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# Soil Mechanics:-

## Ch 1: Introduction:-

### ✻ Defⁿ of soil and Rocks:-

→ Soil is defined as an unconsolidated material which comprises of solid particles & voids. The void may comprise of either air only or water only or both air and water. If the voids are totally occupied by water, then soil is said to be saturated soil and if the voids are totally occupied by air, then the soil is said to be dry soil.

↳ Soil is used as a construction material in various civil engineering project and it supports structural foundations.

↳ Rock is defined as natural aggregate of mineralogical composition bounded by strong & permanent cohesive force.

### ✻ Definition of soil Mechanics & its importance in Civil Engineering:-

→ Soil Mechanics is the branch of science that deals with study of physical properties of soil & behaviour of soil masses subjected to various types of forces.

→ Geotechnical engineering is the sub-discipline of ~~soil engineering~~ civil engineering that includes soil engineering, rock mechanics & geology.

## * Importance of soil Mechanics in Civil Engineering

1) In the field of foundation engineering, as it helps to select different types of foundation according to condition & properties of soil.

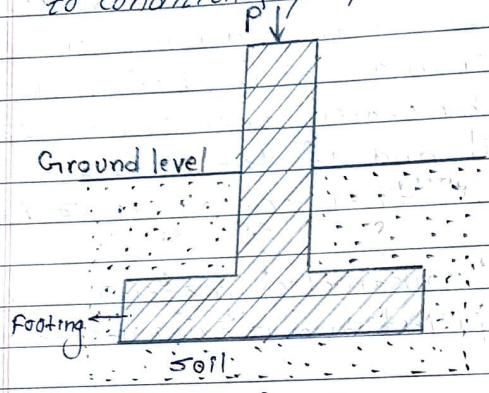



Fig:- shallow foundation

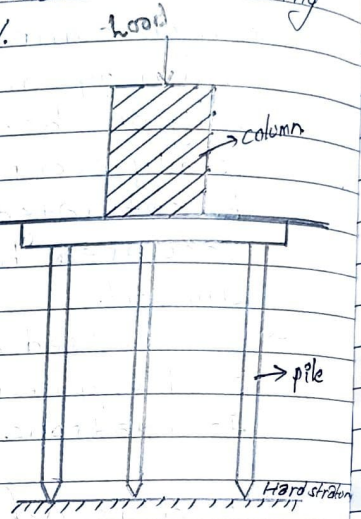
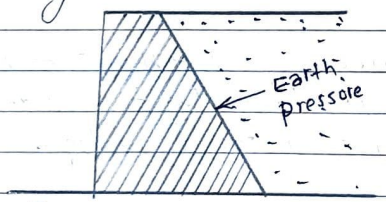
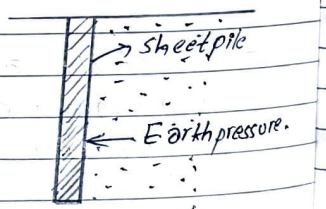



Fig:- pile foundation.

The types of foundation during construction depends on
i) Type of soil strata (seepage, swelling & bearing capacity of soil)
ii) Magnitude of load
iii) Ground water condition

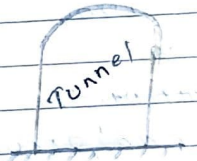
2) Soil Mechanics gives the theories of earth pressure on retaining structures.

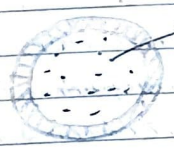



3) Soil mechanics provide the method for checking the stability of slope. If soil surface is not horizontal, there is a component of weight of soil which tends to move it downward & causes instability of slope.
(Excavation slope & embankment slope)

4) It helps to design & construct the underground structures such as tunnels, shafts, conduits etc because soil mechanics gives the value of force exerted by soil on these structures.




5) Soil Mechanics give the idea about design of pavement

6) In the field of construction of earthen dam as it helps in the computation of permeability, seepage force being exerted on soil mass.

7) Miscellaneous soil problems related to soil such as soil heave, soil subsidence, frost heave, shrinkage & swelling of soils etc has been tackled by geotechnical engineering during construction phase. Soil Mechanics gives the idea about these problems.

## ✻ Formation process of soils and its major types.

✻ Formation of soils:-
The soils are formed by disintegration of rocks either physical disintegration or chemical disintegration.

(a) physical disintegration:-
Mechanical disintegration of rocks into small fragments due to different actions of geological agents is called physical disintegration. It occurs due to following physical processes.

a) Temperature change
Due to different coeff of thermal expansion, expansion & contraction of mineral occurs due to change in temp.

(b) Wedging action of ice:-
Water in the pores and minut cracks of rocks gets frozen in very cold climates. The disintegration of rocks takes place due to increase in volume.

(c) Spreading of roots of plants:-
when the roots of trees grow up inside crack & fissures, force acts on rock & break down.

(d) Abrasion:-
Due to abrasion, the rocks are disintegrated into soils. Abrasion is due to movement of geological agent over surface of rocks.

Note:- The soil formed has the properties of parent rock (In physical disintegration)

(b) Chemical disintegration:-
↳ Chemical disintegration / decompasition of rocks due to different chemical process such as hydration, carbocation, Oxidation, solution, hydrolysis etc.
↳ The soil formed donot have properties of parent rocks.

(1) Hydration:-
Water combine with rock & result in formation of new chemical compounds.

(2) Carbocation:-
Carbondioxide combines with water to form carbonic acid. This carbonic acid combines with rocks & disintegrate the rock into soil mass.

(3) Oxidation:-
Oxygen ions combine with rock minerals.
Eg:- Formation of rust.

(4) Solution:-
Some of the rock minerals form a solution with water when they get dissolved in water. Chemical rxn takes place in the solution & soils are formed.

(5) Hydrolysis:-
In this process, water gets dissociated into $H^+$ and $OH^-$ ions.

## ✻ Types of Soils:-

→ On the basis of formation of soils, it can be divided into residual soils & transported soils.

### (i) Residual Soils:-

↳ The soils which remain at the place of formation of soils after disintegration of rocks.

↳ The characteristics of residual soil is similar to parents rock characteristics.

### (ii) Transported soils:-

↳ The soils transported at a far distance from place of formation due to action of different geological agent such as wind, water, glacier etc.

↳ The characteristics of transported soil is different from parent rock characteristics.

↳ Soil deposits after transportation by geological agents.

#### a) Alluvial deposit:-

↳ Soil deposit due to running action of water inside river.

#### b) Lacustrine deposit:-

↳ soil deposit at lakes.

#### c) Marine deposit:-

↳ soil deposit at sea

#### (d) Aeolian deposit:-

↳ soils deposit due to action of wind in desert area.

#### (e) Glacial deposit:-

→ Soil deposit due to geological action of glacier.

#### (f) Colluvial Soil:-

↳ Soil deposits due to action of gravity.

# Chapter-6 :-

## Principle of Effective stress, Capillarity & permeability on soil

### * Total stress :-

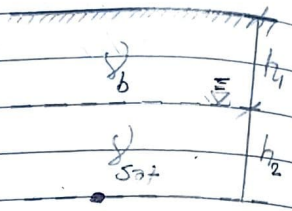
→ The stress carried by soil mass and pore water due to applied load is called total stress.

→ It is denoted by $\sigma$.

→ $\sigma = \gamma_b * h_1 + \gamma_{sat} * h_2$

$= \gamma_b * h_1 + \gamma_{sub} * h_2 + \gamma_w * h_2$

$[\gamma_{sat} = \gamma_{sub} + \gamma_w]$




### * Neutral stress / Pore water pressure :-

→ The stress carried by pore water only due to applied load is called neutral stress

→ It is denoted by $u$.

→ From figure, $u = \gamma_w h_2$

### * Effective Stress :-

→ The stress due to presence of actual soil mass & not due to pressure caused by pore water.

→ The diff$^n$ betn total stress and neutral stress gives the effective stress.

$$\boxed{\sigma' = \sigma - u}$$

→ Effective stress is denoted by $\sigma'$

$$\sigma' = \gamma_b h_1 + \gamma_{sub} * h_2$$

### * Importance of effective stress :-

→ The effective stress controls the engineering properties of soils.

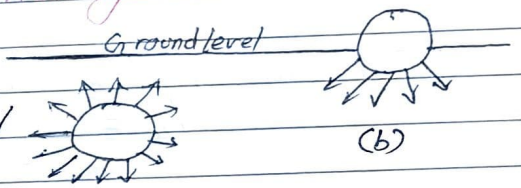
→ Compression and shear strength of soil are dependent on effective stress.

- As effective stress increases, compression of soil occurs. This causes settlement of structure built on soils.
- As the effective stress changes, shear strength also changes. The stability of slopes, earth pressures against retaining structure & bearing capacity of soil depends on shear strength of soil. Hence, these parameters changes with change in effective stress.

→ The permeability of soil is also governed by effective stress. Permeability depends on void ratio & void ratio changes with change in effective stress.

### * Surface tension and Capillarity in soil :-

To undserstand surface tension, let us consider a molecule of water surrounded by other molecules in the body of water. (Fig a). The forces due to Molecular attraction acts all around & the molecule is in equilibrium, however at the free surface (Fig (b)), the pull from the air above is smaller than the pull from water molecules below & equilibrium is disturbed. The forces tend to reduce the surface area of air-liquid surface to a minimum. The surface assumes a curved shape to maintain equilibrium. The interface behaves like a stretched membrane. This means, the surface tension exist at interface.




Effect of surface tension.

→ The phenomenon of rise or fall of pore water due to surface tension is called capillarity action.
→ In fact, the voids or pores of a soil mass, act as a series of capillarity tubes, extended vertically above the water table.
- Adhesion force → Liquid sticks to wall of tube
- Cohesion force → Mutual attraction between similar molecule.

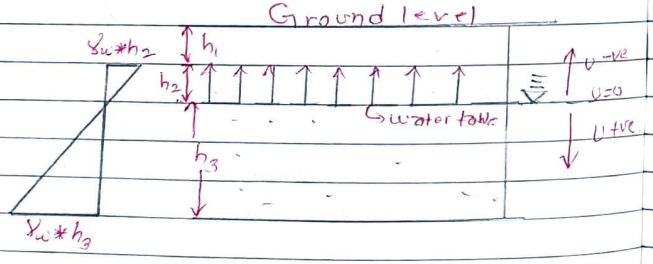
→ When cohesion force is less than adhesion force, the capillary rise takes place whereas when adhesion force is less than cohesion force, capillary fall takes place.
→ The water rises through capillary tube until water column balances the force of ~~attraction of~~ adhesion.

→ According to Terzaghi & Peck,

$$h_c = \frac{C}{e D_{10}}$$

$h_c$ = Capillary rise,
C = Constant (0.1 – 0.5)
$D_{10}$ = Size corresponding to 10% fines.
e = void ratio.

Note:-




## ** Computation of Effective discharge-

### (a) Hydrostatic Condition:-

**Case 1: Water table above the soil surface:-**

At level a-a,
$\sigma = 0,\ u = 0,\ \sigma' = 0$

At level A-A
$\sigma = \gamma_w * H,\ u = \gamma_w * h,\ \sigma' = 0$

At level B-B
$\sigma = \gamma_w * H + \gamma_{sat1} * H_1$
$u = \gamma_w (H + H_1)$
$\sigma' = \sigma - u = \gamma_{sub1} * H_1$

At level C-C
$\sigma = \gamma_w * H + \gamma_{sat1} * H_1 + \gamma_{sat2} * H_2$
$u = \gamma_w (H + H_1 + H_2)$
$\sigma' = \gamma_{sub1} * H_1 + \gamma_{sub2} * H_2$

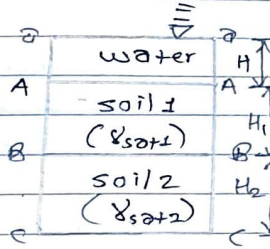


Fig:- Water table above soil.

**Case 2: Water table on the soil surface:-**

At level A-A
$\sigma = 0,\ u = 0,\ \sigma' = 0$

At level B-B
$\sigma = \gamma_{sat1} * H_1,\ u = \gamma_w * H_1$
$\sigma' = \gamma_{sub1} * H_1$

At level C-C
$\sigma = \gamma_{sat1} * H_1 + \gamma_{sat2} * H_2$
$u = \gamma_w * H_1 + \gamma_w * H_2$
$\sigma' = \sigma - u = \gamma_{sub1} * H_1 + \gamma_{sub2} * H_2$

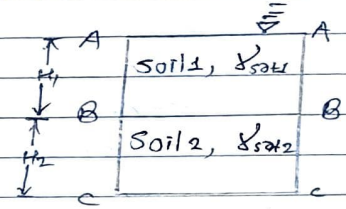


Fig:- Water table on soil surface.

**Conclusion:-**
The effective stress on soil mass is independent of the depth of water above the soil surface.

⑥ Effective stress due to surcharge load.

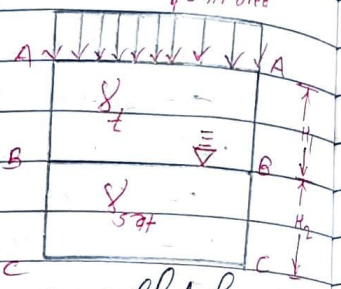


Fig: effect of surcharge.

At level A-A
$\sigma = q$
$U = 0$
$\sigma' = q$

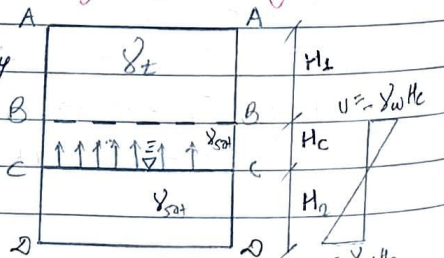
At level B-B
$\sigma = q + \gamma_t * H_1$
$U = 0$
$\sigma' = q + \gamma_t * H_1$

At level c-c
$\sigma = q + \gamma_t * H_1 + \gamma_{sat} * H_2$
$U = \gamma_w * H_2$
$\sigma' = \sigma - U = q + \gamma_t * H_1 + \gamma_{sub} * H_2$

**Conclusion:-**
Hence, when surcharge is applied, effective stress is increased through the depth equal to applied surcharge load.

* Effective stress in soil saturated by capillarity action

If the soil above the water table is saturated by capillary action, the effective stress can be calculated as follows. In this condition, the pore water pressure above the water table is negative.




From figure,

At level A-A,
$\sigma = 0$, $u = 0$, $\sigma' = 0$

At level B-B,
$\sigma = \gamma_t * H_1$, $u = -\gamma_w H_c$ & $\sigma' = \gamma_t H_1 + \gamma_w H_c$

At level c-c
$\sigma = \gamma_t * H_1 + \gamma_{sat} * H_c$
$u = 0$
$\sigma' = \gamma_t * H_1 + \gamma_{sat} * H_c$

At level D-D
$\sigma = \gamma_t * H_1 + \gamma_{sat} (H_c + H_2)$
$u = \gamma_w H_2$
$\sigma' = \sigma - u = \gamma_t H_1 + \gamma_{sat} H_c + \gamma_{sat} H_2 - \gamma_w H_2$
$= \gamma_t H_1 + \gamma_{sat} H_c + H_2 \gamma_{sub}$

Hence, we see that, in the capillary zone (B-B & C-C between the effective stress is greater than total stress.

* Effective stress under Seepage condition:

a) Seepage downwards:-
Consider the downward Seepage of water through pore. At level A-A, there is no head loss. When water flows through soil, the head is loss by 'h' which is shown in figure. The effective stress at different level can be calculated as,

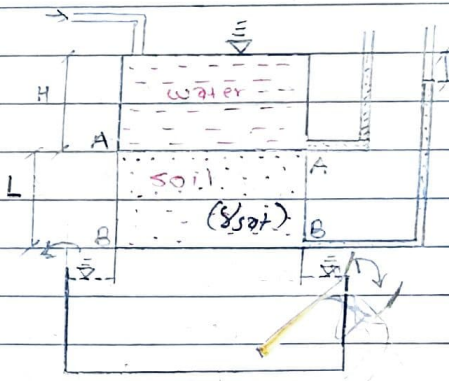


Fig: Seepage downwards.

we consider head

At level A-A,
$\sigma = \gamma_w H$, $u = \gamma_w H$, $\sigma' = 0$

At level B-B
$\sigma = \gamma_w H + \gamma_{sat} L$
$u = \gamma_w (H + L - h)$
$\sigma' = \sigma - u = \gamma_{sat} L - \gamma_w L + \gamma_w h$
$= \gamma_{sub} L + \gamma_w h$

Conclusion:-
The effective stress at level B-B is increased by $\gamma_w h$ for uniform seepage condition compared to hydrostatic condition in case of downward flow.

b) Seepage Upward:-
The figure shows the seepage of water in upward direction. The piezometers at level A-A & B-B indicates the pore water pressure.

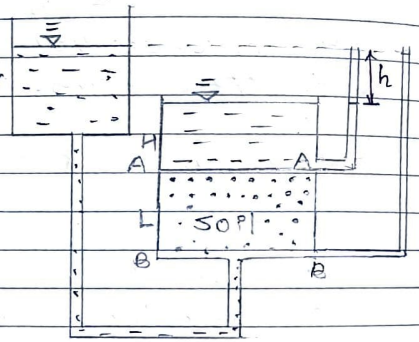



Fig: Seepage upward.

At level A-A
$\sigma = \gamma_w H$
$u = \gamma_w H$ & $\sigma' = 0$

At level B-B
$\sigma = \gamma_{sat} * L + \gamma_w H$
$u = \gamma_w (L + H + h)$
$\sigma' = \sigma - u = \gamma_{sub} L - \gamma_w h$.

Conclusion:
The effective stress at level B-B is decreased by $\gamma_w H$ for uniform seepage condition in upward direction compared to hydrostatic condition.

Therefore, if the upward flow is increased, the stage may be reached at which the effective stress ($\sigma'$) becomes zero. This condition is known as Quick sand condition.

For, Quick sand condition,

$\sigma' = 0$
$\Rightarrow \gamma_{sub} * L - \gamma_w * h = 0$
$\Rightarrow \frac{h}{L} = \frac{\gamma_{sub}}{\gamma_w} = \frac{\gamma_w \left(\frac{G-1}{1+e}\right)}{\gamma_w}$
$\Rightarrow i = \frac{G-1}{1+e}$ ---①

where, $\gamma_{sub} = \gamma_w \left(\frac{G-1}{1+e}\right)$ $G \rightarrow$ sp. gravity & $e =$ void ratio.

$i = h/L$ = hydraulic gradient

eqn ① is required expression for hydraulic gradient in Quick sand condition.

## Types of head:-

When water flows through a saturated soil mass, there is certain resistance for the flow because of the presence of solid matter. However, the laws of fluid mechanics which are applicable for the flow of fluids through pipes are also applicable to the flow of water through soils. According to Bernoulli's eq

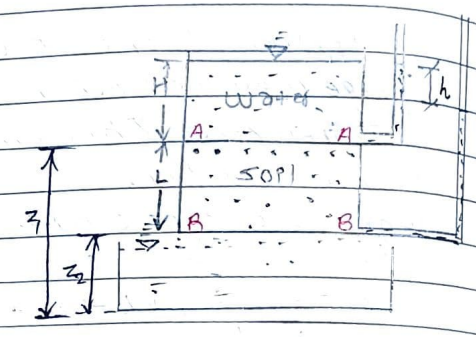
the total head under steady flow condition can be written as,

Total head = Elevation head + velocity head + pressure head

Elevation head = z

velocity head = $v^2/2g$

pressure head = $P/\gamma g$




At level A-A,

D.H = $z_1$

V.H = $v^2/2g$

P.H = H

At level B-B

D.H = $z_2$

V.H = $v_2^2/2g$

P.H = H + L − h

→ The velocity head can be neglected because velocity of flow of water through soil is very small (except in case of gravel).

## ✱ Permeability & its determination (Laboratory & field method)

→ Permeability is the property of the soil by virtue of which the soil mass allows water or any other fluid to flow through it.

→ According to Darcy's law, for laminar flow conditions, the velocity of flow 'v' is directly proportion to hydraulic gradient 'i'.

$$v \propto i$$

$$v = ki$$

where, k = proportionality constant which is called coefficient of permeability.

$$\Rightarrow \frac{Q}{A} = ki$$

$$\Rightarrow Q = kiA \quad \text{---} (1) \quad \text{Discharge eqn.}$$

(i) Laboratory Method:-

(a) Constant head permeability test

b) Varrable head permeability test.

ii) Field Methods:-

a) pumping out tests

b) pumping in tests (bore hole tests).

