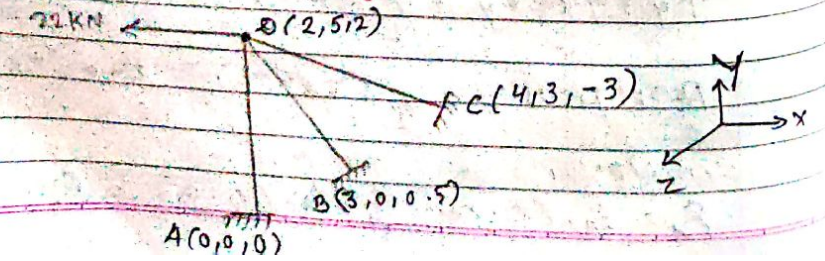
$T_{AC} = t_{AC} \times L_{AC}$

$T_{AD} = t_{AD} \times L_{AD}$.

Where,

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Q Analyses the member forces in the three bar space truss shown in figure using tension coefficient method.



→ Here, A(0,0,0), B(3,0,0.5), C(4,3,-3) & D(2,5,2)

	t_{DA}	t_{DB}	t_{DC}	F
x	-2	1	2	-22
y	-5	-5	-2	0
z	-2	-1.5	-5	0

Eqns

$$\left.\begin{aligned} -2t_{DA} + t_{DB} + 2t_{DC} - 22 = 0 \\ -5t_{DA} - 5t_{DB} - 2t_{DC} = 0 \\ -2t_{DA} - 1.5t_{DB} - 5t_{DC} = 0 \end{aligned}\right\}$$

On solving above eqns,

$t_{DA} = -6.914$ $t_{DB} = 6.6$ $t_{DC} = 0.785$

Then,

$L_{DA} = \sqrt{(0-2)^2 + (0-5)^2 + (0-2)^2} = 5.744$ m

$L_{DB} = \sqrt{(3-2)^2 + (0-5)^2 + (0.5-2)^2} = 5.315$ m

$L_{DC} = \sqrt{(4-2)^2 + (3-5)^2 + (-3-2)^2} = 5.744$ m

Now,

$T_{DA} = t_{DA} \times L_{DA} = -6.914 \times 5.744 = -39.714$ KN

$T_{DB} = t_{DB} \times L_{DB} = 6.6 \times 5.315 = 35.079$ KN

$T_{DC} = t_{DC} \times L_{DC} = 0.785 \times 5.744 = 4.51$ KN

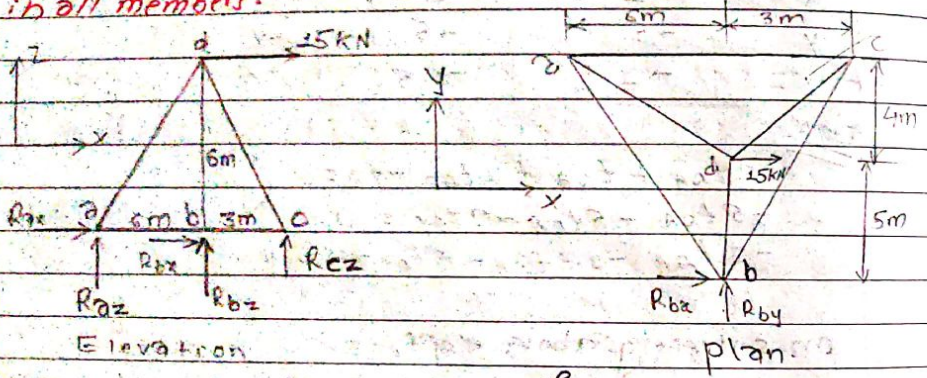
∴ Force in Member,

AD = 39.714 KN (C)

BD = 35.079 KN (T)

CD = 4.51 KN (T)

Q. Pin jointed space truss is shown in figure. Joint 'a', 'b' & 'c' are on horizontal plane & joint 'd' is at a height of 6m from horizontal plane. Calculate force in all members.



→ At joint a, only three member force are unknown.