(a) Constant Head Permeability Method:-

In this method, continuous water supply is done so as to maintain constant head difference 'h'. Let 'V' be the quantity (volume) of water collected in graduated jar in time 't', Then according to

Darcy's law;

$$Q = kiA$$

$$\frac{V}{t} = k \times \frac{h}{L} \times A$$

$$\Rightarrow k = \frac{VL}{tAh}$$

$$\Rightarrow k = \frac{QL}{Ah}$$

which is required expression for coeff. of permeability.

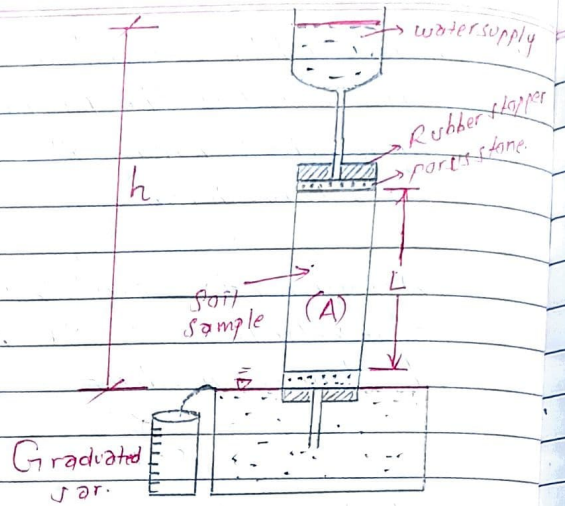



Fig:- Constant head permeability test.

(b) Falling (Variable) head permeability method:

Let, 'h' be the change of head in time 't'. If 'dh' be the small change is head in small time 'dt'. Then, the rate of change of head is given by $-dh/dt$. -ve sign indicates that the head is decreasing with time. Applying Darcy's law,

$$q = kiA$$

$$\Rightarrow a * \left(\frac{-dh}{dt}\right) = k * \frac{h}{L} * A$$

$$\Rightarrow -a \int_{h_1}^{h_2} \frac{dh}{h} = \frac{kA}{L} \int_0^t dt$$

$$\Rightarrow -a * [\ln(h_2) - \ln(h_1)] = \frac{kA}{L} \times t$$

$$\Rightarrow k = 2.303 * \frac{a}{A} * \frac{L}{t} * \log_{10}\left(\frac{h_1}{h_2}\right)$$

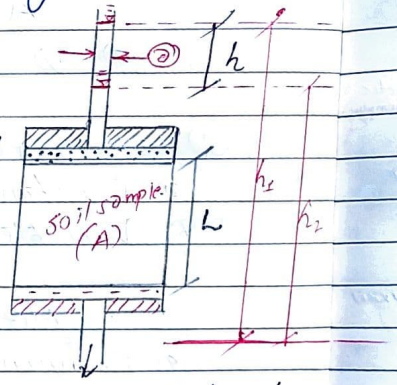



Fig:- Variable head permeability method.

## Field methods:-

(a) Pumping out test in unconfined aquifer:-

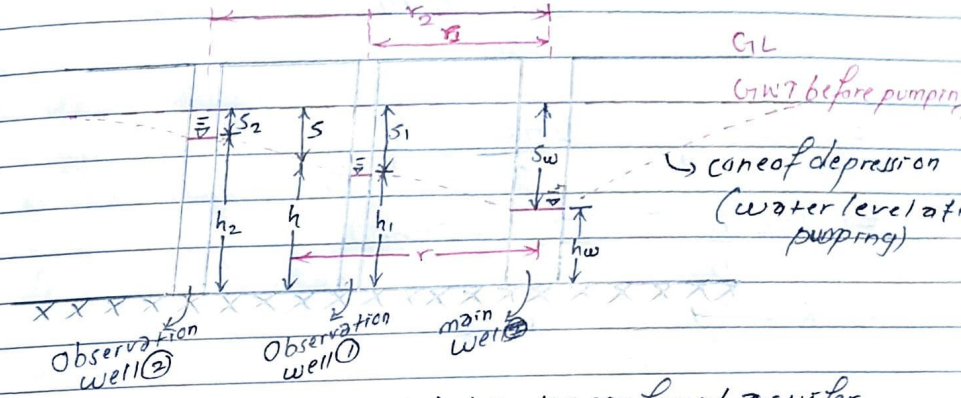



Fig:- pumping out test in unconfined aquifer

Let,

'h' depth of water table at a radius 'r' from main well.

's' is the corresponding drawdown from the original GWT at this point.

$r_1$, $h_1$, $s_1$ & $r_2$, $h_2$, $s_2$ are identical parameters for 1 & 2 respectively.

Applying Darcy's law, we have,

$$q = kiA \quad ---(i)$$

Let, 'dh' be the small head drop for small lateral movement of water 'dr'.

$$i = \frac{dh}{dr} \quad ---- (ii)$$

At any radius 'r' contributing area of flow towards main well is, $\rightarrow A = 2\pi rh$ ---(iii)

From (i), (ii) & (iii), we get,

$$q = k \times \frac{dh}{dr} \times 2\pi rh$$

$$\Rightarrow \frac{dr}{r} = \frac{2\pi k}{q} \times h \times dh$$

$$\Rightarrow \int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k}{q} \int_{h_1}^{h_2} h\,dh$$

$$\Rightarrow [\log_e r]_{r_1}^{r_2} = \frac{2\pi k}{q} \left(\frac{h^2}{2}\right)_{h_1}^{h_2}$$

$$\Rightarrow \log_e \frac{r_2}{r_1} = \frac{2\pi k}{q}(h_2^2 - h_1^2)$$

$$\Rightarrow k = \frac{q}{\pi(h_2^2 - h_1^2)} \ln\left(\frac{r_2}{r_1}\right)$$

(b) Puping out test in confined aquifer:-

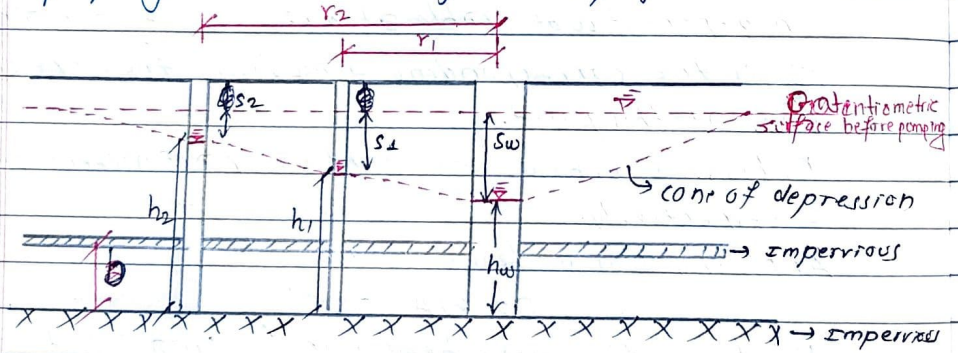



Fig:- well operating in confined aquifer

Let, two observation wells are driven at $r_1$ & $r_2$ distance from center of main well. Let $s_1$ & $s_2$ are the drawdown & $h_1$ & $h_2$ located at $r_1$ & $r_2$ are the water level distance away from the main well.

Let b = height of initial piezometric surface above bottom impervious stratum.

From Darcy's law,

$$q = K \cdot A$$

$$\Rightarrow q = k * \frac{dh}{dr} * 2\pi r b$$

$$\Rightarrow \frac{dr}{r} = \frac{2\pi k b}{q} \times dh$$

$$\Rightarrow \int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k b}{q} \int_{h_1}^{h_2} dh$$

$$\Rightarrow \ln\left(\frac{r_2}{r_1}\right) = \frac{2\pi k b}{q}(h_2 - h_1)$$

$$\Rightarrow k = \frac{q}{2\pi b(h_2 - h_1)} \times \ln\left(\frac{r_2}{r_1}\right)$$

This is required expression for coefficient of permeability in confined aquifer.

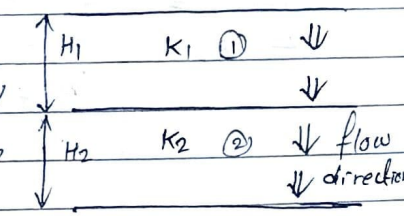
## ✻ Average permeabilities of stratified soil deposit :-

(i) Flow normal to bedding:-

Let,

$K_z$ be the equivalent permeability. If h is the total head loss in overall flow, then

$$h = h_1 + h_2 \quad ---(A)$$




$h_1$ = head loss in flow through layer 1

$h_2$ = head loss in flow through layer 2

For layer ①,

$$q = K_1 \cdot A$$

$$\Rightarrow q = K_1 \times \frac{h_1}{H_1} \times A \Rightarrow h_1 = \frac{qH_1}{K_1 A} \quad ---(1)$$

For layer ②,

$q = kiA$

$\Rightarrow q = k_2 \times \frac{h_2}{H_2} \times A$

$\Rightarrow h_2 = \frac{qH_2}{k_2 A}$ ---- (ii)

For equivalent overall layers,

$q = kiA$

$\Rightarrow q = k_z \times \frac{h}{H_1 + H_2} \times A$

$\Rightarrow h = \frac{q(H_1 + H_2)}{k_z \times A}$ --- (iii)

From eqn (A),

$\frac{q(H_1 + H_2)}{k_z \times A} = \frac{qH_1}{k_1 A} + \frac{qH_2}{k_2 A}$

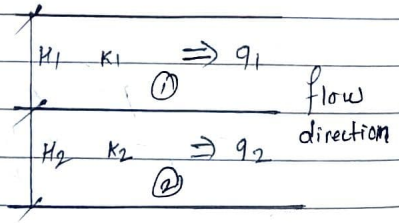
$\Rightarrow \frac{H_1 + H_2}{k_z} = \frac{H_1}{k_1} + \frac{H_2}{k_2}$

$\Rightarrow \frac{1}{k_z} = \frac{1}{(H_1 + H_2)} \left(\frac{H_1}{k_1} + \frac{H_2}{k_2}\right)$

$\Rightarrow k_z = \frac{H_1 + H_2}{\left(\frac{H_1}{k_1} + \frac{H_2}{k_2}\right)}$ $\therefore$ For 'n' layers, $k_z = \frac{\Sigma H_n}{\Sigma H_n / k_n}$

## (ii) Flow in the direction of bedding:-

Consider a soil deposits consisting of two horizontal layers of soils of thickness $H_1$ & $H_2$ having coeff. of permeability $k_1$ & $k_2$ respectively as shown in figure.




Total discharge per unit width is,

$q = q_1 + q_2$ ---- (A)

Using Darcy's law,

for layer ①,

$q_1 = k_1 i A_1$

$\Rightarrow q_1 = k_1 * i * (H_1 * 1)$ ---- (i)

[Taking unit width]

Similarly, for layer ②,

$\Rightarrow q_2 = k_2 * i * (H_2 * 1)$ --- (ii)

For the overall equivalent layers,

$q = k_x * i * (H_1 + H_2) * 1$ --- (iii)

where, $k_x$ be the equivalent permeability in x-direction,

$q = q_1 + q_2$

$\Rightarrow k_x * i * (H_1 + H_2) = k_1 * i * H_1 + k_2 * i * H_2$

$\Rightarrow k_x (H_1 + H_2) = k_1 H_1 + k_2 H_2$

$\Rightarrow k_x = \frac{k_1 H_1 + k_2 H_2}{H_1 + H_2}$

$\Rightarrow k_x = \frac{\Sigma k_n H_n}{\Sigma H_n}$

## ✻ Factors affecting permeability:-

The general expression for coeff. of permeability of a soil is,

$$k = c \left(\frac{\gamma_w}{\mu}\right) \left(\frac{e^3}{1+e}\right) D^2$$

Where, $\mu$ = viscosity of soil, $c$ = constant

$D$ = particle size, $\gamma_w$ = unit weight of water

$e$ = void ratio

$k$ = coeff. of permeability.

→ The factors which affects the permeability of soil are as follows.

1) Particle size:-
→ From above eqn, permeability of soil (k) is directly proportional to square of particle size (D)
→ The permeability of coarse grained soil is very large as compared to fine grained soils.

2) Structure of soil mass
→ The size of flow passage depends upon the structural arrangement. For same void ratio, permeability of flocculated structure soil is more than the dispersed structure soil.

3) Shape of particle:-
→ Angular particles are less permeable than those with rounded particles.

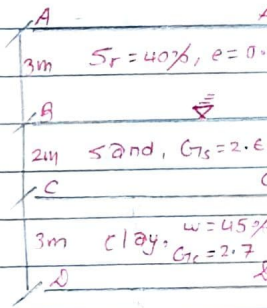
4) Void ratio:-
→ With increase in void ratio, coeff. of permeability increases, as $k \propto \frac{e^3}{1+e}$

5) Properties of pore fluid:-
→ Coefficient of permeability is directly proportional to unit weight of water ($\gamma_w$) & inversely proportional to viscosity ($\mu$).

(6) Degree of saturation:-
The permeability of partially saturated soil is considerably smaller than that of fully saturated soil because presence of air pocket causes blockage of passage.

(7) Impurities in water:-
Any foreign material present in water reduce effective voids & decrease the permeability of soils.

Nm Q For the subsoil condition shown in figure below. Draw a total, neutral & effective stress diagram upto depth of 8m. Neglect capillary flow.




→ For sandy soil,

$$\gamma_t = \frac{\gamma_w (G+Se)}{1+e} = 18.06 \text{ kN/m}^3$$

$$\gamma_{sat} = \frac{\gamma_w (G+e)}{1+e} = 20.31 \text{ kN/m}^3$$

For clay soil,

$$\gamma_{sat} = \frac{\gamma_w (G+e)}{1+e}$$

But, $G.w = S.e$

$$e = G.w \quad [S = 1]$$

$$\gamma_{sat} = 17.67 \text{ kN/m}^3$$

Stress calculation:-

At level A-A,

$$\sigma = 0, \quad U = 0, \quad \sigma' = 0$$

At level B-B,

$$\sigma = \gamma_t \times 3 = 54.18 \text{ kN/m}^2$$

$u = 0$

$\sigma' = 54.18\ kN/m^2$

At level C-C,

$\sigma = \gamma_t \times 3 + \gamma_{sat} \times 2 = 94.8\ kN/m^2$

$u = \gamma_w \times 2 = 20\ kN/m^2$

$\sigma' = \sigma - u = 74.8\ kN/m^2$

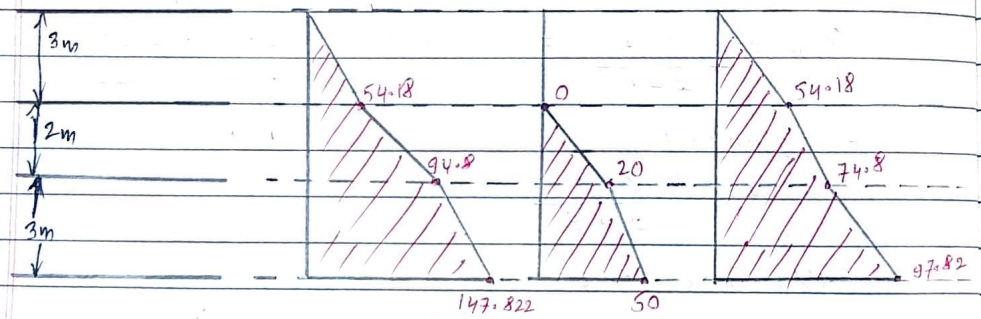
At level D-D,

$\sigma = \gamma_t \times 3 + \gamma_{sat} \times 2 + (\gamma_{sat})_{dry} \times 3$

$= 147.822\ kN/m^2$

$u = \gamma_w \times 5 = 50\ kN/m^2$

$\sigma' = \sigma - u = 97.82\ kN/m^2$




Total stress distb | pore water press$^r$ distbn | effective stress diagram.

# Ch9: Seepage Analysis:-

## Introduction

→ Seepage is the flow of water under gravitational force in a permeable medium. Flow of water takes place from a point of high head to point of low head. The flow is generally laminar.

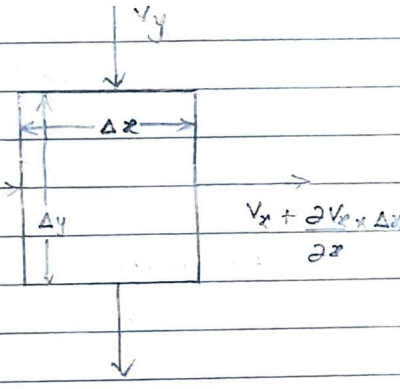
→ A path followed by a water particle is represented by a flow line.

→ At certain points on different flow lines, the total head will be the same. The lines connecting points of equal head can be drawn. These lines are known as equipotential lines.

## * Two dimensional flow Laplace equation:-

Consider a soil element of size $\Delta x \times \Delta y$. Let, the thickness of element be unity. Let $V_x$ be the velocity of fluid entering from x-direction & $V_y$ be the velocity of fluid entering from y-direction. Then, velocity of fluid exiting from x-direction is given by $V_x + \frac{\partial V_x}{\partial x} \times \Delta x$ & the velocity of fluid exiting from y-direction is given by,

$$V_y + \frac{\partial V_y}{\partial y} \times \Delta y$$




Now, Entering discharge is,

$$Q_{entry} = V_x * (\Delta y * 1) + V_y * (\Delta x * 1)$$

$$= V_x \Delta y + V_y \Delta x$$

Exiting discharge,

$$Q_{exit} = \left(v_x + \frac{\partial v_x}{\partial x} \times \Delta x\right) * (\Delta y \times 1) + \left(v_y + \frac{\partial v_y}{\partial y} \times \Delta y\right) * (\Delta x \times 1)$$

$$= v_x \Delta y + \frac{\partial v_x}{\partial x} \times \Delta x \Delta y + v_y \Delta x + \frac{\partial v_y}{\partial y} \times \Delta x \times \Delta y$$

From continuity eqn,

$$Q_{entry} = Q_{exit}$$

$$\Rightarrow v_x \Delta y + v_y \Delta x = v_x \Delta y + \frac{\partial v_x}{\partial x} \times \Delta x \Delta y + v_y \Delta x + \frac{\partial v_y}{\partial y} \times \Delta x \times \Delta y$$

$$\Rightarrow \Delta x \Delta y \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) = 0$$

$$\Rightarrow \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \quad ------ (1)$$

According to Darcy's law,

$$v_x = k_x i_x = k_x \times \frac{\partial h}{\partial x}$$

$$v_y = k_y i_y = k_y \times \frac{\partial h}{\partial y}$$

From (1),

$$\frac{\partial\left(k_x \times \frac{\partial h}{\partial x}\right)}{\partial x} + \frac{\partial\left(k_y \times \frac{\partial h}{\partial y}\right)}{\partial y} = 0$$

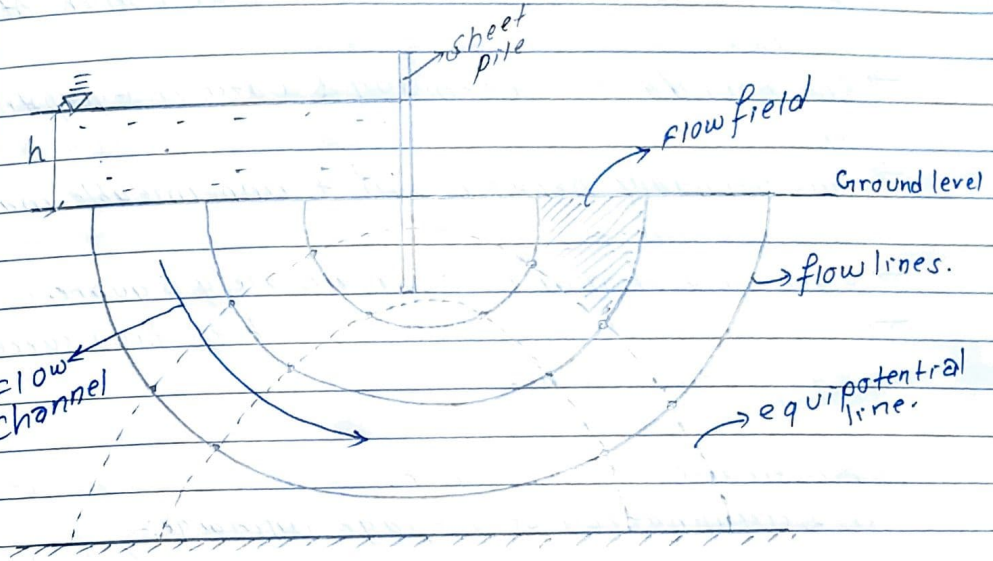
$$\Rightarrow k_x \times \frac{\partial^2 h}{\partial x^2} + k_y \times \frac{\partial^2 h}{\partial y^2} = 0$$

For isotropic soil mass, $k_x = k_y$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

Which is reqd Laplace eqn for 2D flow.