Take,

a (0,0,0), b (6, ~~-9~~ -9, 0), c (9, 0, 0), d (6, -4, 6)

Then,

At joint d

	t_{da}	t_{db}	t_{dc}	F
x	-6	0	3	15
y	4	-5	4	0
z	-6	-6	-6	0

Eqns

$$-6t_{da} + 0t_{db} + 3t_{dc} + 15 = 0$$
$$4t_{da} - 5t_{db} + 4t_{dc} = 0$$
$$-6t_{da} - 6t_{db} - 6t_{dc} = 0$$

on solving,

$t_{da} = 1.67$ $\quad t_{db} = 0$ $\quad t_{dc} = -1.67$

$L_{da} = \sqrt{36+16+36} = 9.38\,m$

$L_{db} = \sqrt{0+25+36} = 7.81\,m$

$L_{dc} = \sqrt{9+16+36} = 7.81\,m$

$T_{da} = 1.67 \times 9.38 = 15.66\,KN$

$T_{db} = 0$ & $T_{dc} = -1.67 \times 7.81 = -13.04\,KN$

At joint c

	t_{ca}	t_{cd}	t_{cb}	F
x	-9	-3	-3	0
y	0	-4	-9	~~Rby~~ 0
z	0	6	0	R_{cz}

eqns

$$-9t_{ca} - 3 \times (-1.67) + t_{cb} = 0$$
$$-9t_{ca} + t_{cb} = -5.01 \quad \text{——— (1)}$$

~~$-9(-1.67) - 9t_{cb} = R_{by}$~~ (2)

~~$0t_{ca} + 0t_{cb} = -10.02$~~ (3)

~~On solving~~ ×

$$0t_{ca} - 4t_{cd} - 9t_{cb} = 0$$
$$-4 \times (-1.67) = 9t_{cb} \Rightarrow t_{cb} = 0.742\,KN/m$$
$$R_{cz} = 6 \times (-1.67) = -10.02\,KN$$

From (1), $t_{ca} = 0.639$

Then,

$L_{ca} = \sqrt{(-9)^2+0^2+0^2} = 9\,m$

$L_{cb} = \sqrt{(-3)^2+(-9)^2} = 9.486\,m$

$T_{ca} = 0.639 * 9 = 5.751\,KN$ $\quad T_{cb} = 9.486 \times 0.742 = 7.033\,KN$

At joint a

	t_{ab}	t_{ad}	t_{ac}	F
x	6	6	9	R_{ax}
y	-9	-4	0	0
z	0	6	0	R_{az}

Eqns

$-9t_{ab} - 4 * 1.67 = 0$

$t_{ab} = -0.742$

$R_{az} = 6 * 1.67 = 10.02\ kN$

$6 \times (-0.742) + 6 \times 1.67 + 9 \times 0.639 = R_{ax}$

$\Rightarrow R_{ax} = 11.319\ kN$ | $L_{ab} = \sqrt{36+81} = 10.816\ m$

$T_{ab} = -8.026\ kN$

Now,

$(+\rightarrow) \Sigma F_x = 0$

$\Rightarrow 15 + R_{ax} + R_{bx} = 0$

$\Rightarrow R_{bx} = -15 - 11.319 = -26.319\ kN$

$(+\uparrow) \Sigma F_y = 0$

$\Rightarrow R_{by} = 0$

$(+\uparrow) \Sigma F_z = 0$

$\Rightarrow R_{az} + R_{bz} + R_{cz} = 0$

$\Rightarrow 10.02 - 10.02 + R_{bz} = 0 \Rightarrow R_{bz} = 0.$

∴

Member	Memberforce(kN)
ab	8.026 (C)
bc	7.038 (T)
cd	13.04 (C)
da	15.66 (T)
db	0
ca	5.751 (T)

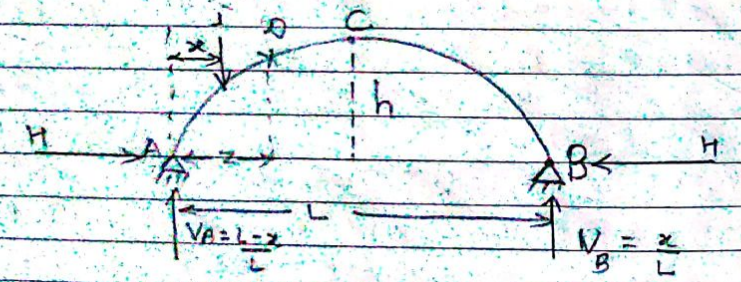
Support Rxn
$R_{ax} = 11.319\ kN$
$R_{bx} = -26.319\ kN$
$R_{by} = 0$
$R_{az} = 10.02\ kN$
$R_{bz} = 0$
$R_{cz} = -10.02\ kN$

ILD for arch:-

Consider the three hinged arch of span 'L' & rise 'h' shown in figure. The influence line diagram for following case will be discussed

1) Horizontal thrust, H
2) Moment at section 'D'
3) Normal thrust at 'D'
4) Radial shear at 'D'

Where, D is a point at a distance 'z' from left support 'A'.



Let, x be the distance of unit load from support at A.