## * Flow net, their characteristics & Applications:




→ The path followed by a water particle during seepage is known as flow lines.

→ The line joining point of equal head is known as equipotential line.

→ The net formed by intersecting possible flow lines & equipotential lines is known as flow net. It gives graphical representation of seepage flow through soil mass.

→ The space betn any two adjacent flow lines is called flow channel.

→ The space between two adjacent flow line & two successive equipotential lines is called field.

## Characteristics:-

→ Flow lines & equipotential lines cut each other at right angle.
→ The boundary between soil & water is equipotential line.
→ Any boundary between soil & impermeable material is flow line.
→ Each field should be approximately square.
→ The same potential drop occurs betn two successive equipotential line.

## ✻ Application of flow net:-

(i) Determination of seepage discharge:-

For a flow channel, $\left(\Delta h = h/N_d\right)$

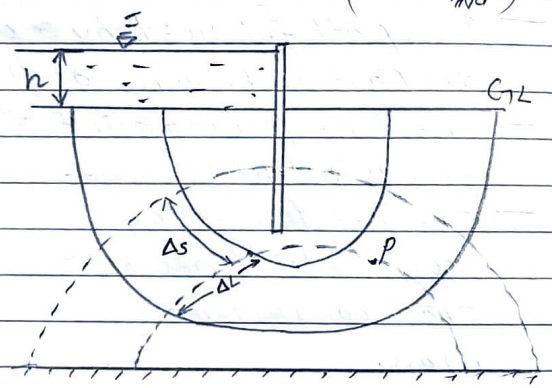
$\Delta q = kiA$

$= k \times \frac{\Delta h}{\Delta s} \times (\Delta L \times 1)$

$= \frac{kh}{N_d \times \Delta s} \times \Delta L$

$= \frac{kh}{N_d} \times \frac{\Delta L}{\Delta s}$

$= \frac{kh}{N_d} \quad [\Delta L \simeq \Delta s]$




If $N_f$ is the number of flow channels,

$q = N_f \times \Delta q$

where, $N_d$ = No. of potential drops.

$q = kh \times \frac{N_f}{N_d}$

(ii) Determination of seepage pressure,

Seepage pressure at point p is,

$P_s = h_p \times \gamma_w$

where,

$h_p$ = Total head at that point

$= h - n \times \Delta h$

$= h - n \times \frac{h}{N_d}$

where, n = number of equipotential drops upto point 'p'.

(iii) Determination of exit gradient.

The average value of exit gradient for any flow field is given by,

$i = \frac{\Delta h}{\Delta s}$

where, $\Delta h$ = Loss of head = $h/N_d$

$\Delta s$ = Length of flow field

## ✻ Rate of seepage, Uplift pressure & exit gradient.

a) Rate of Seepage:-

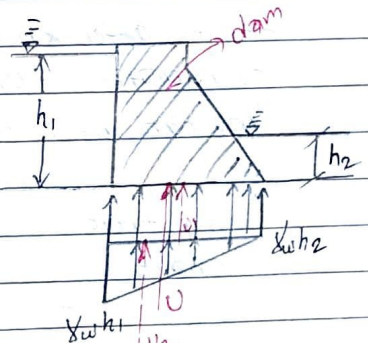
$q = Kh \frac{N_f}{N_d} = Kx_0$ where $x_0 = \sqrt{b^2+h^2} - b$

b) Exit gradient:-

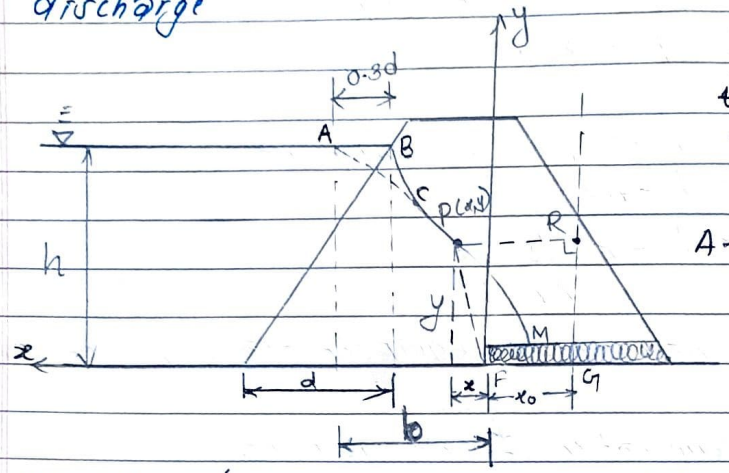
$i = \frac{\Delta h}{\Delta s}$ & $\Delta h = \frac{h}{N_d}$

c) Uplift pressure:-

$U = \underbrace{\gamma_w h_2 \times B}_{U_1} + \underbrace{\frac{1}{2}(\gamma_w h_1 - \gamma_w h_2) \times B}_{U_2}$



## ✻ Seepage through earthdam / Analysis of Seepage discharge




pheratricline → topmost seepage line B–P–M

A–C–M → true parabolic line.

Let, P(x,y) be any point on the parabolic pheratic line.

$$PF = PR$$

$$\sqrt{x^2+y^2} = x + x_0 \quad ----(I)$$

Squaring both sides we get,

$$x^2+y^2 = (x+x_0)^2$$

$$x^2+y^2 = x^2 + 2xx_0 + x_0^2$$

$$y = \sqrt{2xx_0 + x_0^2} \quad ----(II)$$

From (I),

$$x_0 = \sqrt{x^2+y^2} - x$$

When $x=b$, we have $y=h$,

$$x_0 = \sqrt{b^2+h^2} - b \quad ----(III)$$

Hence, after knowing $x_0$ from eqn (III), we can compute coordinates of pheratic line from eqn (II). Hence, pheratic line can be constructed.

Computation of seepage discharge,

we have,

$$q = kiA$$

$$q = k \times \left(\frac{dy}{dx}\right) \times y \times 1$$

$$q = k \times \frac{d\left(\sqrt{2xx_0+x_0^2}\right)}{dx} \times (y \times 1)$$

$$q = k \times \frac{1}{2\sqrt{2xx_0+x_0^2}} \times 2x_0 \times y$$

$$q = k \times \frac{1}{2\sqrt{2xx_0+x_0^2}} \times 2x_0 \times \sqrt{2xx_0+x_0^2}$$

$$\boxed{q = kx_0}$$

Hence, the seepage discharge is known.

## * Failure due to piping & its prevention – protective filter:-

As the water seeps from upstream to downstream portion such that the exit gradient at the downstream portion exceeds the critical exit gradient then the seepage water starts to erode the downstream material. This erosion gradually continues towards the upstream end leading to the formation of hollow pipe along the seepage path. This is called piping & consiquent failure of structure due to this phenomenon is called piping failure.

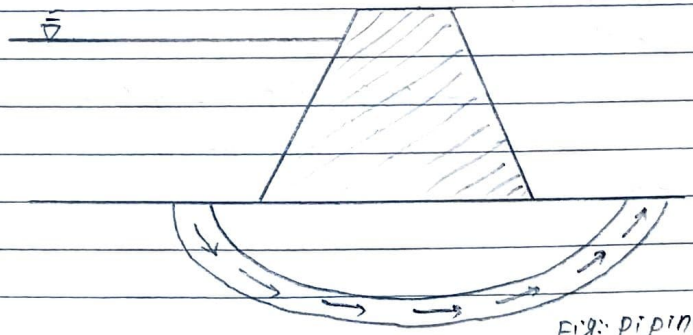


Fig: piping Mechanism

→ piping can be prevented by protective filter. protective filter consists of sandy, gravely, aggregate material which can permit the flow of water through it without erosion of the filter material itself. The requirement of filter material is,

$$\frac{D_{15}\text{ (filter)}}{D_{85}\text{ (protected material)}} < 5$$

$$4 < \frac{D_{15}\text{ (filter)}}{D_{15}\text{ (protected material)}} < 20$$

$$\frac{D_{50}\text{ (filter)}}{D_{50}\text{ (protected material)}} < 25$$

* Seepage through anisotropic condition:-

For isotropic condition,

$(k_x = k_y)$ → equivalent permeability

$q = k_e \times h \times \frac{N_f}{N_d}$ $\quad k_e = k_x = k_y$

$q = k_e x_0$

For anisotropic condition,

$(k_x \neq k_y)$

$q = k_e h \times \frac{N_f}{N_d}$ where, $k_e = \sqrt{k_x \times k_y}$

$q = k_e x_0$

Q A 30m high dam has a top width of 10m & a free board of 5m. The dam is made of fine sand having coefficient of permeability of $4.5 \times 10^{-4}$ cm/sec. A 60m long horizontal drainage blanket is placed near the d/s of dam. Determine the quantity of seepage if bottom width of dam is 185m

→ we have,

$q = k x_0$

Given,

$k = 4.5 \times 10^{-6}$ m/sec

$h = 25$m

top width = 10m

bottom width = 185m

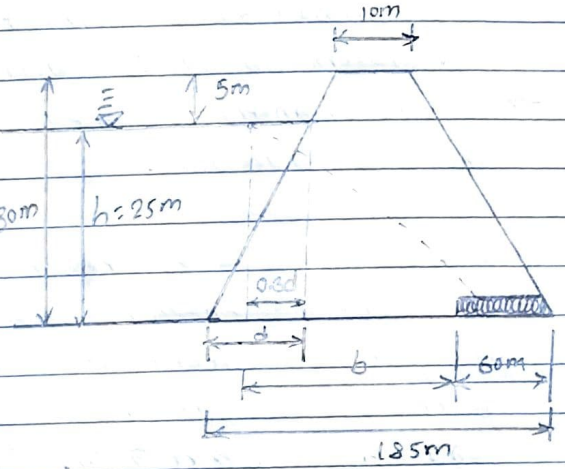
Here,

$x_0 = \sqrt{b^2 + h^2} - b$

Let,

U/s slope of dam surface = 1V : 2.5H

Then, $d = 62.5$ m.




Now,

$185 = 60 + b + (d - 0.3d)$

$b = 81.25$ m

Then,

$x_0 = \sqrt{81.25^2 + 25^2} - 81.25 = 3.759$ m

Therefore,

$q = k x_0$

$= 1.69 \times 10^{-5} \left(\frac{m^3/sec}{m}\right)$

ch-8

# Vertical Stress below applied load:-

## ✻ Concept of stress distribution on soil:-

→ Stresses are induced in a soil mass due to
i) self weight of soil
ii) Added load (Superimposed load of buildings, dams etc)

→ These stresses are required for the stability analysis of soil mass, the settlement analysis of foundations & the determination of earth pressures.

→ In this chapter, we are concerned with stresses due to applied load only.

## ✻ Boussinesq's & Westergaard's theory:-

### * Boussinesq's theory:-

As per this theory, the stress developed at any point in the soil mass due to externally applied load can be calculated. The load could be of any type eg: concentrated line load, strip load, circular load etc.

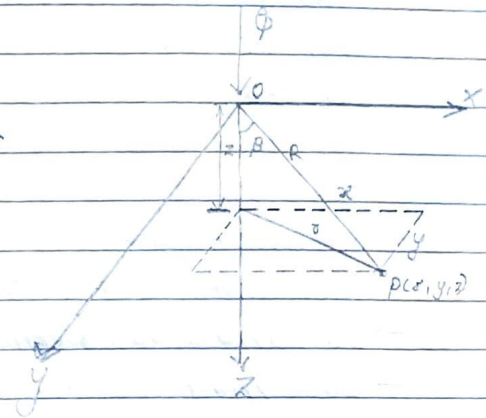
ASSUMPTIONS:-

- The soil mass is linearly elastic medium.
- Soil mass is homogeneous.
- Soil mass is isotropic.
- Soil mass is semi-infinite
- Soil is weightless & free from residual stresses before the application of load.

→ Figure shows the horizontal surface of the elastic continuum subjected to point load 'Q' at point 'O'. The origin of coordinates is taken at 'O'. Using logarithmic stress function for solution of elasticity problem, Boussinesq proved that polar stress ($\sigma_R$) at point p(x, y, z) is given by,

$$\sigma_R = \frac{3}{2\pi} \times \frac{Q \cos\beta}{R^2} \quad \text{-----(1)}$$

where,
R = polar distance betn O & P
β = Angle which line OP makes with vertical.




From figure,

$$r = \sqrt{x^2+y^2}$$

$$R = \sqrt{r^2+z^2}$$

$$= \sqrt{x^2+y^2+z^2}$$

The vertical stress at any point 'p' is given by,

$$\sigma_v = \sigma_R \cos^2\beta$$

$$= \frac{3}{2\pi} \times \frac{Q\cos\beta}{R^2} \times \cos^2\beta \quad \text{[using eqn (1)]}$$

$$= \frac{3}{2\pi} \times \frac{Q}{R^2} \times \left(\frac{z}{R}\right)^3 \quad \left[\cos\beta = \frac{z}{R}\right]$$

$$= \frac{3Qz^3}{2\pi (r^2+z^2)^{5/2}}$$

$$= \frac{3Q \times z^5}{2\pi z^2 \left[1+(r/z)^2\right]^{5/2} \times z^5}$$

$$= \frac{3Q}{2\pi z^2} \times \frac{1}{\left[1+(r/z)^2\right]^{5/2}}$$

$$\sigma_v = I_B \times \frac{Q}{z^2} \quad \text{----(A)}$$

where,

$$I_\theta = \frac{3}{2\pi} \times \frac{1}{\left[1 + \left(r/z\right)^2\right]^{5/2}}$$

= Boussinesq's influence factor.

→ Hence, vertical stress at any point below the ground surface due to concentrated load on the ground surface can be obtained with the help of eqn (A).

* Westergaard's theory:

Assumptions:

→ Soil mass is elastic

→ Soil mass is semi-infinite

→ Soil mass consists of numerous closely spaced horizontal sheets of negligible thickness & comprising of infinitely rigid materials.

* The thin sheets are closely spaced & permit only downward displacement of soil mass as a whole without any lateral displacements. Therefore, westergaard's solution represents more closely the actual sedimentary deposit.

→ As per westergaard's theory,

$$\sigma_z = \frac{c/2\pi}{\left[c^2 + \left(r/z\right)^2\right]^{3/2}} \times \frac{Q}{z^2}$$

where,

z = depth below concentrated load

where, c depends upon poisson's ratio ($\mu$) & given by,

$$c = \sqrt{\frac{1-2\mu}{2-2\mu}}$$

value of $\mu$ varies from 0 to 0.5

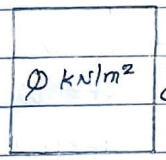
* Approximate method of vertical stress distribution:

The methods discussed in proceeding sections are relatively more accurate, but are time consuming. Sometimes, the engineer is interested to estimate the vertical stresses approximately for preliminary designs. Following approximate methods are used to calculate vertical stress distribution.

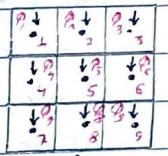
- Equivalent Point-Load Method
- Two to one load distribution method.
- Sixty degree distribution.

→ Equivalent point load method:

Let 'Q' kN/m² be the intensity of uniform load acting on the area as shown in the figure. We may divide the area into different segments as shown below fig(b).



fig(a)


fig(b)

Now, the uniform load intensity (Q kN/m) when multiplied with the area of newly formed segment (1, 2, 3, 4, 5, 6, 7, 8, 9). We get the total load $Q_1, Q_2 \dots Q_9$ on each segments acting through their centroid. Now, the vertical stress at any point below the overall loaded area is equal to the sum of vertical

stresses due to these ~~eq~~ equivalent individual point loads.

i.e, $\sigma_v = \frac{1}{z^2} \sum_{i=1}^{n} \phi_i I_{Bi}$

$= \frac{\phi_1 I_{B1} + \phi_2 I_{B2} + \dots + \phi_n I_{Bn}}{z^2}$

## ✱ Pressure Bulb Concept: (Isobars)

An isobar is a curve joining the points of equal stress intensity. In other words, an isobar is a contour of equal stress.

An isobar is a spatial curved surface of the shape of an electrical bulb or an onion. The curved surface is symmetrical about the vertical axis passing through point load. They are termed as pressure bulb.

## * Computation for drawing pressure bulbs (isobar)

Let '$\phi$' be the intensity of load on the surface. Let us draw an isobar joining all the points below the ground surface for which stress equals to $0.1\phi$ (i.e pressure bulb will be drawn for $\sigma_v = 0.1\phi$)

We have,

As per Boussinesq's theory,

$$\sigma_v = I_B \times \frac{\phi}{z^2}$$

$$\Rightarrow I_B = \sigma_v \times \frac{z^2}{\phi}$$

$$\Rightarrow I_B = \frac{0.1\phi \times z^2}{\phi}$$

$$\Rightarrow I_B = 0.1 z^2 \quad \text{------} \quad (A)$$

Also,

$$I_B = \frac{3}{2\pi} \times \frac{1}{\left[1 + (r/z)^2\right]^{5/2}} \quad \text{----} \quad (B)$$

For significant depth, $r = 0$

Hence,

$$I_B = \frac{3}{2\pi} \times \frac{1}{\left[1 + (r/z)^2\right]^{5/2}}$$

$$\Rightarrow 0.1 z^2 = \frac{3}{2\pi} \times \frac{1}{\left[1 + (0/z)^2\right]^{5/2}}$$

$$\Rightarrow z = 2.185 \text{ m}$$

Table for radial distance (r)

| z (assume) | $I_B$ (from eqn A) | r/z (from eqn B) | r (from column 1 & 3) |
|---|---|---|---|
| 0.25 | | | |
| 0.5 | | | |
| 0.75 | | | |
| 1 | | | |
| 1.25 | | | |
| 1.5 | | | |
| 1.75 | | | |
| 2 | | | |
| 2.185 | | | |

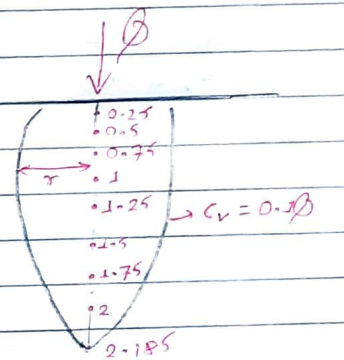



Fig: pressure bulb diagram

## Vertical stress distribution diagram

### (a) Vertical stress distribution on horizontal plane:

→ The vertical stress at various points on a horizontal plane at a particular depth 'z' can be obtained by using the equation below.

$$\sigma_v = I_B \times \frac{\emptyset}{z^2} \quad ---(A)$$

where, $I_B = \frac{3}{2\pi} \times \frac{1}{[1+(r/z)^2]^{5/2}}$ ---(B)

→ Let us determine the stresses at a depth of 2m.

$$\sigma_v = I_B \times \frac{\emptyset}{z^2} = I_B \times \frac{\emptyset}{2^2} = 0.25 I_B \emptyset \quad ---(1)$$

→ Computation of $\sigma_v$.

| r (assume) | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| $r/z = r/2$ | 0 | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 2 |
| $I_B$ [From eq(B)] | 0.4775 | 0.4103 | 0.2733 | 0.1565 | 0.0844 | 0.0454 | 0.0251 | 0.0085 |
| $\sigma_v$ [From (1)] | 0.1194$\emptyset$ | 0.1026$\emptyset$ | 0.0683$\emptyset$ | 0.0390$\emptyset$ | 0.0211$\emptyset$ | 0.0113$\emptyset$ | 0.0063$\emptyset$ | 0.0021$\emptyset$ |

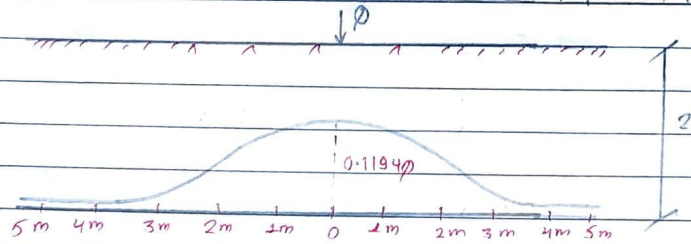



Fig: vertical stress distribution on horizontal plane

→ The diagram is symmetrical about vertical axis. The maximum stress occurs just below the load (r=0) & it decreases rapidly as the distance 'r' increases.

### (b) Vertical stress distribution in vertical plane:

→ In this case, the radial distance 'r' is constant & the depth changes.

→ The vertical stress at various points in vertical plane at a particular depth 'z' can be obtained using the eqⁿ below.

$$\sigma_v = I_B \times \frac{\emptyset}{z^2} \quad ---(A)$$

where, $I_B = \frac{3}{2\pi} \times \frac{1}{[1+(r/z)^2]^{5/2}}$ ---(B)

→ Let us determine the stresses at a radial distance r=1m.

| z | 0.25 | 0.5 | 1 | 1.5 | 2 | 2.5 | 5 |
|---|---|---|---|---|---|---|---|
| $r/z$ | 4 | 2 | 1 | 0.667 | 0.5 | 0.4 | 0.2 |
| $I_B$ | 0.0004 | 0.0085 | 0.0844 | 0.1904 | 0.2733 | 0.3294 | 0.4329 |
| $\sigma_v$ | 0.0064$\emptyset$ | 0.034$\emptyset$ | 0.0844$\emptyset$ | 0.0845$\emptyset$ | 0.0683$\emptyset$ | 0.0527$\emptyset$ | 0.017$\emptyset$ |

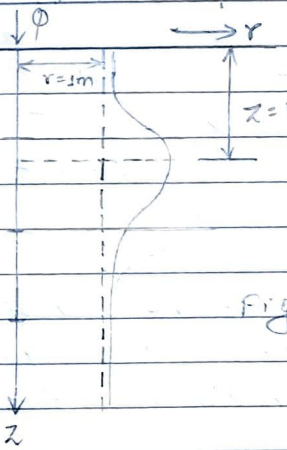



Fig:- Vertical stress distribution in vertical plane.

→ From diagram, it is seen that vertical stress first increases & then decreases after reaching to maximum value. The maximum vertical stress occurs at $r/z = 0.817$.

## Vertical Stress distribution beneath the loaded Area:-

### (a) Vertical stress due to line load:-

→ The vertical stresses in a soil mass due to vertical line load can be obtained using Boussinesq's solution.

→ Let the vertical line load of intensity q kN/m acting on soil mass along y-axis.

→ Consider a load acting on a small length dy. The load can be taken as a point load of q×dy.

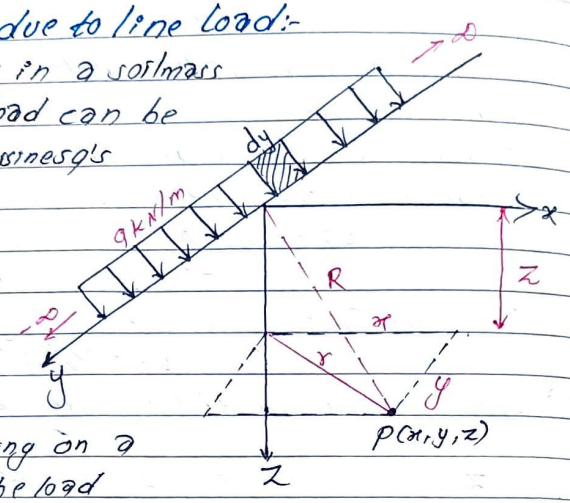



Fig:- Line load acting along y-axis.

→ According to Boussinesq's theory,

$$\sigma_v = \frac{3Q}{2\pi z^2} \times \frac{1}{[1+(r/z)^2]^{5/2}}$$

$$= \frac{3Q}{2\pi z^2} \times \frac{1 \times z^5}{(r^2+z^2)^{5/2}}$$

$$= \frac{3Q}{2\pi} \times \frac{z^3}{(x^2+y^2+z^2)^{5/2}} \quad \text{----(A)}$$

→ Here, for small strip of line load of length dy' resultant load ($Q$) = q×dy

Hence, small vertical stress ($\Delta\sigma_v$) due to line load of length dy' is,

$$\Delta\sigma_v = \frac{3 \times q\,dy}{2\pi} \times \frac{z^3}{(x^2+y^2+z^2)^{5/2}}$$

→ The total vertical stress due to line load at point 'P' is,

$$\sigma_v = \int_{-\infty}^{\infty} \Delta\sigma_v$$

$$= \int_{-\infty}^{\infty} \frac{3q\,dy}{2\pi} \times \frac{z^3}{(x^2+y^2+z^2)^{5/2}}$$

$$= \frac{3qz^3}{2\pi} \int_{-\infty}^{\infty} \frac{dy}{(x^2+y^2+z^2)^{5/2}}$$

put, $x^2+z^2 = v^2$,

$$\sigma_v = \frac{3qz^3}{2\pi} \int_{-\infty}^{\infty} \frac{dy}{(v^2+y^2)^{5/2}}$$

Let, $y = v\tan\theta$

$$\frac{dy}{d\theta} = v\sec^2\theta$$

when $\theta \to -\infty$, $y \to \infty$, $y \to -\infty$, $\theta \to -\pi/2$

$\theta \to \infty$, $y \to \pi/2$ $y \to \infty$, $\theta \to \pi/2$

$$\therefore \sigma_v = \frac{3qz^3}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{v\sec^2\theta\,d\theta}{(v^2+v^2\tan^2\theta)^{5/2}}$$

$$= 2\times\frac{3qz^3}{2\pi} \int_{0}^{\pi/2} \frac{v\sec^2\theta\,d\theta}{v^5 \times \sec^5\theta}$$

$$= 2\times\frac{3qz^3}{2\pi v^4} \int_{0}^{\pi/2} \cos^3\theta\,d\theta$$

$$= 2\times\frac{3qz^3}{2\pi v^4} \int_{0}^{\pi/2} [\cos\theta - \cos\theta\cdot\sin^2\theta]\,d\theta$$

$$= 2\times\frac{3qz^3}{2\pi (x^2+z^2)^2} \times \left[\sin\theta - \frac{\sin^3\theta}{3}\right]_0^{\pi/2}$$

$$= \frac{2 \times 3 q z^3}{(x^2+z^2)^2 \times 2\pi} \times \frac{2}{3}$$

$$= \frac{2q z^3}{\pi (x^2+z^2)^2}$$

$$= \frac{2q z^3}{\pi z^2 \left(1+(x/z)^2\right)^2}$$

$$= \frac{2q}{\pi z \left[1+(x/z)^2\right]^2}$$

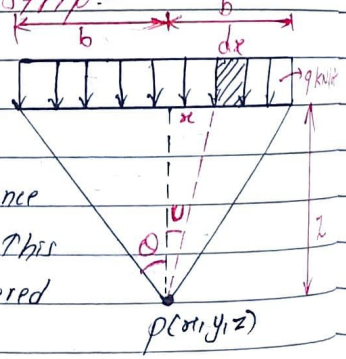
$$\therefore \sigma_v = \frac{2q}{\pi z} \times \frac{1}{\left[1+(x/z)^2\right]^2} \quad \text{----(1)}$$

(b) Vertical stress due to strip load:-

The expression for vertical stress at any point 'p' under a strip load can be developed from eqn (1) of line load. The expression will depend upon whether the point 'p' lies below the centre of strip load or not. Consider length of strip will be unity.

(1) Point 'p' below the centre of strip:-

Figure shows a strip load of width '2b' & intensity q kN/m². Let us consider a small elementary width 'dx' at a distance 'x' from the centre of the load. This small load of q x dx can be considered as a line load of intensity q.




Now, from eqn of line load,

$$\Delta \sigma_z = \frac{2q'}{\pi z} \times \frac{1}{\left[1+(x/z)^2\right]^2}$$

$$\Delta \sigma_z = \frac{2q\,dx}{\pi z} \times \frac{1}{\left[1+(x/z)^2\right]^2}$$

Total vertical stress is,

$$\sigma_z = \frac{2q}{\pi z} \int_{-b}^{b} \frac{1}{\left[1+(x/z)^2\right]^2} \cdot dx = \frac{4q}{\pi z} \int_{0}^{b} \frac{1}{\left[1+(x/z)^2\right]^2} dx$$

$$\text{put } x/z = \tan u$$

$$dx = z \sec^2 u\, du$$

when $x = 0$, $u \Rightarrow 0$

$x = b$, $u \Rightarrow \alpha$

Then,

$$\sigma_z = \frac{4q}{\pi z} \times \int_0^{\alpha} \frac{z \sec^2 u\, du}{(1+\tan^2 u)^2}$$

$$\sigma_z = \frac{4q}{\pi z} \times z \int_0^{\alpha} \cos^2 u\, du$$

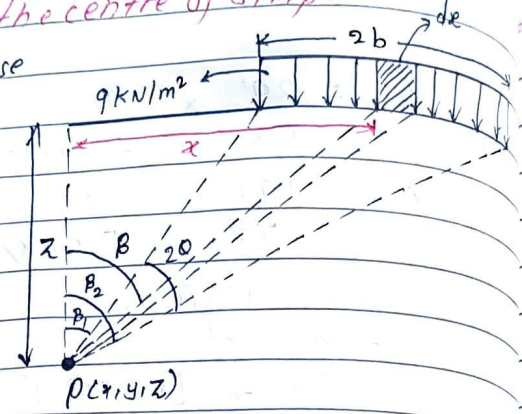
$$\sigma_z = \frac{4q}{\pi} \times \int_0^{\alpha} \frac{1+\cos 2u}{2}\, du$$

$$\sigma_z = \frac{4q}{\pi} \times \frac{1}{2} \left[\alpha + \frac{\sin 2\alpha}{2}\right]$$

$$\sigma_z = \frac{q}{\pi} \left[2\alpha + \sin 2\alpha\right]$$

(2) Point 'p' not below the centre of strip:-

Figure shows the case when the point 'p' is not below the centre of strip. The extremities of the strip makes angles of $\beta_1$ & $\beta_2$ at 'p'. The load $q\,dx$ acting on small length 'dx' can be considered as line load. The vertical stress at 'p' is given by,




$$\Delta\sigma_z = \frac{2q\,dx}{\pi z} \times \left\{\frac{1}{1+(x/z)^2}\right\}^2 \quad ---(1)$$

Assume,

$$x/z = \tan\beta. \Rightarrow dx = z\sec^2\beta\,d\beta$$

The total stress at 'p' is,

$$\sigma_z = \int_{\beta_1}^{\beta_2} \frac{2q\,dx}{\pi z}\left\{\frac{1}{1+(x/z)^2}\right\}^2$$

$$= \int_{\beta_1}^{\beta_2} \frac{2q}{\pi z} \times \left[\frac{1}{1+\tan^2\beta}\right]^2 \times z\sec^2\beta\,d\beta$$

$$= \frac{2q}{\pi}\int_{\beta_1}^{\beta_2} \cos^2\beta\,d\beta$$

$$= \frac{2q}{\pi} \times \int_{\beta_1}^{\beta_2} \frac{1+\cos 2\beta}{2}\,d\beta$$

$$= \frac{q}{\pi}\left[\beta + \frac{\sin 2\beta}{2}\right]_{\beta_1}^{\beta_2}$$

$$= \frac{q}{\pi}\left[\beta_2 + \frac{\sin 2\beta_2}{2} - \beta_1 - \frac{\sin 2\beta_1}{2}\right]$$

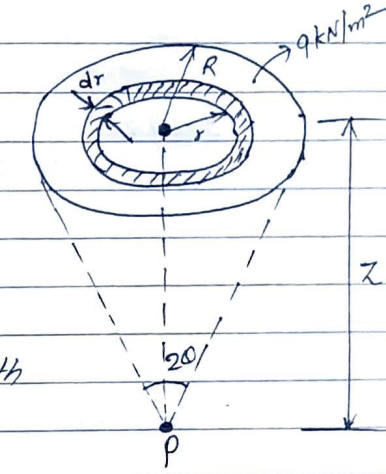
$$= \frac{q}{\pi}\left[\beta_2 - \beta_1 + \frac{2\sin\beta_2\cdot\cos\beta_2}{2} - \frac{2\sin\beta_1\cdot\cos\beta_1}{2}\right]$$

$$= \frac{q}{\pi}\left[2\theta + \sin\beta_2\cos\beta_2 - \sin\beta_1\cos\beta_1\right]$$

(c) Vertical stress due to Circular loaded Area:-

→ Consider point 'p' at a depth 'z' below the centre of uniformly loaded circular area. Let the intensity of load be 'q' per unit area & 'R' be radius of loaded area.

→ Boussinesq's solution can be used to determine $\sigma_z$. The load on the elementary ring of radius 'r' & width 'dr' is equal to $(q * 2\pi r * dr)$.




Then,

$$\Delta\sigma_z = \frac{3(q*2\pi r\,dr)}{2\pi z^2} \times \left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

The vertical stress due to entire load is given by,

$$\sigma_z = 3qz^3\int_0^R \frac{r\,dr}{(r^2+z^2)^{5/2}}$$

Let,

$r^2+z^2 = U$ when $r=0$, $U=z^2$

$2r\,dr = dU$ $r=R$, $U=R^2+z^2$

$r\,dr = dU/2$

$$\sigma_z = 3qz^3 \int_{z^2}^{R^2+z^2} \frac{du}{2u^{5/2}}$$

$$= 3qz^3 \times \frac{1}{2} \times \left[ \frac{u^{-5/2+1}}{-5/2+1} \right]_{z^2}^{R^2+z^2}$$

$$= -qz^3 \left[ \frac{1}{(R^2+z^2)^{3/2}} - \frac{1}{(z^2)^{3/2}} \right]$$

$$= qz^3 \left[ \frac{1}{z^3} - \frac{1}{(R^2+z^2)^{3/2}} \right]$$

$$= qz^3 \left[ \frac{1}{z^3} - \frac{1}{z^3} \left\{ \frac{1}{1+(R/z)^2} \right\}^{3/2} \right]$$

$$= q \left[ 1 - \left\{ \frac{1}{1+(R/z)^2} \right\}^{3/2} \right]$$

$$= q \times I_c$$

where,

$$I_c = \left[ 1 - \left\{ \frac{1}{1+(R/z)^2} \right\}^{3/2} \right]$$

= Influence coefficient for circular area.

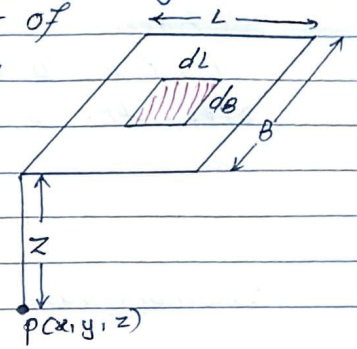
## (d) Vertical Stress Under Corner of Rectangular Area:-

The vertical stress under a corner of rectangular area with a uniformly distributed load of intensity 'q' can be obtained from Boussinesq's solution.

Let 'L' & 'B' be the ~~length~~ & width of rectangular area.

'p' be the point at depth 'z' where stress due to rectangular loaded area is to be determined.

Let dL & dB be length & width of small strip. Then, From Boussinesq's theory,




$$\Delta\sigma_z = \frac{3Q}{2\pi z^2} \left[ \frac{1}{1+(r/z)^2} \right]^{5/2}$$

$$= \frac{3Q}{2\pi} \times \frac{z^3}{(x^2+y^2+z^2)^{5/2}}$$

For small strip dL x dB,

$$\Delta\sigma_z = \frac{3q \times dL \times dB}{2\pi} \times \frac{z^3}{(x^2+y^2+z^2)^{5/2}}$$

On integration,

$$\sigma_z = \int_0^L \int_0^B \frac{3q}{2\pi} \times \frac{z^3}{(x^2+y^2+z^2)^{5/2}} \times dB \times dL$$

The integral is quite complicated. Newmark was able to perform it. which is discussed further in next topic.

## ✱ Newmark's solution & Fadum's chart:-

We have,

For a point 'p' below the corner of rectangular loaded area, the value of vertical stress is given by,

$$\sigma_z = \int_0^L \int_0^B \frac{3q}{2\pi} \times \frac{z^3}{(x^2+y^2+z^2)^{5/2}} \times dL \times dB \quad \text{--- (i)}$$

Newmark gave the solution of above expression as,

$$\sigma_z = q \times I_N$$

where,

$$I_N = \frac{1}{4\pi}\left[\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\right)\times\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right] \quad \text{--- (ii)}$$

= Newmarks influence coefficient

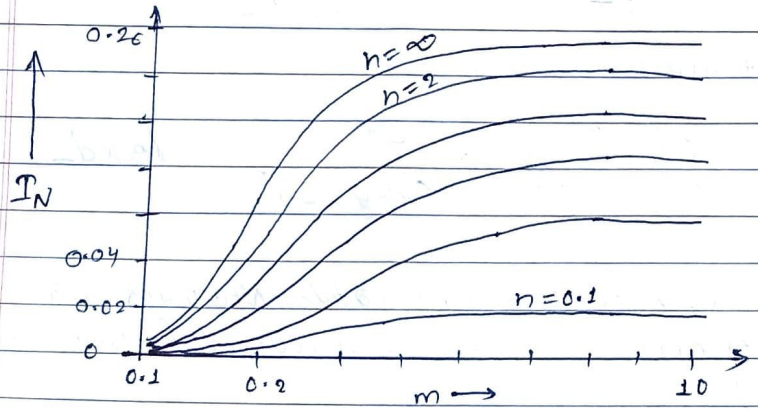
where,

$m = B/z$ & $n = L/z$

$B$ = width of rectangular loaded area

$L$ = length of rectangular loaded Area.

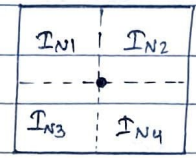
Formula for eq$^n$ (ii) is also too long. So, Fadum gave charts for determination of $I_N$.




Fadum's chart.

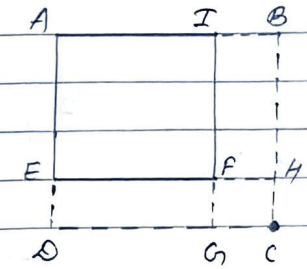
## ✱ Vertical stress at any point under a rectangular Area:-

(a) Case(i)



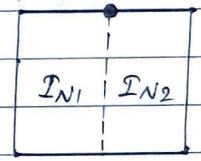

$$\sigma_z = q \times [I_{N1} + I_{N2} + I_{N3} + I_{N4}]$$

(b) Case(ii)



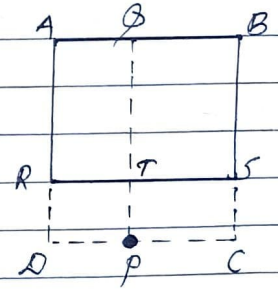

$$\sigma_z = q\,[(I_N)_{AIGD} - (I_N)_{EIGD} - (I_N)_{EAIH} + (I_N)_{FHGI}]$$

(c) Case(iii)




$$\sigma_z = q\,(I_{N1} + I_{N2})$$

(d) Case(iv)




$$\sigma_z = q\,[(I_N)_{APPD} + (I_N)_{BPCP} - (I_N)_{RTPD} - (I_N)_{TSCP}]$$

Q. A water tower has a circular foundation of 10m diameter. If the total weight of tower including foundation $2 \times 10^5$ kN. Calculate the vertical stress at a depth of 3m below the centre of foundation.

→ Given,

diameter of circular foundation (d) = 10m

weight of foundation ($\phi$) = $2 \times 10^5$ kN

depth (z) = 3m.

We know, the vertical stress below 'z' from centre of circular foundation is,

$$\sigma_z = q\left[1 - \left\{\frac{1}{1+(R/z)^2}\right\}^{3/2}\right]$$

Here, $A = \frac{\pi d^2}{4} = \frac{\pi \times 10^2}{4} = 25\pi\ m^2$

$$q = \frac{\phi}{A} = \frac{2 \times 10^5}{25\pi} = 2546.47\ kN/m^2$$

Then,

$$\sigma_z = 2546.47\left[1 - \left\{\frac{1}{1+(5/3)^2}\right\}^{3/2}\right]$$

$$= 2199.66\ kN/m^2$$

Q. A water tower weighing 15000kN is to be considered as concentrated load acting on ground surface. Compute the vertical stress at a depth of 8m below the surface. Also compute the vertical stress at a distance of 7m away from centre of water tower.

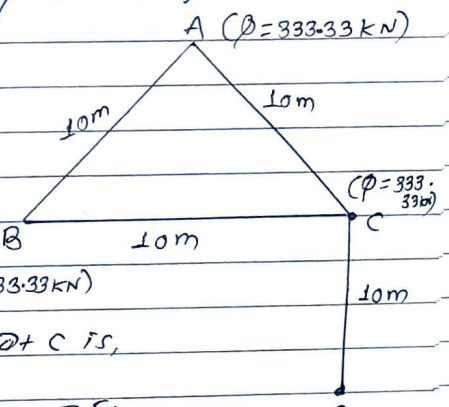
→ Given

| case (i) | case (ii) |
|---|---|
| $\phi = 15000$ kN | $\phi = 15000$ kN |
| $z = 8$m | $z = 8$m |
| $r = 0$ | $r = 7$m |
| Then, $\sigma_z = \frac{3\phi}{2\pi z^2} \times \left[\frac{1}{1+(r/z)^2}\right]^{5/2}$ | Then, $\sigma_z = \frac{3\phi}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$ |
| $= \frac{3 \times 15000}{2\pi \times 8^2} \times 1$ | $= \frac{3 \times 15000}{2\pi \times 8^2}\left[\frac{1}{1+(7/8)^2}\right]^{5/2}$ |
| $= 111.906\ kN/m^2$ | $= 27\ kN/m^2$ |

Q. A water tank on three pillars is in the form of an equilateral triangle in plan & sides of the triangle are 10m. The total weight of water tower is 1000kN. Find the vertical stress 10m below the ground surface under anyone of the legs.