ILD for H:

When unit load moves from A to C,

$\Sigma M_C = 0$ [Right part]

$H \times h - \frac{x}{L} \times \frac{L}{2} = 0$

$H = x/2h$

At A, $x = 0$, $H = 0$

At C, $x = L/2$, $H = L/4h$

When unit load moves from C to B,

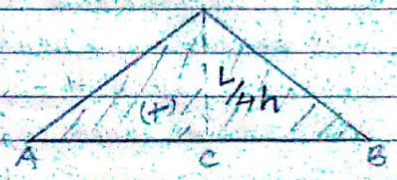
$\Sigma M_C = 0$ [Left part]

$\frac{L-x}{L} \times \frac{L}{2} - H \times h = 0$

$H = \frac{L-x}{2h}$

At C, $x = L/2$, $H = L/4h$

At B, $x = L$, $H = 0$.

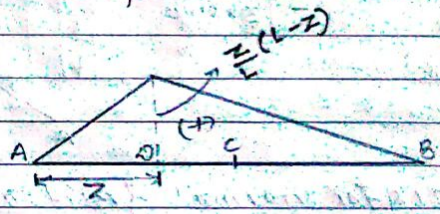


ILD for H.

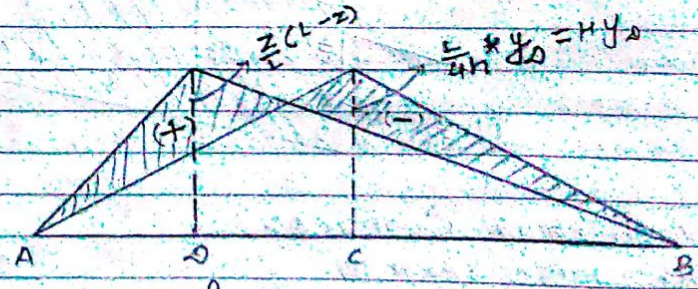
ILD for M_D

BM in arch = Beam moment − Hy

The ILD for beam for moment at D is,



The ILD for moment at D in arch can be obtained by subtracting ILD for beam horizontal thrust * y from beam moment ILD.

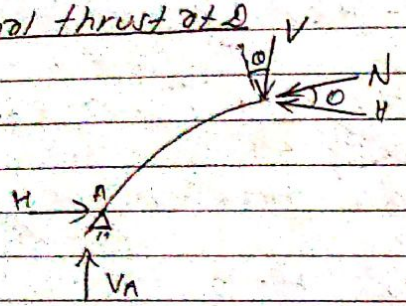


ILD for M_D.

$\frac{L}{4h} * y_D = \frac{L}{4h} * \frac{4hz}{L^2}(L-z) = \frac{z}{L}(L-z)$

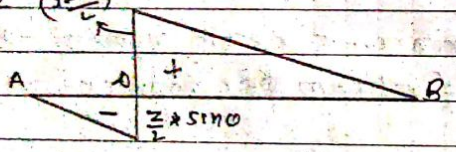
The ordinate for Hy_D is equal to beam moment ordinate at D.

ILD for normal thrust at D

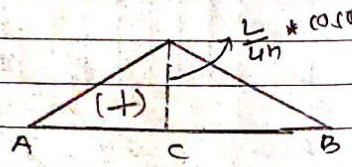


The Normal thrust is, $N_D = H\cos\theta + V\sin\theta$.

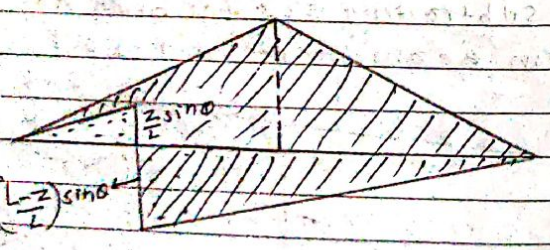
ILD for $V\sin\theta$



ILD for $H\cos\theta$



To Obtain ILD for normal trust at D (N_D), we have to combined the ILD for $V\sin\theta$ & $H\cos\theta$.



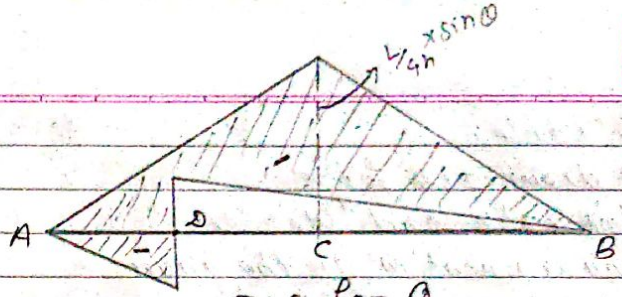
ILD for N_D

ILD for radial shear at D:-

The radial shear is,

$$Q_D = V\cos\theta - H\sin\theta$$

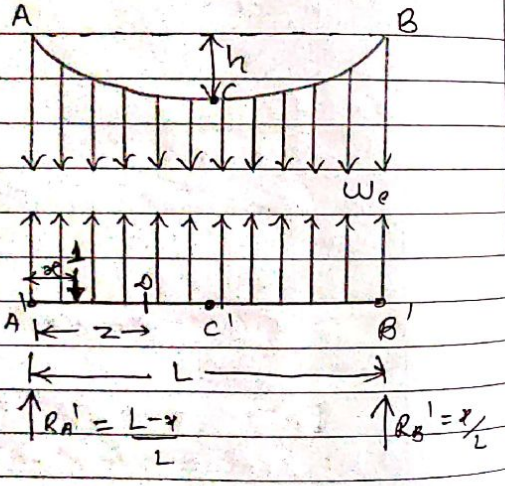
ILD for Q_D can be obtained by subtracting ILD for $H\cos\theta$ from $V\sin\theta$.



ILD for Q_D

ILD for cable:

We will draw ILD for
i) Equivalent cable load 'We'
ii) Bending moment 'M' in the girder.
iii) Horizontal force in the cable
iv) Shear force in cable.



i) ILD for We:

When unit load moves from A' to C',

$\Sigma M_{C'} = 0$ [Right part]

⇒ Moment due to unit load + Moment due to $w_e = 0$

⇒ $\frac{x}{1} \times \frac{1}{2} - \frac{w_e l^2}{8} = 0$

⇒ $w_e = \frac{4x}{l^2}$

At A', $x = 0$, $w_e = 0$

At C', $x = l/2$, $w_e = 2/L$

When unit load moves from C' to B',

$\Sigma M_{B'} = 0$ [Left part]

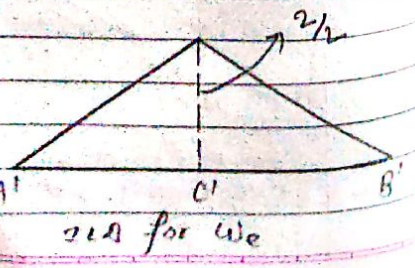
⇒ Moment due to unit load + Moment due to $w_e = 0$

⇒ $\frac{1-x}{1} \times \frac{1}{2} - \frac{w_e \times l^2}{8} = 0$

⇒ $w_e = \frac{4(1-x)}{l^2}$

At C', $x = l/2$, $w_e = 2/L$

At B', $x = L$, $w_e = 0$.

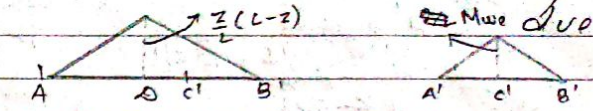


ILD for We

ii) ILD for MD

MD = ~~BM due~~ Moment due to unit load + Moment due to We

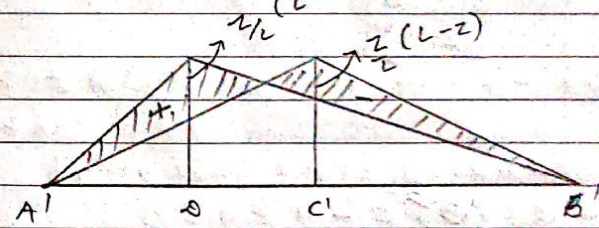
ILD for MD = ILD for moment due to unit load + ILD for moment due to We.



Moment due to $w_e = -\frac{w_e * x (l-x)}{2}$

At D, $x = z$, so, $M_{w_e} = -\frac{w_e * z (l-z)}{2}$

$= -\frac{x}{L} * \frac{z}{x} (l-z) = \frac{-z}{L}(L-z)$

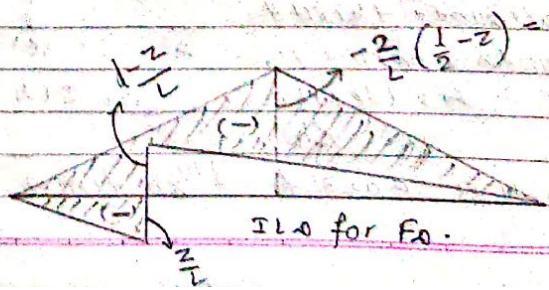


ILD for MD

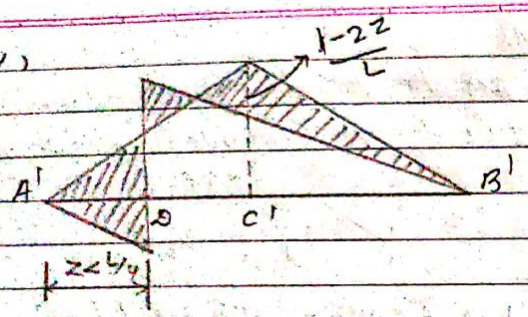
iii) ILD for shear force at D, (F_D)