→ Vertical stress at depth 'z' is,

$$\sigma_z = \frac{3\phi}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$




(a) Vertical stress at point 'p' due to load $\phi$ (1000/3) at C is,

$$(\sigma_z)_c = \frac{3 \times 333.33}{2\pi \times 10^2} \times \left[\frac{1}{1+(0/10)^2}\right]^{5/2}$$

$$= 1.591\ kN/m^2$$

(b) vertical stress at 'p' due to load $(\phi_{/3}) = (1000/3)$ at point A,

$$(\sigma_z)_A = \frac{3 \times 333.33}{2\pi \times 10^2} \times \left[\frac{1}{1+(10/10)^2}\right]^{5/2}$$

$$= 0.2813 \text{ kN/m}^2$$

(c) Similarly vertical stress at 'p' due to load $\phi_{/3}$ at point B,

$$(\sigma_z)_B = (\sigma_z)_A = 0.2813 \text{ kN/m}^2$$

∴ Total stress at point p is,

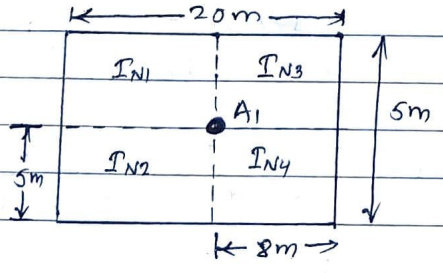
$$(\sigma_z)_p = (\sigma_z)_p \text{ due to load at A} + (\sigma_z)_p \text{ due to load at B} + (\sigma_z)_p \text{ due to load at C}$$

$$= 0.2813 + 0.2813 + 1.591$$

$$= 2.1536 \text{ kN/m}^2$$

Q. A footing is given in plan as shown in figure below. The load per unit area is 300 kN/m². Determine the intensity of vertical stress at a point directly 5m below the point A1.

→




We know, vertical stress at depth 'z' of corner of rectangular loaded area is,

$$\sigma_z = q \times I_N$$

$$= q\,[I_{N1} + I_{N2} + I_{N3} + I_{N4}]$$

| For $I_{N1}$ | For $I_{N2}$ | For $I_{N3}$ | For $I_{N4}$ |
|---|---|---|---|
| L = 12m | L = 12m | L = 8 m | L = 8m |
| b = 2.5m | b = 2.5m | b = 2.5 m | b = 2.5m |
| z = 5m | z = 5m | z = 5 m | z = 5m |
| $m = b/z = 0.5$ | m = 0.5 | m = 0.5 | m = 0.5 |
| $n = L/z = 2.4$ | n = 2.4 | n = 1.6 | n = 1.6 |
| $I_{N1} = 0.115$ | $I_{N2} = 0.115$ | $I_{N3} = 0.09$ | $I_{N4} = 0.09$ |

Value of $I_N$ is found using Fadum's chart.

Then,

$$\sigma_z = 300\,[0.115 + 0.115 + 0.09 + 0.09]$$

$$= 123 \text{ kN/m}^2$$

Q. A point load of 132 kN is applied on the ground surface. Construct a pressure bulb when stress imposed becomes 20% of applied load.

→ we have,

From Boussinesq's theory,

$$\sigma_v = I_B \times \frac{\phi}{z^2}$$

According to question, $\sigma_v = 20\%$ of $\phi = 0.2\phi$

Then,

$$0.2\phi = I_B \times \phi/z^2$$

$$\Rightarrow I_B = 0.2z^2 \quad ---- (1)$$

Also,

$$I_B = \frac{3}{2\pi} \times \left[ \frac{1}{1 + (r/z)^2} \right]^{5/2} \quad \text{---(ii)}$$

For significant depth $r = 0$. Then,

$$I_B = 3/2\pi$$

From eqn (i)

$$0.2z^2 = 3/2\pi$$

$$z = \sqrt{\frac{3}{2\pi \times 0.2}} = 1.545 \text{ m}$$

Computation of 'r' for pressure bulb diagram.

| z | 0.2 | 0.4 | 0.6 | 0.8 | 1 | 1.2 | 1.4 | 1.545 |
|---|---|---|---|---|---|---|---|---|
| $I_B = 0.2z^2$ | 0.008 | 0.032 | 0.072 | 0.128 | 0.2 | 0.288 | 0.392 | 0.4774 |
| r/z [From (ii)] | 2.032 | 1.395 | 1.0636 | 0.832 | 0.645 | 0.473 | 0.286 | 0 |
| $r = (r/z) \times z$ | 0.406 | 0.558 | 0.638 | 0.665 | 0.645 | 0.567 | 0.4 | 0 |

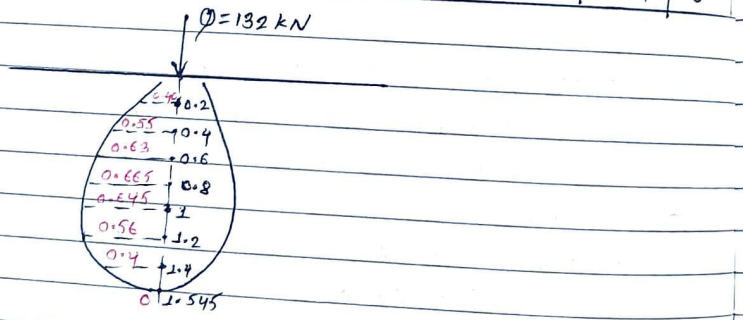



Fig: pressure bulb diagram.

## ✻ Introduction to clay minerals:-

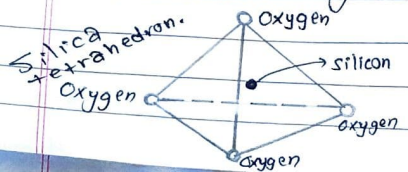
→ The behaviour of fine grained soils depends upon nature & characteristics of minerals present in the soil.
→ Mainly, surface activity of clay soil depends upon minerals present in clay.
→ The clay minerals impart cohesion & plasticity.
→ The study of clay mineral is essential for understanding the behaviour of clay soils.
→ When size of particle is less than 2μ, then it represents clay soils.

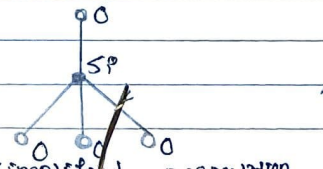
## ✻ Basic Structural units of clay minerals:-

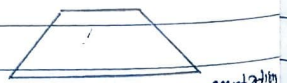
→ clay minerals are composed of two basic structural units.
i) Tetrahedral unit
ii) Octahedral Unit

### (i) Tetrahedral Unit:-

A tetrahedral unit consists of silicon atom ($Si^{4+}$) surrounded by four oxygen atoms ($O^{2-}$), forming a shape of tetrahedron. Oxygen atoms are at the tips of tetrahedron, whereas the silicon atom is at its centre. A individual tetrahedron cannot exist in nature. A number of tetrahedron unit combine to form a sheet, with oxygen atoms at the base of all tetrahedron in a common plane, & all the tips pointing in the same direction. Each oxygen at the base is shared by two tetrahedron.



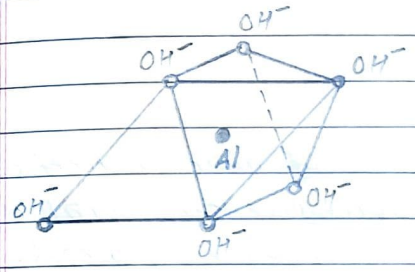



Symbolic representation of silica sheet

### (ii) Octahedral Unit:-

→ An octahedral Unit consists of six hydroxyls ($OH^-$) forming a configuration of octahedron & having one aluminium atom at the centre. As the aluminium has three positive charges, an octahedral unit has 3 negative charges. Because of net negative charge, an octahedral unit cannot exist in isolation.
→ Several octahedral units combine to form a gibbsite sheet.



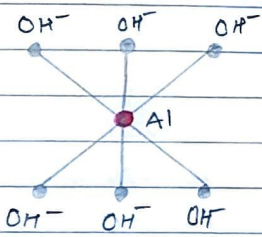




Fig: Gibbsite.

## ✻ Types of clay minerals:-

↳ kaolinite
↳ Montmorillonite
↳ Illite

### (a) kaolinite Mineral:-

→ The basic structural unit consists of an alumina sheet (gibbsite) (G) combined with silica sheet (S).
→ Tips of silica sheet and one base of alumina sheet form a common interface.
→ The total thickness of structural unit is about 7A°
[1A° = $10^{-10}$m = $10^{-7}$mm]
→ kaolinite mineral is formed by stacking, one over the other, several such basic structural units.

→ The structural units join together by hydrogen bond, which develops between the oxygen of silica sheet & hydroxyls of alumina sheet.
→ As the hydrogen bond is strong, so mineral is stable.

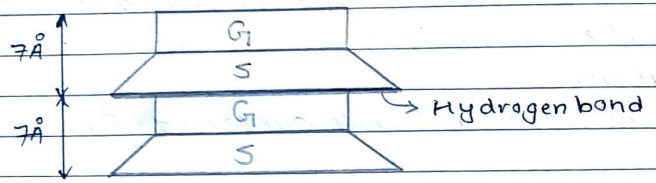
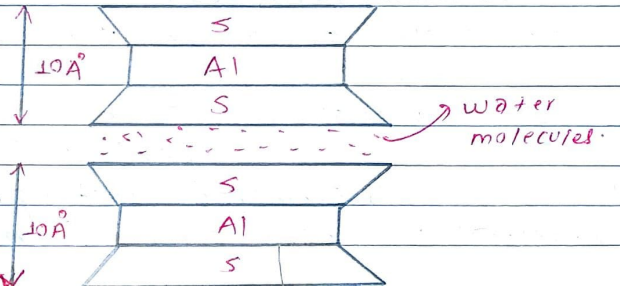



Fig: Kaolinite Mineral

(b) Montmorillonite Mineral:-

→ The basic structural unit consists of alumina sheet sandwiched between two silica sheets. Successive structural units are stacked one over another, like leaves of a book.
→ The thickness of each structural unit is about 10Å
→ The two successive structural units are joined together by a link between oxygen ions of the two silica sheets. This link is due to vanderwaal forces.
→ Due to presence of water molecules, the soil containing a large amount of minerals montmorillonite exhibits high shrinkage & high swelling characteristics.




(c) Illite Minerals:-

→ The basic structural unit is similar to that of the mineral montmorillonite. However, the mineral has properties different from montmorillonit due to following reasons.
- Illite mineral has a larger negative charge than montmorillonite
- The link between different structural units is through non-exchangable pottassium ion ($K^+$) & not through water.
- Bonds are stronger than that of montmorillonite.
- Swelling is less than montmorillonite but greater than kaolinite.
- The space betⁿ structural unit is much smaller than Montmorillonite.

→ The properties of Illite Minerals is in between kaolinite & Montmorillonite mineral.

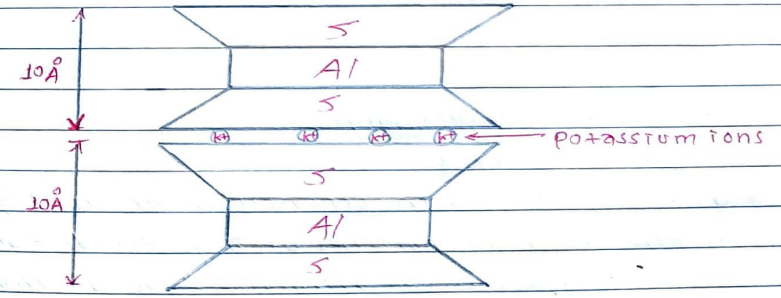



Fig: Illite Minerals.

## ✻ Clay particles Interactions:-

→ The forces between soil particles may be of two types
(i) Gravitational force
(ii) Surface force

→ Gravitational forces being proportional to mass are important in case of coarse grained soil only.

→ For fine grained soil (like clay), surface forces dominate the gravitational forces.

→ Surface force may also be either attractive or repulsive

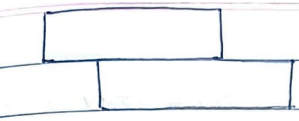
(a) attractive force:-
- Vander waal forces
- Hydrogen bond
- Cation linkage
- dipole cation dipole linkage
- Water dipole linkage
- Ionic bond etc.

(b) Repulsive force:-
- Due to similar charges
- Cation repulsion.

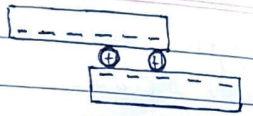
→ For a given type of clay in suspension, the net force between the adjacent particles at a given distance is algebric sum of repulsive & attractive force at that distance.

→ The interparticle force decreases with increasing distance from mineral surface.

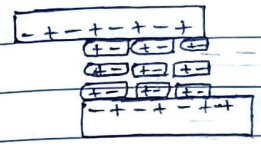
→ When total potential energy decreases, they approach each other, the particle will experience attraction & will flocculate. & with increase in P.E, particles will dispersed.



Hydrogen bond



cation Linkage.



Water dipole linkage.

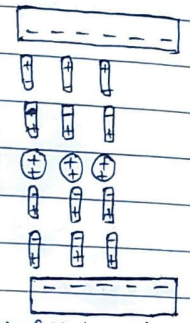
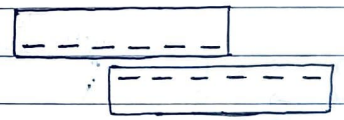
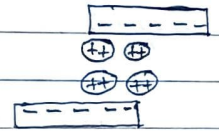


Fig:- Dipole-cation-dipole linkage.

Fig:- Interparticle Attractive forces.



particle repulsion due to similar charges



cation Repulsion

Fig:- Interparticle Repulsive forces.

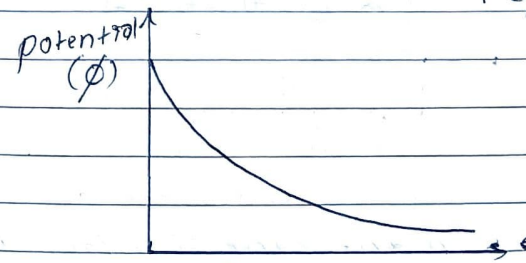
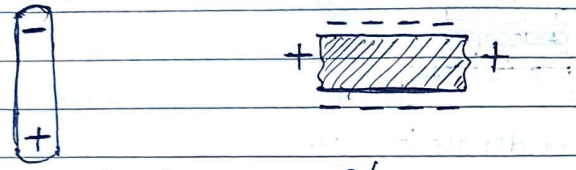



Fig:- potential ~~from~~ versus distance from mineral surface.

## Diffuse double layer:-

→ Normally clay soils are associated with water & its properties are significantly influence with the presence of water.

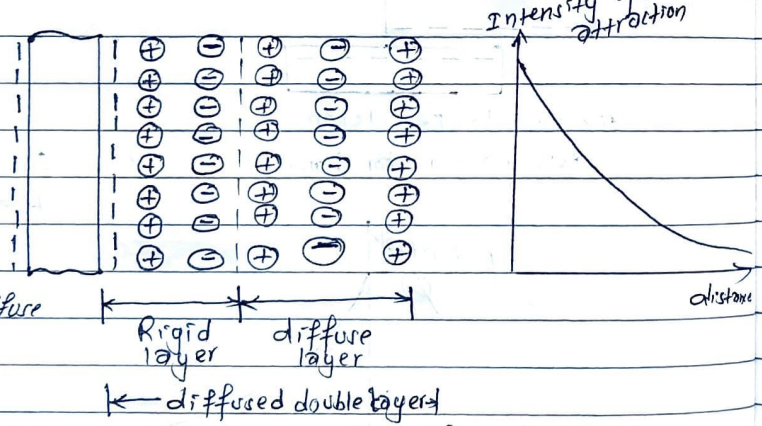


Water molecule  Clay particle

→ Water molecule is dipole, since hydrogen atoms are not symmetrically oriented around oxygen atom. The molecule act as bar magnet.

→ The faces of clay mineral carry a net negative charge except at edge.

→ The layer extending from clay particles surface to limit of attraction is known as diffuse double layer.




→ The water held in the zone of diffuse double layer is known as adsorbed water.

→ Adsorbed water imparts plasticity to the clay.

→ kerosene being non-polar liquid doesnot cause plasticity to the clay.

## Soil Structures:-

→ The geometric arraggement of soil particles with respect to one another is known as soil structure.

→ The soils in nature have different structures depending upon the particle size & Mode of formation. The following types of structure are usually found.

(i) Single grained structure
(ii) Honey-comb structure
(iii) Flocculated structure
(iv) Dispersed structure.
(v) Coarse grained skeleton
vi) Clay matrix structure.

### (a) Single grained Structure:-

→ This kind of structure is found in coarse grained soils.

→ Grains which are cohesionless and makeup soils like gravel & sand, form this kind of structure.

→ These grains are large enough & gravity is the dominant force acting on them. When these particles get deposited they acquire an equilibrium position by each particle being in contact with other surrounding particles. Soil structure so formed is called single grained structure.

→ Closely the particles are packed together leaving very less void space betn them, denser is the structure. When soil particles are loosely packed, leaving large void spaces between them, makeup loose soil structure.

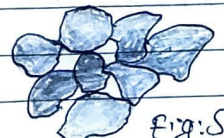


Fig: Single grained structure.

(b) Honey Comb structure:-

→ Sometimes smaller soil particles of silt sized when depositing join with one another & form a bridge like structure. They contain large voids between those bridges & make the soil very loose in nature. Such a formed structure is called honey-combed structure.

→ As soil in honey-comb structure is loose they can support load only static condition. When they are subjected to vibration or shock, the structure collapses & large deformation takes place & soil achieves relatively denser state.

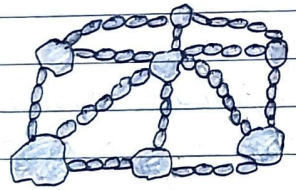


Fig: Honeycomb structure

(c) Flocculated structure:-

→ Clay particles are very small, flaky in shape & they have large surface area & because of that their surface forces are more prominent in them than gravitational force. These particles have negative charge on the surface & positive charges on edges. They combine with each other by joining negative surface of particle to positive edge of the other. This way they form a soil structure which is called flocculated structure.

→ Soils with flocculated structure have large amount of voids, so, their void ratio is high. These soils are less sensitive to vibrations as they formed a strong electrical bond.

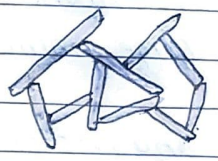


Fig: Flocculated structure

(d) Dispersed Structure:-

→ When clay soils are remoulded, their flocculated structure changes and particles change their orientation from edge to face orientation to face to face orientation. This kind of formed soil structure is called dispersed soil structure.

→ These soils have relatively lower volume of voids consequently low void ratio.

→ When soils are in dispersed structure, there will be low shear strength, high compressibility & low permeability.

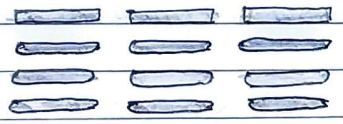


Fig: Dispersed Structure

* When both coarse grained & fine grained particles are present in the soil, they make up two kind of soil structure.
  - Coarse grained skeleton
  - Cohesive Matrix structure.

(e) Coarse Grained Skeleton:-

→ In coarse grained skeleton, coarse grained particles are in large amount than fine grained particles & coarse particles remain in direct contact with other coarse particles forming a framework or a skeleton.

→ The space bet^n these large grains is occupied by fine particles.

→ These soils are stable and less compressible. They can take heavy loads without much deformation.

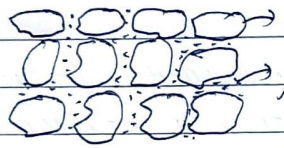



Fig: Coarse grained skeleton

(f) Cohesive Matrix Structure:-

→ In cohesive matrix structure, fine grained particles are in large amount than coarse grained particles.

→ Coarse grains appear embeeded in fine grains if there is no direct particle to particle contact bet^n them.

→ Their behaviour is similar to that of an ordinary clay deposit.