V_D = SF due to unit load + SF due to We

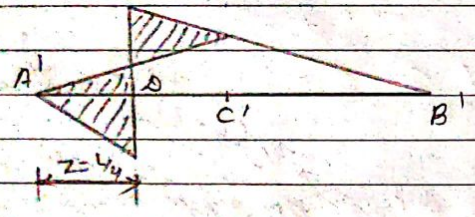
SF at D due to $w_e = -w_e\left(\frac{1}{2} - z\right)$ → $-w_e\left(\frac{1}{2} - x\right)$



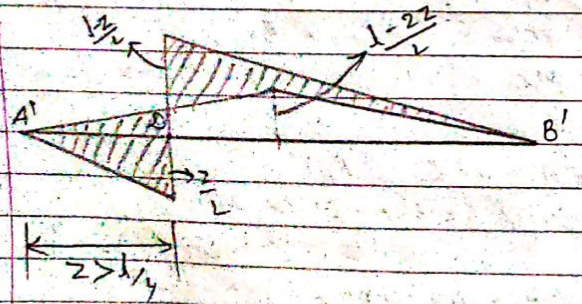
i) $z < l/4$,



ii) $z = l/4$



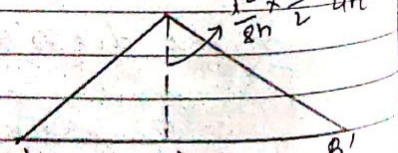
iii) $z > l/4$.



iv) ILD for Horizontal thrust H.

The horizontal thrust is,

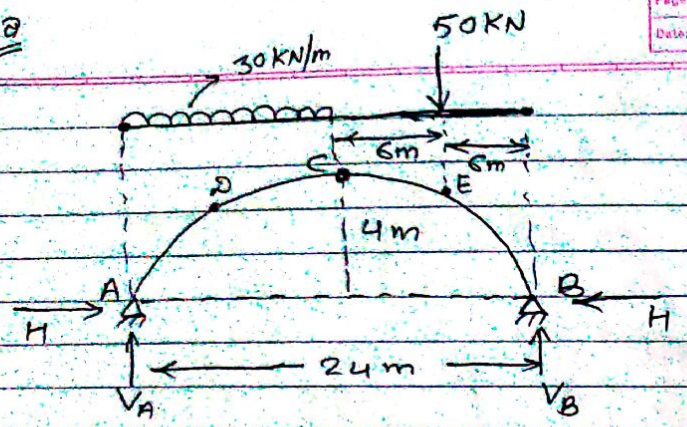
$$H = \frac{W \times l^2}{8h}$$



ILD for H.

The ILD for H is equal to $l^2/8h$ times. ILD for w

2013 Fall 5a



Calculation of rxn :-

$+\circlearrowright \Sigma M_A = 0$

$\Rightarrow 30\times12\times6 + 50\times18 - V_B\times24 = 0 \Rightarrow V_B = 127.5\,KN$

$(+\uparrow)\Sigma F_y = 0$

$\Rightarrow V_A - 30\times12 - 50 + 127.5 = 0 \Rightarrow V_A = 282.5\,KN$

$+\circlearrowright \Sigma M_C = 0$ [Taking right part only]

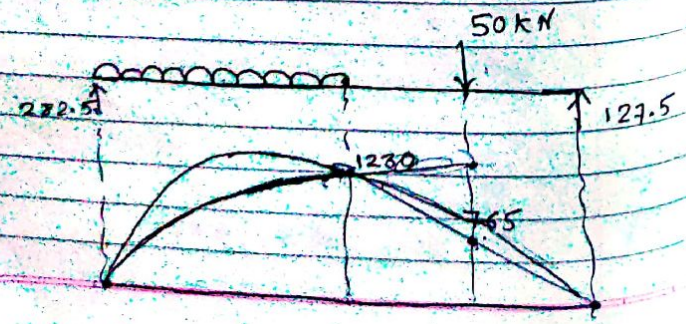
$\Rightarrow 282.5\times12 - 30\times12\times6 - H\times4 = 0$

$\Rightarrow H = 307.5\,KN$

We know,

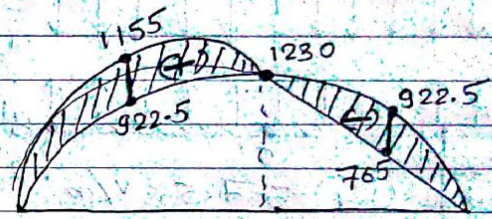
Arch Moment = Beam moment − Hy

1) BMD for Beam,



$y_D = y_E = \frac{4\times4\times6}{24^2}(24-6) = 3m$

$Hy_D = Hy_E = 3\times307.5 = 922.5\,KNm$



$M_D = 1155 - 922.5$

$= 232.5\,KNm$

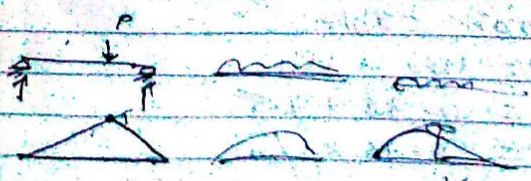
$M_E = 922.5 - 765 = 157.5\,KNm$

Check

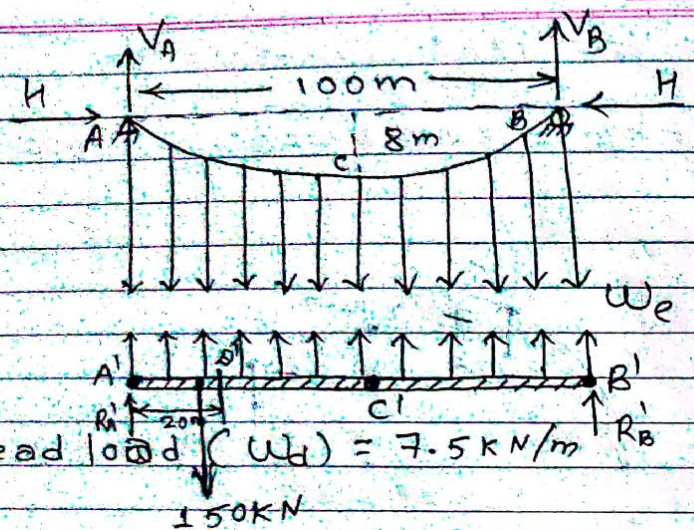
$M_D = 282.5\times6 - 30\times6\times3 - 307.5\times3$

$= 232.5\,KNm$

$M_E = 127.5\times6 - 307.5\times3 = -157.6$



6(a)



$+\circlearrowleft \Sigma M_{A'} = 0$

$150 \times 16 - R_B' \times 100 = 0 \Rightarrow R_B' = 24\,kN$

$R_A' = 126\,kN$

$+\circlearrowleft \Sigma M_C' = 0$ [Taking left only]

BM due to given loading + BM due to w_e = 0

$\Rightarrow 126 \times 50 - 150 \times (50-16) - \frac{w_e \times x}{2}(1-x) = 0$

$\Rightarrow 1200 - \frac{w_e \times L}{4} \times \frac{L}{2} = 0$

$\Rightarrow w_e = 0.96\,kN/m$

Total UDL on main cable

$w = w_e + w_d$

$= 0.96 + 7.5$

$= 8.46\,kN/m$

$(SF)_{20m\,from\,left} = R_A' - 150 - w_e\left(\frac{l}{2} - x\right)$

$= 126 - 150 - 8.46 \times (50 - 20)$

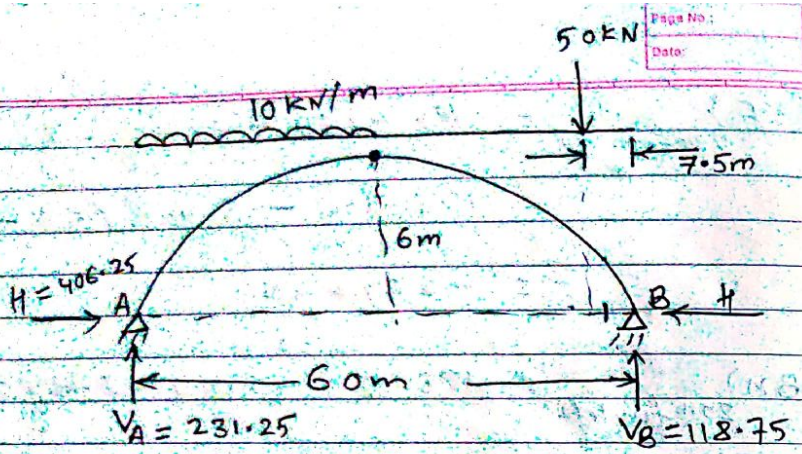
$= -277.8\,kN$

$(BM)_{20m\,fm\,left} = 126 \times 20 - 150 \times 4 - \frac{8.46 \times 20}{2}(100-20)$

$= -4848\,kNm$

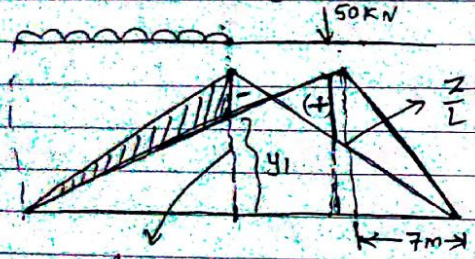
2351156

4 (a)