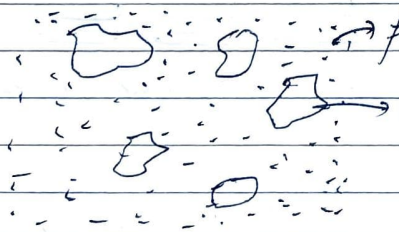


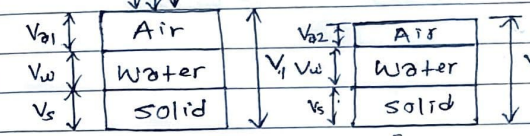

Fig: Cohesive Matrix structure.

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ch-5

# Soil Compaction:-

The phenomenon of packing of soil particles to increase dry density by reducing air voids after the application of dynamic load or by mechanical means.




Before compaction ~~Bef~~ After compaction.

$V_1 > V_2$, $V_{a2} < V_{a1}$

* Compaction involves:-
→ Expulsion of air from voids
→ Rearrangement of soil solid.
→ Increase of denseness
→ Reduction in permeability of soil
→ Reduction in potential of swelling & shrinkage.
→ Reduction in compressibility
→ Increase in shear strength & bearing capacity of soil
→ Increase in dry density.
→ Decrease in volume.

* Objective of Compaction:-
→ Increase dry density of soil.
→ Increase shear strength & bearing capacity of soil
→ Decreasing the undesirable settlement of structure.
→ Control undesirable volume changes.
→ decrease permeability of soil.
→ Increasing stability of slope.

## * Degree of compaction:-

→ Degree of compaction is measured in terms of dry density ($\gamma_d$).

→ Amount of particles packed in unit volume of soil.

$$\gamma_d = \frac{\gamma}{1+w}$$

$\gamma_d$ → dry density.
$w$ → % water content
$\gamma$ → Bulk density.

If $\gamma_d > 90\%$ → Maxm compaction acheived.

## ※ Lab Method for determining dry density.

| Standard Proctor's test | Modified proctor's test |
|---|---|
| → Take 2.5 kg of soil sample passing through 4.75 mm sieve. | → Take 5 kg of soil sample passing through 4.75 mm sieve. |
| → place the sample in standard mould of size, drameter- 100 mm, height - 127.3 mm & capacity - 1000 cc | → place the sample in standard mould of size, diameter 150 mm, height - 127.3 mm & capacity 2249 cc. |
| → Fill mould by 3 layers with collar above. | → Fill mould by 5 layers with collar above. |
| → Each layer is compacted by std. Rammer of weight 2.5 kg & free fall of 310 mm. | → Each layer is compacted by std. Rammer of weight 4.9 kg and free fall of 457.10 mm. |
| → Compaction energy, $E_n = \frac{NnWh}{V} * 100\%$ | → Energy required for compaction is 2700 KJ/m³ about 4.5 times greater than standard proctor test. |
| Energy required for compaction 592 KJ/m³. | |

In compaction energy formula,
N → Number of blow per layer
n → Number of layer
W → weight of rammer
h → Height of free fall
V → volume of mould.

## * Observation & Calculation:-

Mass of mould + base (empty) = (1)
Mass of mould + Compacted soil = (2)
Mass of compacted soil = (2) − (1) = M (say)
Volume of Mould = $\frac{\pi D^2}{4} \times h = V$

Density of compacted soil = $\frac{M}{V} = \gamma$

Dry density ($\gamma_d$) = $\frac{\gamma}{1+w}$

$w$ → water content %
Mass of empty can = (I)
Mass of can + wet soil = (II)
Mass of can + oven dried soil = (III)
(After 24 hour)
Mass of water ($M_w$) = (II) − (III)
Mass of soil solid = $M_s$ = (III) − (I)
Then,

$$w = \frac{M_w}{M_s} \times 100\%$$

Finally,

$\gamma_d = - - -$
$w = - - -$

## * Compaction Curve:

→ The graph of dry density ($\rho_d$) versus water content gives compaction curve.

→ It is observed that $\rho_d$ initially increases with increase in water content till maximum density $(\rho_d)_{max}$ is attained. With further increase in water content, the dry density decreases. The water content corresponding to maximum dry density is known as Optimum water content (OWC) or optimum moisture content (OMC).

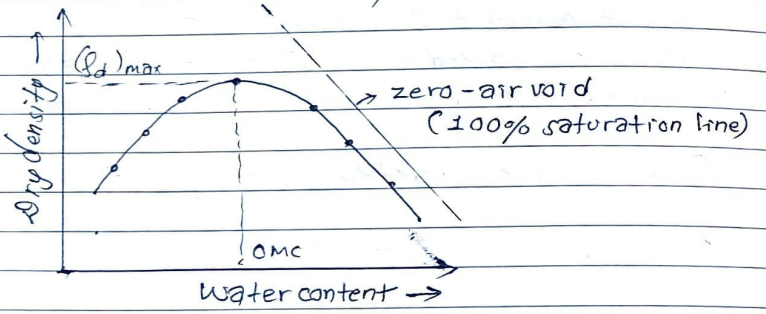
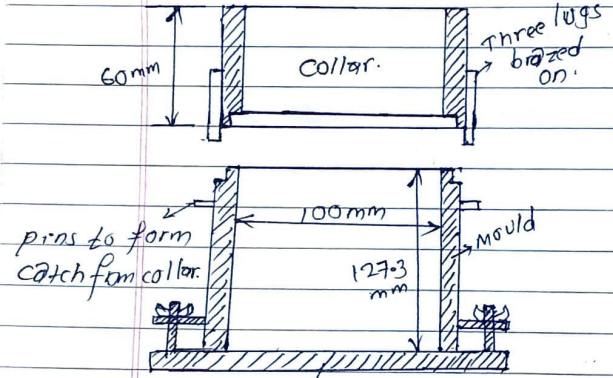
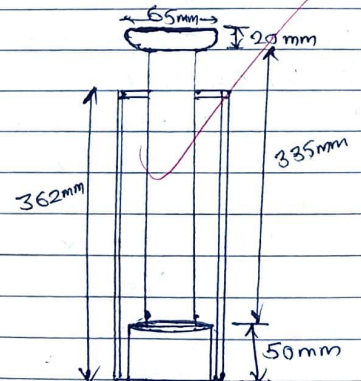



Fig: Compaction curve.




## * Zero Air Void Line (100% saturation line)

Practically, it is impossible to reduce completely the air voids (up to zero) but theoretically it has been assumed soil having zero-air voids.

According to void line equation,

$$\rho_d = \frac{(1-\eta_a)\cdot G\rho_w}{1+wG} \quad \text{----(II)}$$

where, $\eta_a$ = % air voids = $\frac{V_a}{V_v} = 0$ [$\because V_a = 0$]

$$\rho_d = \frac{G\rho_w}{1+wG} \quad \text{----(II)}$$

Again, from saturation line equation, $\rho_d = \frac{G\rho_w}{1+\frac{wG}{S}}$

$$S = \text{Degree of saturation} = \frac{V_w}{V_v} = \frac{V_w}{V_a + V_w}$$

Since, $V_a = 0$, $S = 1 = 100\%$

$$\rho_d \text{ (theoretical)} = \frac{\rho_w G}{1+wG} \quad \text{----(III)}$$

Finally, Eqn (II) = Eqn (III),

ZAVL = 100% saturation line

If we plot,

$\rho_d$ (theoretical) → y axis

$w$ → x axis.

The linear curve is obtained (ZAVL)

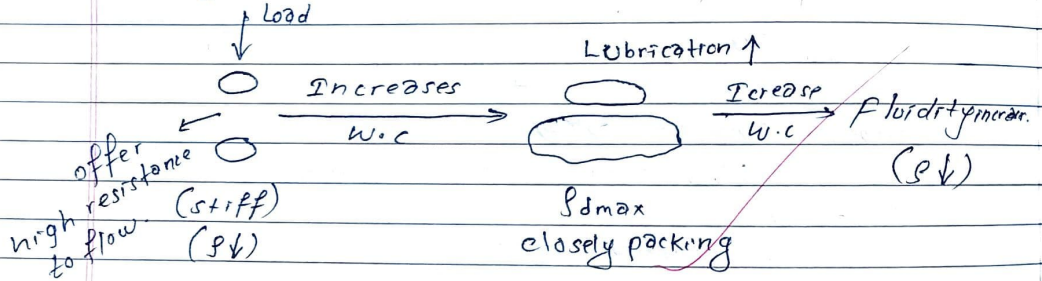
Saturation lines:- For a compacted soil the line showing relation between the water content and the dry density of the soil for a constant degree of saturation (S %) is known as saturation line.

## ※ Factors Affecting Compaction:-

(a) Water Content:-
(b) Amount of compaction
(c) Type of soil being compacted
(d) Method of Compaction
(e) Admixtures etc.

### (a) Water Content:-

When water content is low, the soil is stiff and offers more resistance to compaction. We know, the compaction of soil depends upon dry density of soil. When water content increases, dry density increases till optimum water content is reached. At this condition dry density is maximum. With further increase the water content behind OWC, the dry density decreases.




### (b) Amount of compaction:-

→ With increase in amount of compactive force, the maximum dry density increases & OWC decreases.
→ For a certain increase in compactive force, the increase in dry density becomes smaller.

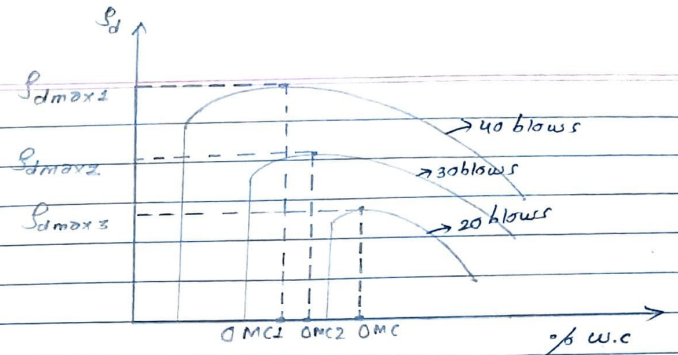
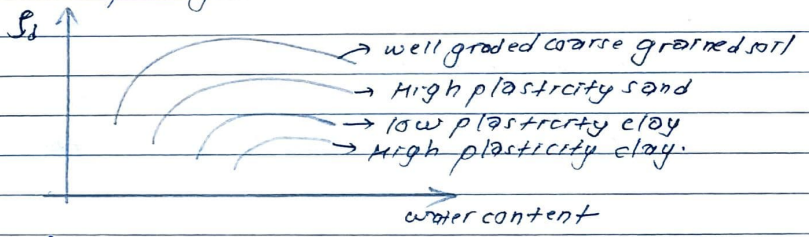



Fig: Effect of increasing compactive effort.

### (c) Type of soil being compacted:-

The dry density depends upon type of soil being compacted. ~~The~~ In general, coarse grained soil can be compacted to higher dry density than fine grained soils.




### (d) Method of Compaction:-

The dry density achieved not only depends upon amount of compaction but also depends upon method of compaction. The following factors determine the dry density of soil during compaction.

i) Types of compacting equipment
ii) Area of contact
iii) Time of contact
iv) Nature of effect ~~compaction~~ produced by compacted equipments. (Pressure, kneading, Impact, vibration)

→ Types of compacting equipments.

a) Roller
   ↳ Smooth wheel Roller
   ↳ Sheep foot Roller
   ↳ Pneumatic tired roller

b) Vibrator

c) Rammer

→ Smooth wheel Roller
- Pressure action
- Coarse soil
- Crushed gravel / sub-base
- Eg: Road.

→ Sheep foot Roller
- Kneading action
- Fine grained soil
- Eg: Dam construction

→ Pneumatic Tired Roller
- pressure + kneading action
- Both coarse grained + fine grained soil
- Eg: Airfield Construction.

→ Vibrator
- Vibration Action
- Sand
- confining area
- Oil storage tank

→ Rammer
- Impact action
- Sand
- Retaining wall

(e) Admixtures:-

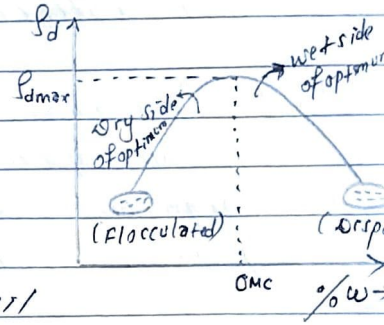
The compaction characteristics of soils are improved by adding other materials, known as admixtures. The most commonly used admixtures are lime, cement & bitumen. The dry density achieved depends upon the type & amount of admixtures.

## * Effect of Compaction on engineering behaviour of fine grained soil:-

The engineering properties of soils are improved by compaction. The effect of compaction on various properties of soil are described below.

(a) Soil Structure:-

The water content at which the soil is compacted plays an important role in the engineering properties of soil. Soils compacted at water content less than OMC generally have flocculated structure whereas the soil compacted at a water content more than OMC generally have dispersed structure.




(b) Permeability:-

The permeability of soil decreases with increase in water content. On dry side of optimum, permeability increases & on wet side of optimum, permeability decreases because w.c increases.

(c) Pore water pressure:-

The soil ~~compted~~ compacted on dry side of optimum has low pore water pressure whereas the soil compacted on wet side of optimum has high pore water pressure.

(d) Compressibility:-

On dry side of optimum, the flocculated structure is

developed which offers greater resistance to compression. So, the soils on the dry side are less compressible.

(e) Strength:
On dry side of optimum, strength is maximum whereas on wet side of optimum, strength is minimum due to more water content.

(f) Swelling:
A soil compacted dry of the optimum water content has high water deficiency and more random orientation of particle. So, in dry side of optimum, swelling is more.

(g) Shrinkage:
Soils compacted dry of optimum shrink less on drying compared with those compacted wet of the optimum.

## ✻ Relative Compaction:-

→ The ratio of dry density achieved in the field to the maximum dry density obtained from standard / modified proctor's test is known as relative compaction.
Thus,

$$R.C = \frac{\gamma_d \text{ in the field}}{(\gamma_d)_{max} \text{ in laboratory}} \times 100$$

→ If Relative compaction is greater than 95%, it is assumed that, good compaction is achieved.

## ✻ Compaction Control in field:

The compaction control is done by measuring the dry density & water content of the compacted soil in field. Dry density can be calculated by using core cutter or sand replacement method whereas water content can be calculated by using proctor's needle method.

(a) Method Specification (Destructive test)
(i) Sand Replacement method
(ii) Core cutter method

(b) End product Specification
(i) Water content placement test / proctor's needle method

## ✻ Sand Replacement method:

The sand replacement method consists of making of hole into the ground. The excavated soil is weighted. The volume of hole is determined by replacement with sand. Knowing the weight of excavated soil & volume of hole, the ensitu density can be easily determined. When ensitu density is known, dry density can be calculated as,

$$\gamma_d = \frac{\gamma_t}{1+w}$$

where, $\gamma_t$ = field (insitu) density of soil
$w$ = water content.

Procedure:-
1. The site is cleaned & square tray with a central hole in it is placed on the cleaned surface.

2. The hole of diameter equal to diameter of the hole in the tray & depth about 10-15cm is made in the ground.
3. The excavated soil in a tray was collected & weighted & its water content is determined.
4. The pouring cylinder is filled with sand to about 3/4 of capacity & its weight was taken. The density of sand is known already.
5. The cylinder is then placed over the hole & tap is opened. The sand is allowed to run to fill the excavated hole and conical end.
6. When no further flow of sand is taken place, the tap is closed & bottle with remaining sand is weighed. The weight of sand filling the cone of cylinder was taken separately.

Here,

Mass of cone = $(M_1)$ ~~gm~~

Mass of cylinder with sand = $M_2$ gm

Mass of cylinder after pouring in hole = $M_3$ gm

Mass of soil collected on tray = $M_4$ gm

Volume of soil $(V_s) = \frac{M_2 - M_3 - M_1}{\rho_{sand}}$

Field density $(\rho_t) = \frac{M_4}{V_s}$

& dry density of soil $(\rho_d) = \frac{\rho_t}{1+w}$

For w, Mass of empty can = $m_1$

Mass of can with soil = $m_2$

Mass of can with dry soil = $m_3$, Then, $w = \frac{m_2 - m_3}{m_3 - m_1} \times 100\%$

## ✱ Core cutter method:

→ It is a field method for determination of dry density. A core cutter consists of an open, cylindrical barrel with a hardened sharp cutting edge. A dolley is placed over the cutter & it is rammed into the soil. The soil extruding above the edge of cutter is removed. The mass of cutter filled with soil is taken. A representative sample is taken for water content determination. The volume of soil is equal to the internal volume of the cutter which can be determined from the dimensions of the cutter. Then,

Field density $(\rho) = \frac{M_2 - M_1}{V}$ Then, $\rho_d = \frac{\rho_t}{1+w}$

where, $M_2$ = Mass of cutter, with soil

$M_1$ = Mass of empty cutter

$V$ = Internal volume of cutter

→ This method is suitable for soft, fine grained soils.

Procedure:-

a) The mass of clean core cutter was noted $(M_1)$

b) The ground was cleaned & leveled where density to be determined.

c) The cylinder cutter was pressed into the soil to its full depth by steel Rammer.

d) The core cutter was removed from soil layer excavating the soil surrounding the cutter.

e) The mass of cutter with soil sample is taken. For detm of water content, process is similar to that of sand replacement method.

## * Proctor's needle method:-

The water content can be determinged directly using proctor's needle. It consists of Handle, stem, sliding ring, needle pointer. The equipment is provided with a series of needle points of different cross-sectional areas (0.25, 0.5, 1 & 2.5 $cm^2$) The graduated needle shank measures the depth of penetration in cm. Graduated stem measure the penetration resistance. The sliding ring on the stem indicates the maximum resistance recorded during the test. The proctor's needle is rotated at the speed of 1.25 cm/sec upto depth of 7.5cm. From the known area of needle, penetration resistance per unit area (R) is recorded. A number of such measurements are made in laboratory & curve is obtained betn the penetration resistance (R) ($kN/cm^2$) & water content (w%) as shown in fig. The penetration resistance decreases with increase in water content. The moisture content is obtained from calibration curve.

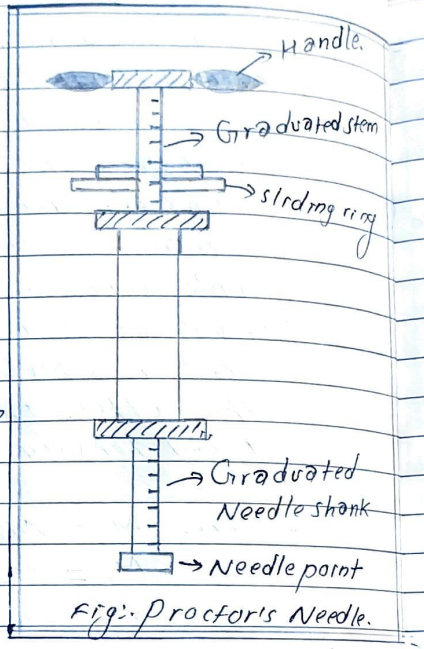
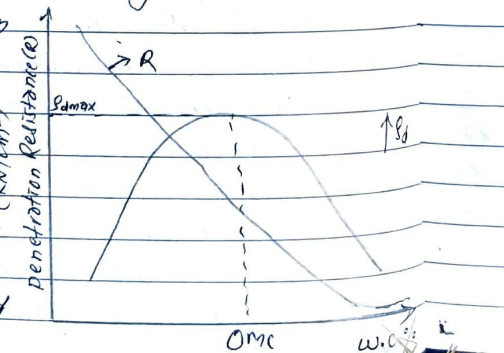


Fig:- Proctor's Needle.




## * Compaction of sand / Bulking of sand:-

→ The behaviour of fine grained soil is totally different from coarse grained soil.

→ Sand posses large void but less in number.

→ With increase in water content, capillarity tension increases as a result apparent cohesion is increased & there is formation of honeycombed structure due to which volume increases. The increased volume is maxm at 4 to 6% of w.c and after this on further increases the water content volume decreases.

→ This property of increase in volume & decrease in dry density of sand is known as bulking of sand.

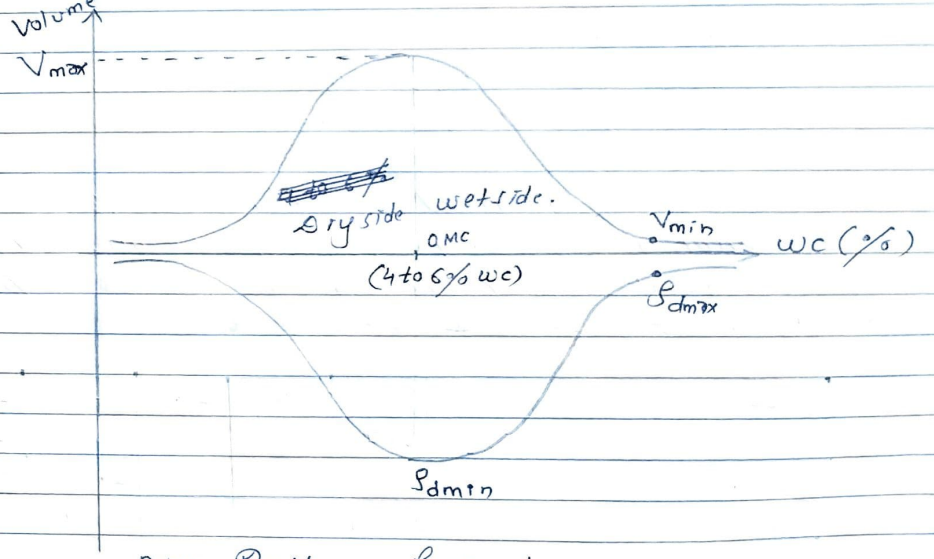


Fig:- Bulking of sand.