Arch Moment = Beam moment − Hy

i)



$\frac{z}{L}(L-z) = \frac{53}{60}(60-53)$

$= 6.183$

$\frac{L}{4h} \times y = 6.183$

From similar Δ,

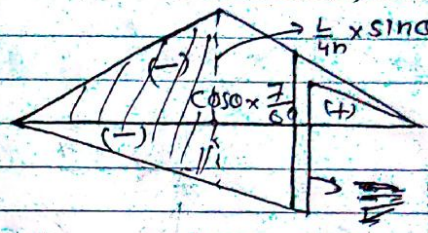
$y_1 = 6.183 \times \frac{30}{53} = 3.4998$

∴

$BM = -10 \times \left(\frac{1}{2} \times 30 \times (6.183 - 3.4998)\right) + 50 \times \left[\frac{6.183 \times 52.5}{53} - 6.183 \times \frac{17.5}{30}\right]$

$= -173.534$ kNm

ii) Radial shear (Q) = $V\cos\theta - H\sin\theta$

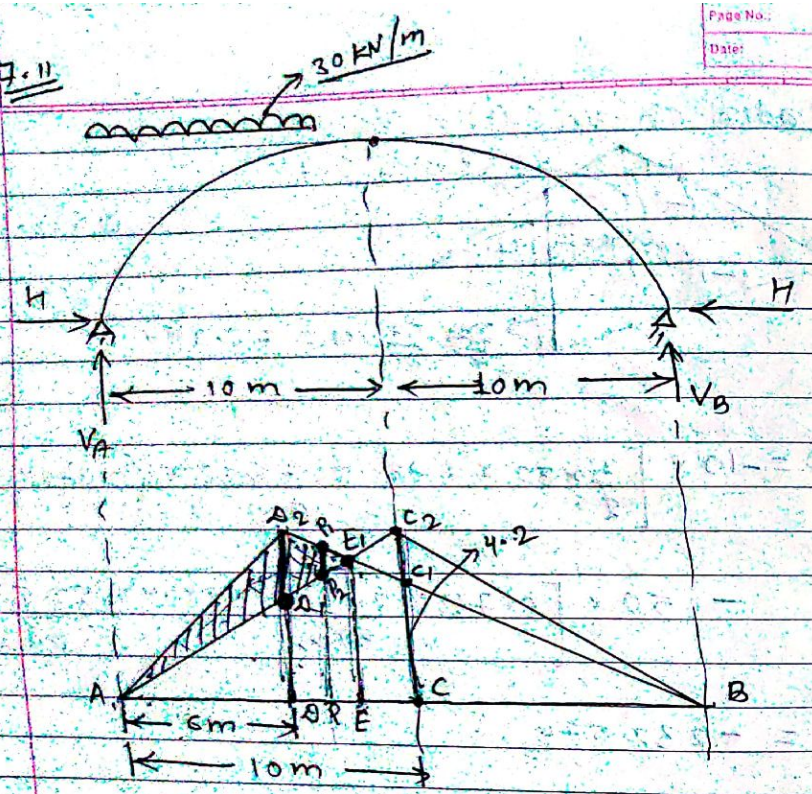


$\theta = 17.05°$

$Q = -10 \times \left[\frac{1}{2} \times 30 \times 0.733 + \frac{1}{2} \times 30 \times \left(0.844 \times \frac{30}{53}\right)\right] - 50 \times \left[0.733 \times \frac{7.5}{30} + 0.844 \times \frac{52.5}{53}\right]$

$= -232.574$

Book 7.11



$$\frac{EE_1}{AE} = \frac{CC_2}{AC} = \frac{DD_2}{BD}$$

$$\Rightarrow \frac{EE_1}{AE} = \frac{4.2}{10} = 4.2$$

$$\Rightarrow EE_1 = CC_2 \times \frac{AE}{AC} = DD_2 \times \frac{BE}{BD}$$

$$\Rightarrow \frac{AE}{AC} = \frac{BE}{BD}$$

$$\Rightarrow \frac{AE}{10} = \frac{BE}{14}$$

$$\Rightarrow \frac{AE}{BE} = \frac{10}{14}$$

$$\Rightarrow \frac{AE}{20 - AE} = \frac{10}{14}$$

$$\Rightarrow 14AE = 200 - 10AE$$

$$\Rightarrow 24AE = 200$$

$$\Rightarrow AE = 8.33 m$$

$$BM = 30 \times \left[\frac{1}{2} \times 6 \times 4.2 - \frac{1}{2} \times 6 \times 4.2 \times \frac{6}{10}\right] + 30 \times [\text{Area of trapezium}]$$

$$\text{Area of trapezium} = \left(\frac{1}{2} \times 14 \times 4.2 - \frac{1}{2} \times 12 \times 4.2 \times \frac{12}{14}\right) - \left(\frac{1}{2} \times 8 \times 4.2 \times \frac{8}{10} - \frac{1}{2} \times 6 \times 4.2 \times \frac{6}{10}\right)$$

$$= 1.92$$

~~BM =~~

$$\Delta AD_2B = \frac{1}{2} \times 20 \times 4.2 = 42 m^2$$

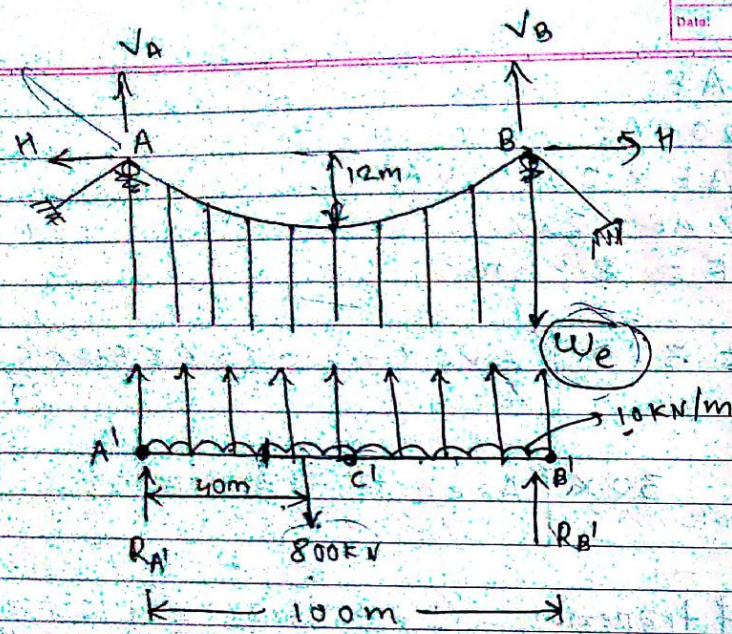
$$\Delta AE_1B = \frac{1}{2} \times 20 \times \left(4.2 \times \frac{8.33}{10}\right) = 34.986 m^2$$

$$PP_1 = 4.2 \times \frac{12}{14} = 3.6m \qquad PP_2 = 4.2 \times \frac{8}{10} = 3.36m$$

$$P_1P_2 = 0.24m$$

$$\text{Area of } \Delta P_1P_2E_1 = \frac{1}{2} \times 0.24 \times 0.33 = 0.0396 m^2$$

$$\text{Req}^d \text{ Area} = 42 - 34.986 - 0.0396 = 6.9744 m^2$$



$+\circlearrowright \Sigma M_{B'} = 0$

$\Rightarrow 800 \times 40 + 10 \times 100 \times 50 - R_{B'} \times 100 = 0$

$R_{B'} = 820\,KN$

$R_{A'} = 980\,KN$

$+\circlearrowright \Sigma M_{C'} = 0$ [Taking ~~right~~ left part only]

$\Rightarrow R_{A'} \times 50 - 800 \times 10 - \frac{w_e \times 100^2}{8} - 10 \times 50 \times 25 = 0$

$\Rightarrow 980 \times 50 - 8000 - \frac{w_e \times 100^2}{8} = 0$

$\Rightarrow w_e = 22.8\,KN/m$

$V_A = V_B = \frac{wL}{2} = \frac{22.8 \times 100}{2} = 1140\,KN$

$H = \frac{wl^2}{8h} = \frac{22.8 \times 100^2}{8 \times 12} = 2375\,KN$

$T_{max} = \sqrt{V_A^2 + H^2} = 2634.43\,KN$

Maximum shear force occurs at support.

$V_{max} = 1140\,KN.$

$(SF)_{30m} = R_{A'} - 10 \times 30 - w_e\left(\frac{1}{2} - x\right)$

$= 980 - 300 - 22.8\,(100 - 30)$

$= -916\,KN$

$(BM)_{30m} = R_{A'} \times 30 - 10 \times 30 \times 15 - \frac{w_e \times 30}{2}(100 - 30)$

$= 960\,KNm$

$\sigma_{allow} = 120\,N/mm^2$

$= 120 \times 10^{-3}\,KN / 10^{-6}\,m^2$

$= 120 \times 10^3\,KN/m^2$

$\sigma_{allow} \geq \frac{T_{max}}{Area} = \frac{2634.43}{\frac{\pi \times d^2}{4}} = 120 \times 10^3$

$d = 0.1671\,m = 167.1\,mm$