## Numericals

Formula:- $\gamma_d = \frac{\gamma}{1+w}$, $\gamma_d = \frac{(1-n_a) G \gamma_w}{1+wG}$ → in terms of % air voids

$\gamma_d = \frac{G\gamma_w}{1+wG}$ (ZAVL) $\gamma_d = \frac{G\gamma_w}{1+\frac{wG}{S}}$ → in terms of degree of saturation

$\gamma_d = \frac{G\gamma_w}{1+e}$ → In terms of void ratio

$S_e = w.G$

* Standard proctor test was conducted on soil sample & following observation were made.

| % w | 7.7 | 11.5 | 14.6 | 17.5 | 19.5 | 21.2 |
|---|---|---|---|---|---|---|
| Mass of wet soil (kg) | 1.7 | 1.89 | 2.03 | 1.99 | 1.96 | 1.92 |

Calculate,

(i) & raw compaction curve

(ii) Maxm dry density & OMC

(iii) ~~Opt~~ 80% & 100% saturation line

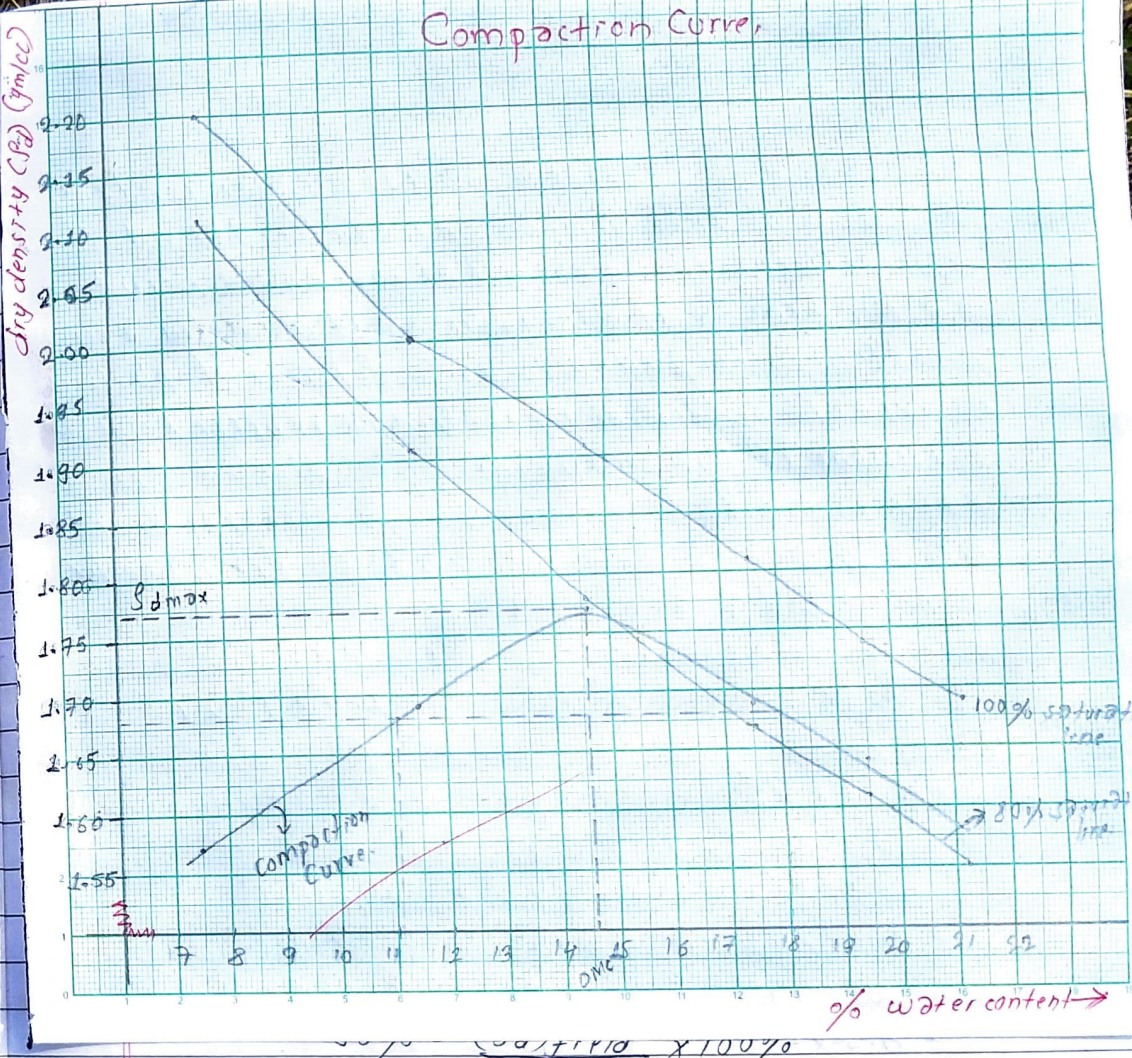
(iv) ZAVL

v) If it is assumed to achieve 95% RC in field, what is the range of wc to be allowed?

vi) Would 20% voids curve be same as 80% saturation line?

→ Here, $G = 2.65$ & $V = 1000$ cc

| % w | 7.7 | 11.5 | 14.6 | 17.5 | 19.5 | 21.2 |
|---|---|---|---|---|---|---|
| m (kg) | 1.7 | 1.89 | 2.03 | 1.99 | 1.96 | 1.92 |
| $\rho = m/v$ (gm/cc) | 1.7 | 1.89 | 2.03 | 1.99 | 1.96 | 1.92 |
| $\rho_d = \rho/(1+w)$ | 1.578 | 1.695 | 1.771 | 1.693 | 1.6401 | 1.584 |
| $\rho_d (80\%) = \frac{G\rho_w}{1+\frac{Gw}{S}}$ | 2.111 | 1.918 | 1.786 | 1.677 | 1.61 | 1.556 |
| $\rho_d (100\%)$, $S=1$, $ZAVL = G\rho_w/(1+wG)$ | 2.2 | 2.031 | 1.9107 | 1.810 | 1.747 | 1.696 |




$\frac{(\rho_d)_{field}}{(\rho_{dmax})_{lab}} \times 100\%$

$\Rightarrow (\rho_d)_{field} = 1.68$

Then,

From graph, range of wc = (11.1 – 18)%

(vi) For 20% voids, $\rho_d = \frac{(1-n_a) G\rho_w}{1+wG}$

At $w = 7.7\%$, $\rho_d = 1.7607$ gm/cc

Also, for 80% saturation line, $\rho_d = \frac{G\rho_w}{1+\frac{wG}{S}}$

At $w = 7.7\%$ & $S = 0.8$, $\rho_d = 1.918$ gm/cc

Hence 20% void line & 80% S.L are not same.

* A borrow area soil has a natural water content of 10% and bulk density of 1.8 gm/cc. The soil is used for embankment to compacted at 20% moisture content to a dry density of 1.9 gm/cc. Determine the amount of water to be added to 1 $m^3$ of borrow of soil. How many cubic meters of excavation is required for 1 $m^3$ of compacted embankment.

→ For borrow area

$w = 10\%$

$\gamma = 1.8$ gm/cc

$\gamma_d = \frac{\gamma}{1+w} = 1.63$ gm/cc

Then,

$W_s = \gamma_d * V$

$= \frac{1.63 \times 10^{-3}}{10^{-6}} \times 1$

$= 1630$ kg

$W_w = w \times W_s$

$= 0.1 \times 1630$

$= 163$ kg

For embankment

$w = 20\%$, $\gamma_d = 1.9$ gm/cc

$W_s = \gamma_d * Vol$

$= \frac{1.9 \times 10^{-3}}{10^{-6}} \times 1$

$= 1900$ kg

$W_w = w \times W_s$

$= 0.2 \times 1900$

$= 380$ kg

∴ Amount of water to be added $= 380 - 163$

$= 217$ kg

Total excavation reqd for 1 $m^3$ $= \frac{(W_s)_{embankment}}{(W_s)_{borrow}} = \frac{1900}{1630}$

$= 1.165\ m^3$

* A sample of soil is prepared by mixing quantity of dry soil with 11% by weight of water. Find the weight of this wet mixture which will be required through static compaction in a cylindrical sample (Mould) 150mm diameter and 125mm deep with 5% air voids. Also find the dry density & void ratio of soil.

→ Given,

$w = 11\% = 0.11$

Volume of sample $(V) = \frac{\pi \times 0.15^2}{4} \times 0.125$

$= 2.2089 \times 10^{-3}\ m^3$

% air voids $(n_a) = 5\%$

Weight of wet mixture $(W) = ?$

dry density $(\gamma_d) = ?$

void ratio $(e) = ?$

Now,

$\gamma_d = \frac{(1-n_a) G \gamma_w}{1+wG} = \frac{(1-0.05) \times 2.7 \times 10}{1+0.11 \times 2.7}$

$\gamma_d = 19.77\ KN/m^3$

And,

$\gamma_d = \frac{\gamma}{1+w} \Rightarrow \gamma = 19.77 * (1+0.11)$

$\Rightarrow \gamma = 21.95\ KN/m^3$

Again,

$\gamma_d = \frac{G\gamma_w}{1+e} \Rightarrow 21.95 = \frac{2.7 \times 10}{1+e}$

$\Rightarrow e = 0.3657$

$W = \gamma * V = 21.95 * 2.2089 \times 10^{-3} = 0.04848\ KN$

* An earthen embankment of $10^6 m^3$ volume is to be constructed with a soil having void ratio of 0.80 after compaction. There are three borrow pit marked by A, B & C having soil with void ratio of 0.90, 1.50 & 1.80 respectively. The cost of excavation & transporting the soil is Rs 0.25, 0.23 & 0.18 per $m^3$ respectively. Calculate the volume of soil to be excavated from each pit, which pit is most economical?

→ For borrow pit,

| A | B |
|---|---|
| $e_A = 0.90$ | $e_B = 1.50$ |
| $C_A$ = Rs $0.25/m^3$ | $C_B$ = Rs $0.23/m^3$ |
| $V_A = (1+e_A) * V_s$ | $V_B = (1+e_B) * V_s$ |
| $= (1+0.9) * 0.55 \times 10^6$ | $= 1375000\ m^3$ |
| $= 1055555.56\ m^3$ | |
| $T.C_A$ = Rs 263888.8 | $T.C_B$ = Rs 316250 |

| C | Embankment |
|---|---|
| $e_C = 0.80$ | $e = 0.80$ |
| $C_C$ = Rs $0.18/m^3$ | $V = 10^6 m^3$ |
| | $V = (1+e) V_s$ |
| $V_C = (1+e_C) \times V_s$ | $10^6 = (1+0.8) V_s$ |
| $= 1540000\ m^3$ | $V_s = 0.55 \times 10^6 m^3$ |
| $T.C_C$ = Rs 277200 | |

Since, the cost of excavation for borrow pit A is less than B and C. So, borrow pit 'A' is most economical.

## ** Consolidation:-

→ Consolidation is the phenomena of gradual reduction in volume of soil mass due to squeezing of pore water after the application of static load.

→ The property of soil by virtue of which volume decrease occurs under applied pressure is termed as compressibility.

→ Water is almost incompressible. The compression of soils can occur due to expulsion of water in voids.

→ Consolidation may occur due to one or more of the following reasons.
- Due to structural load <whatever> acted on soil mass.
- Due to lowering of G.W.T.
- Due to self weight of soil (recently placed fill)

→ Consolidation is time dependent phenomenon, especially in clays and it is continuous for long time. It is the property of fine grained soil because the fine grained soil mass has large void ratio & higher w.c.

→ Consolidation may be i) Initial Compression
ii) primary consolidation
iii) Secondary Consolidation.

### (i) Initial Compression:-

The reduction in volume of soil just after application of load is known as initial compression/initial consolidation. For saturated soils, the initial compression is mainly due to compression of solid particles.

### (ii) Primary Consolidation:-

→ After initial consolidation, further reduction in volume occurs due to expulsion of water from voids.
→ When saturated soil is subjected to a pressure, initially all pressure is taken by water (pore water pressure). A hydraulic gradient develops & water starts flowing out & decrease in volume occurs.
→ This process is natural & time dependent.
→ In coarse grained soil, primary consolidation occurs quickly due to high permeability.
→ About 90% consolidation is primary consolidation.

### (iii) Secondary Consolidation:-

→ After primary consolidation, the reduction in volume continues at very slow rates.
→ Secondary consolidation is the condition of soil in which water is completely removed/ expulsed from voids but this is almost impossible.
→ Rearrangement of soil solids.
→ About 10% consolidation is secondary consolidation.

### ✻ Difference betn compaction & consolidation:-

| Compaction | Consolidation |
|---|---|
| a) process of expulsion of air from voids to increase dry density of soil. | a) process of expulsion of water from voids to increase effective stress. |
| b) The soil involved in compaction is partially saturated. | b) the soil involved in consolidation is fully saturated. |
| c) Compaction is quick process than consolidation. | c) Consolidation is slow process than compaction. |
| d) Dynamic loading is applied for compaction. | d) Static loading is responsible for consolidation. |
| e) Compaction is artificial process (This means dynamic loading is applied by human) | e) Consolidation is natural process. |
| f) Applies to cohesive as well as cohesionless soils. | f) Applies to cohesive soils only. |

### ✻ Terzaghi's Spring Analogy for demonstration of consolidation:

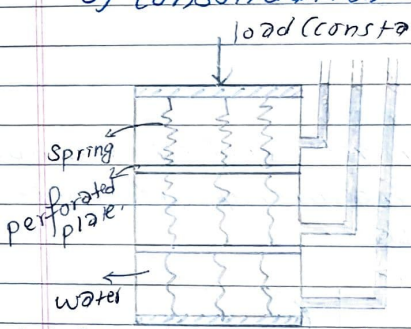


Fig a: just after apply load

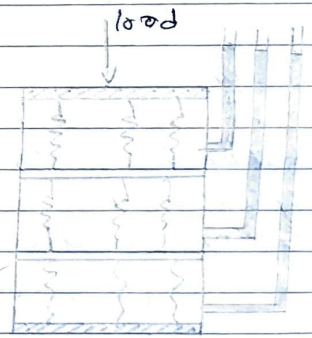


Fig b: All pressure resist by water

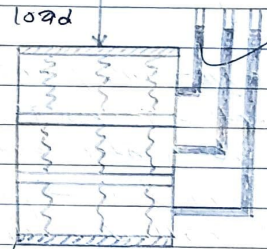


Fig c:- Spring Compress &

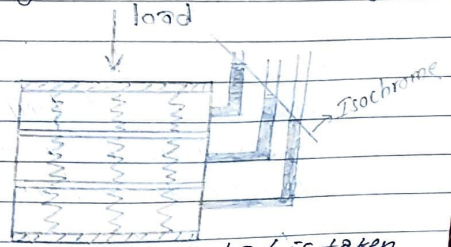
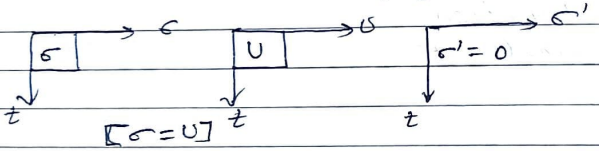


Fig d: Totally load is taken

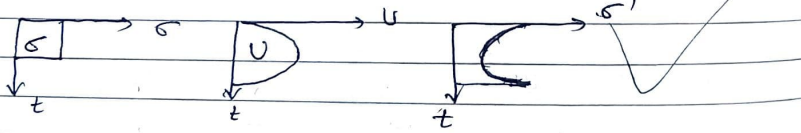
Consider a cylinder fitted with piston. A piezometer is connected on the side of cylinder. The cylinder is filled with water and contains a spring of specified stiffness.

At stage 1, t = 0, total system is at rest just applying the load. Hydrostatic condition is set up which is shown in fig 'a'.

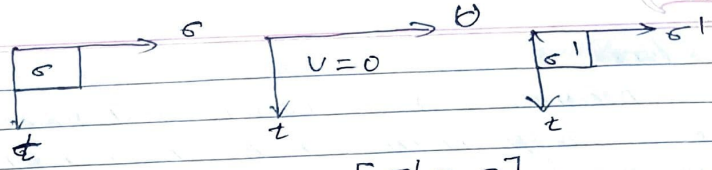
At stage 2, for few time, the load applied is resist by pore water pressure only. No load is taken by spring but water level on piezometer rises up which is shown in figure 'b'.




At stage 3, t >>, the water get expulsed & volume of cylinder is decreased. The spring gets compressed & start to take a load. (fig c)




At stage 4; t >>>, with increase in time, pore water pressure is totally zero & effective stress gets increased. The volume of water decreases & water level on prezometers have same head. The line ~~of~~ joining same heads of diff^n prezometer is called isochrome.



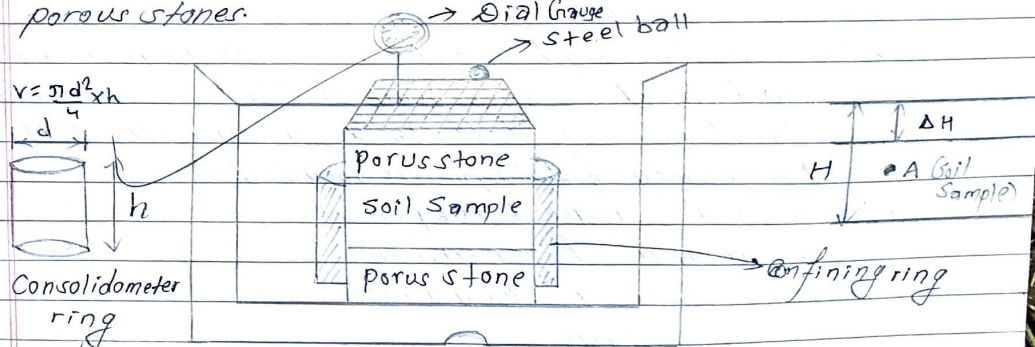

[σ' = σ]

Fig 'a' shows initial consolidation, Fig (b) & (c) shows primary consolidation in which water expulsion increases & effective stress also increases. Fig (d) shows the secondary consolidation in which water is totally expulsed & only spring takes the applied load completely.
This is the one dimensional Terzaghi's spring analogy.

* One-dimensional Laboratory Consolidation test:-

→ This test is performed to determine the compressibility characteristics of soil. (i.e to find rate of consolidation)

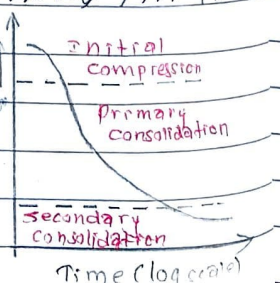
→ The test is performed in the consolidation test apparatus known as consolidometer / oedometer. It consists of a loading device and cylindrical container. The soil specimen is placed in the cell between top & bottom porous stones.




Oedometer assemby.

Procedure

- The specimen shall be 60mm in diameter & 20mm thick. The specimen shall be prepared either from undisturbed samples or from compacted representative samples.
- The soil specimen is placed inside a metal ring with two porous stones, one at top of specimen & other at the bottom. The load on the specimen is applied through a lever arm & compression is measured by a micrometer dial gauge.
- An initial setting load of 5 kN/m² shall be applied until there is no change in dial gauge reading for two consecutive hours or for a maximum 24 hours.
- Load is applied to reach 10 kN/m² & dial gauge readings are recorded after various time intervals 10 sec, 20 sec, 30 sec, 1 min, 2 min, 4 min, 8 min, 16 min, 1 hr, 2 hr, 4 hr, 8 hr, 16 hr, 24 hr.
- The load is doubled to 20 kN/m² & dial gauge reading are taken as a previous steps.
- Above procedure is repeated for loads 40, 80, 160, 320 & 640 kN/m²
- After the last load has been on for required period, the load should be decreased to 1/4 th the value of last load & allowed to stand for 24 hour till the load intensity of 10 kN/m² is reached.
- The general shape of the plot of deformation of specimen against time for load increment is shown in figure.




* Determination of void ratio at various load increment.

→ The results of a consolidation test are plotted in the form of a plot between the void ratio and the effective stress. So, it is necessary to determine the void ratio at various load increment.

→ There are two methods of determining void ratio at various load increment.

(a) Height of solid method:

As we know, the height of solids ($H_s$) is,

$$H_s = \frac{V_s}{A} \quad ----(i)$$

where, $V_s$ = volume of solids

$A$ = cross sectional area of specimen.

Also, $V_s = \frac{M_s}{\rho_s} = \frac{M_s}{G\rho_w}$

From (i), $H_s = \frac{M_s}{G\rho_w} \times \frac{1}{A} \quad ----(ii)$

From defn of void ratio,

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s} = \frac{(A \times H) - (A \times H_s)}{A \times H_s} = \frac{H - H_s}{H_s} \quad ---(iii)$$

where, $H$ = total height of soil specimen.

Therefore,

$$H = H_0 \pm \Sigma \Delta H$$

where, $H$ = Initial height

$\Delta H$ = change in height.

Observation

| Load (kN/m²) | final dial gauge reading (mm) | $\Delta H$ (mm) | $H_0 = H_0 \pm \Sigma \Delta H$ | $e = \frac{H - H_s}{H_s}$ | $\Delta e$ |
|---|---|---|---|---|---|
| | | | | | |

(b) Change in void ratio method:-

→ Assuming the specimen is fully saturated, the final void ration ($e_f$) at the end of test is computed from the relation,

$$S \times e = w \cdot G_s$$

$$e = w \cdot G_s, \quad [S = 1, \text{saturated condition}]$$

$$e_f = w_f \times G_s$$

where, $w_f$ = final water content at the end of test.

→ For intermediate stage,

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s} = \frac{V}{V_s} - 1$$

$$V = V_s(1+e)$$

$$\Rightarrow A \times H = V_s(1+e) \quad \text{---(i)}$$

$$\Rightarrow A \times dH = V_s \times de \quad \text{---(ii)} \quad [\text{partial differentiation}]$$

Dividing eqn (ii) by (i),

$$\frac{dH}{H} = \frac{de}{1+e}$$

$$\Rightarrow de = \frac{(1+e)}{H} \times dH$$

$$\Rightarrow \Delta e = \frac{1+e}{H} \times \Delta H \qquad \Rightarrow \Delta e = \frac{1+e_f}{H_f} \times \Delta H$$

→ Observation table:-

| Applied pressure | Dial gauge readings | Change in thickness $\Delta H$ | $H = H_0 \pm \Sigma \Delta H$ | $\Delta e = \frac{1+e_f}{H_f} \times \Delta H$ | e |
|---|---|---|---|---|---|
| 1 (kN/m²) | | (mm) | | | |
| | | | | | |
| | | | | | |

## ✱ Some definitions & terminologies:-

(a) Coefficient of Compressibility ($a_v$):-

It is defined as the decrease in void ratio per unit increase in effective stress.

i.e, $$a_v = \frac{-\Delta e}{\Delta \bar{\sigma}} \quad \text{---(i)}$$

-ve sign indicates that, the void ratio decreases with increase in effective stress.

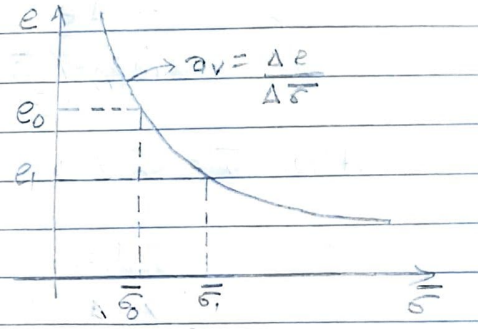



Fig: plot of e versus $\bar{\sigma}$

(b) Coefficient of volume change ($m_v$):-

It is defined as the volumetric strain per unit increase in effective stress. Thus,

$$m_v = \frac{-\Delta V / V_0}{\Delta \bar{\sigma}}$$

where, $V_0$ = Initial volume

$\Delta V$ = Change in volume

$\Delta \bar{\sigma}$ = change in effective stress.

As we know,

$V = V_s(1+e)$

$V_0 = V_s(1+e_0)$ ---- Initial condition

$V_1 = V_s(1+e_1)$ ---- final condition

Then, $\Delta V = V_1 - V_0 = \Delta e \times V_s$

$$\therefore m_v = \frac{-\Delta e \times V_s}{V_s(1+e_0) \times \Delta \bar{\sigma}} \Rightarrow \boxed{m_v = \frac{a_v}{1+e_0}}$$

(c) Coefficient of Compression / Compression Index ($C_c$)

It is defined as the slope of linear portion of the void ratio versus $\log \bar{\sigma}$ plot.

Thus,

$$C_c = \frac{-\Delta e}{\log_{10}\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)}$$

Also, $\bar{\sigma}_1 = \bar{\sigma}_0 + \Delta\bar{\sigma}$

$$C_c = \frac{-\Delta e}{\log\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)}$$

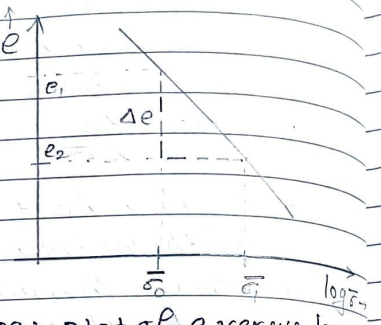



Fig: plot of e versus $\log\bar{\sigma}$

→ The compression index is extremely useful for determination of settlement in the field.

→ Also, we can calculate $C_c$ from Liquid limit.

For undisturbed soil,

$$C_c = 0.009\,(W_L - 10)$$

For remoulded soil,

$$C_c = 0.007\,(W_L - 10)$$ , $W_L$ → Liquid limit

(d) Recompression Index / Coefficient of recompression ($C_r$)

Recompression is the compression of soil which had already been loaded & unloaded.

The load during recompression is less than the load to which the soil has been subjected previously.

$C_r$ is the slope of recompression curve.

$$C_r = \frac{-\Delta e}{\log\left(\frac{\bar{\sigma}_1 + \Delta\bar{\sigma}}{\bar{\sigma}_1}\right)}$$

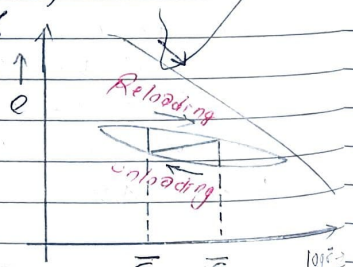


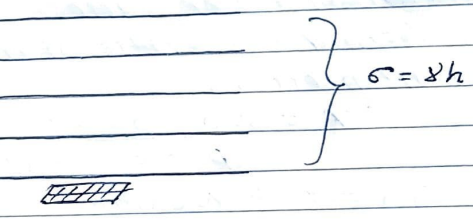

Fig: loading, unloading & reloading plot.

## Consolidation states:-

(a) Normally Consolidated clay:- <NC>

→ A soil is said to be normally consolidated when the existing effective stress ($\bar{\sigma}$) is the maxm that it has ever experienced in its stress history.

→ It has never been subjected to stress higher than the present stress level.

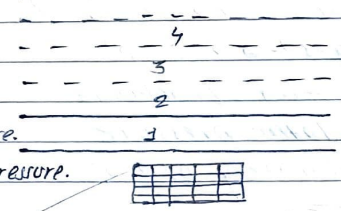



(b) Over consolidated clay <OC>

→ A soil is said to be overconsolidated when the soil has experienced higher pressure in past in excess of present pressure.

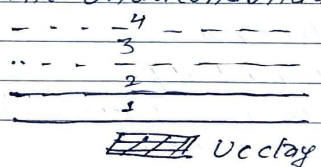
→ Causes:-

- Removal of overburden (excavation, erosion)
- Removal of past structure.
- Increase in pore water pressure.




(c) Under Consolidated clay <UC>

→ Soil which are yet to be consolidated under the overburden pressure ($\bar{\sigma}$). As in case of recently placed fill are reffered to the underconsolidated clay.



(iv) Preconsolidation Pressure:-

→ The maximum pressure to which an overconsolidated soil had been subjected in the past is known as preconsolidation pressure.

→ It is the mean effective stress that has acted in the clay many times in the history.

* Over-consolidation Ratio:-

It is defined as the ratio of max^m past pressure to the present pressure.

$$OCR = \frac{\text{Max}^m \text{ past pressure}}{\text{present pressure}} = \frac{\bar{\sigma}'_{max}}{\bar{\sigma}'_v}$$

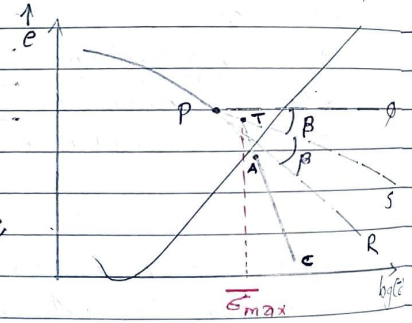
If, OCR = 1 ⇒ NC clay

OCR > 1 ⇒ OC clay

OCR < 1 ⇒ UC clay.

✻ Casagrande's graphical method of det^m of preconsolidation pressure

→ Casagrande suggested a simple graphic construction to determine the pre-consolidation pressure ($\bar{\sigma}_{max}$) from laboratory plot of e versus log $\bar{\sigma}$.




→ After preparing e-log $\bar{\sigma}$ curve, by visual observation establish point 'p', at which curve has maximum radius of curvature.

→ Draw horizontal line pQ

→ Draw line pR, tangent at p.

→ Draw line ps which is bisector of line pQ & pR which makes angle 'β' with both line

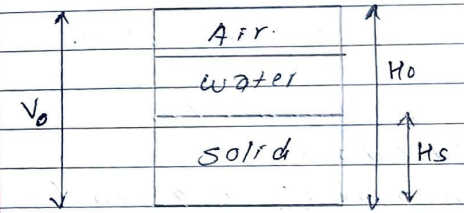
→ project straight line AC of the curve to the back to intersect bisector line ps.

→ Then, draw vertical line from point of intersection of ps & AC (i.e at T) & the effective stress corresponding to this line gives preconsolidation pressure which is shown in figure.

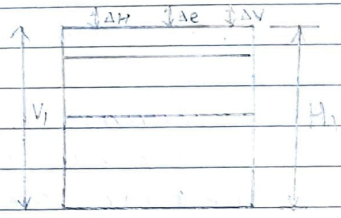
✻ Determination of Consolidation Settlement:-

There are different methods of determining consolidation settlement. ~~by using~~ They are,

(a) By change in void ratio method:-




Before consolidation




After consolidation.

Let,

$H_0$ = Height of soil specimen before consolidation

$H_s$ = Height of soil solid.

Let, $V_s$ = volume of solid = 1.

Then, $V_v = e_0$, where $e_0$ = Initial void ratio = $\frac{V_v}{V_s}$

we have,

$$\frac{\Delta V}{V_0} = \frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0}$$

$$\Delta H = \frac{\Delta e}{1+e_0} \times H_0 \quad ---- (1) \quad [\text{Master eqn}]$$

(b) By using coefficient of compressibility ($a_v$) & coefficient of volume change ($m_v$).

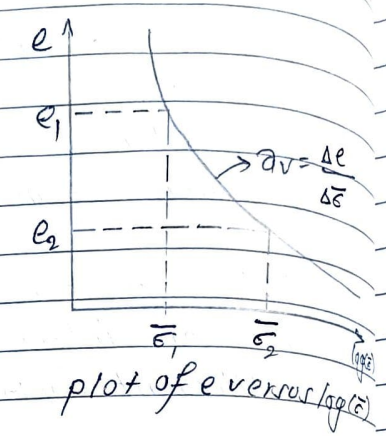
→ We know,

$$a_v = \frac{\Delta e}{\Delta \bar{\sigma}}$$

$$\Rightarrow \Delta e = a_v \times \Delta \bar{\sigma} \quad ---(i)$$

Also,

$$\Delta H = \frac{\Delta e}{1+e_0} \times H_0 \quad ---(ii)$$



plot of e versus log($\bar{\sigma}$)

From (i) & (ii),

$$\Delta H = \frac{a_v \times \Delta \bar{\sigma}}{1+e_0} \times H_0$$

$$\Rightarrow \Delta H = m_v \Delta \bar{\sigma} \times H_0 \quad ---(iii)$$

We can calculate, settlement ($\Delta H$) using eqn (iii).

(c) By using coefficient of compression ($C_c$):-

(a) For normally consolidated soil,

We know, the compression index ($C_c$) is given by

$$C_c = \frac{\Delta e}{\log_{10}\left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0}\right)}$$

$$\Rightarrow \Delta e = C_c \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0}\right) \quad ---(i)$$

Also, $\Delta H = \frac{\Delta e}{1+e_0} \times H_0$

$$\Rightarrow \Delta e = \frac{\Delta H}{H_0}(1+e_0) \quad ---(ii)$$

Equating (i) & (ii)

$$C_c \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0}\right) = \frac{\Delta H}{H_0}(1+e_0)$$

$$\Rightarrow \frac{C_c * H_0}{1+e_0} \times \log\left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0}\right) = \Delta H$$

$$\Rightarrow \Delta H = C_c * \frac{H_0}{1+e_0} * \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0}\right) \quad ---(iii)$$

(b) For over-consolidated soil,

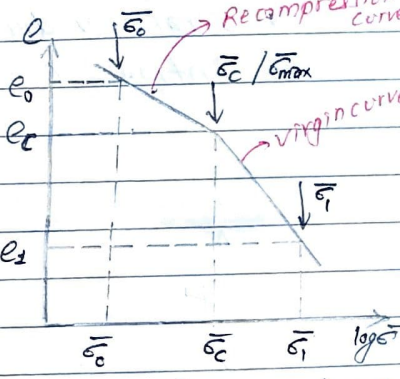
→ The final settlements are small in case of over consolidated clay as $C_r < C_c$.

→ Two cases will arise for over-consolidated soil.

i) When $(\bar{\sigma}_0 + \Delta \bar{\sigma}) < \bar{\sigma}_{max}$ / $\bar{\sigma}_c$ [preconsolidation pressure]

$$S_c = \frac{C_r}{1+e_0} \times H_0 \times \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0}\right)$$

(ii) When $(\bar{\sigma}_0 + \Delta \bar{\sigma}) > \bar{\sigma}_c$, then settlement is computed into two parts.



plot of e versus log$\bar{\sigma}$

- For stress increases from $\bar{\sigma}_0$ to $\bar{\sigma}_c$

$$S_1 = C_r \times \frac{H_0}{1+e_0} \log\left(\frac{\bar{\sigma}_c}{\bar{\sigma}_0}\right)$$

(Recompression index)

- When stress increases from $\bar{\sigma}_c$ to $\bar{\sigma}_1$

$$S_2 = C_c \times \frac{H_0}{1+e_0} \log\left(\frac{\Delta \bar{\sigma} + \bar{\sigma}_0}{\bar{\sigma}_c}\right)$$

(Compression index)

The total settlement for overconsolidated soil is,

$$S_c = S_1 + S_2$$

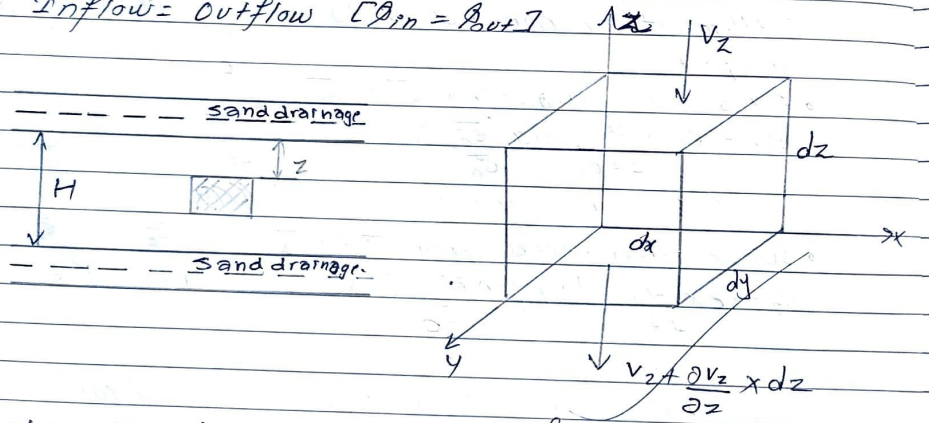
$$= C_r \times \frac{H_0}{1+e_0} \log\left(\frac{\bar{\sigma}_c}{\bar{\sigma}_0}\right) + C_c \times \frac{H_0}{1+e_0} \log\left(\frac{\Delta \bar{\sigma} + \bar{\sigma}_0}{\bar{\sigma}_c}\right)$$

## ✻ TERZAGHI's theory of 1-D Consolidation:-

→ Terzaghi gave the theory for the determination of rate of consolidation of a saturated soil mass subjected to static load.

→ Assumptions:-

- Soil is homogeneous & isotropic, semiinfinite.
- Soil is fully saturated.
- Darcy's law is valid ($V = Ki$)
- The flow through soil is one-dimensional [vertical dir$^{n}$ only]
- The coefficient of permeability ($k$), coeff of volume change ($m_v$) & coeff. of compressibility ($a_v$) remains constant throughout entire period of consolidation.
- Strain produced due to stress is too small.
- Inflow = outflow [$Q_{in} = Q_{out}$]




→ Let us consider a saturated clay of thickness 'H' lie beth two layers of sand which serve as two drainage faces. Consider, $v_z$ be the velocity of entry & velocity of exit will be $v_z + \frac{\partial v_z}{\partial z} \times dz$.

Then, at any time 't', the pore water pressure 'u' is,

$$u = \gamma_w \times h$$

$$\Rightarrow h = \frac{u}{\gamma_w}$$ where, $h$ = Hydraulic head, $\gamma_w$ = Unit weight of water.

According to Darcy's law,

$$V_z = ki = k \times \frac{\partial h}{\partial z}$$

$$\Rightarrow V_z = k \times \frac{\partial (u/\gamma_w)}{\partial z}$$

$$\Rightarrow V_z = \frac{k}{\gamma_w} \times \frac{\partial u}{\partial z} \quad ---- (ii)$$

we have,

$$q = A * \text{velocity}$$

Then, $q_{in} = (dx \times dy) \times V_z$

& $q_{out} = (dx \times dy) \times \left(V_z + \frac{\partial V_z}{\partial z} \times dz\right)$

Then, $\Delta q = q_{out} - q_{in}$

$$\Delta q = \frac{\partial V_z}{\partial z} \times dx\,dy\,dz \quad ---- (iii)$$

From coefficient of volume change,

$$m_v = -\frac{\Delta V}{V_0} \times \frac{1}{\Delta \bar{\sigma}}$$

$$\Delta V = -m_v \times V_0 \times \Delta \bar{\sigma} \quad --- (iii)$$

we have,

$$\Delta \sigma = \Delta \bar{\sigma} + u$$

$$\Rightarrow \frac{\partial}{\partial t}(\Delta \sigma) = \frac{\partial}{\partial t}(\Delta \bar{\sigma}) + \frac{\partial u}{\partial t}$$

$$\Rightarrow \frac{\partial (\Delta \bar{\sigma})}{\partial t} = -\frac{\partial u}{\partial t} \quad ---- (iv)$$

Being small term $\frac{\partial}{\partial t}(\Delta\sigma)$ get neglected.

From eqn (iii) & (iv)

$$\frac{\partial}{\partial t}(\Delta v) = m_v \times v_0 \times \frac{\partial u}{\partial t}$$

$$\Rightarrow \Delta q = m_v \times v_0 \times \frac{\partial u}{\partial t} \quad ----- (v)$$

Also, from eqn (ii) & (v),

$$\frac{\partial v_z}{\partial z} \times dx\,dy\,dz = m_v \times v_0 \times \frac{\partial u}{\partial t}$$

$$\Rightarrow \frac{\partial v_z}{\partial z} = m_v \left(\frac{\partial u}{\partial t}\right) \quad --- (vi) \qquad [v_0 = dx\,dy\,dz]$$

From eqn (i) & (vi),

$$\frac{\partial}{\partial z}\left(\frac{k}{\gamma_w} \times \frac{\partial u}{\partial z}\right) = m_v \times \frac{\partial u}{\partial t}$$

$$\Rightarrow \frac{k}{\gamma_w} \times \frac{\partial^2 u}{\partial z^2} = m_v \times \frac{\partial u}{\partial t}$$

$$\Rightarrow \frac{\partial u}{\partial t} = \frac{k}{m_v \gamma_w} \times \frac{\partial^2 u}{\partial z^2}$$

$$\Rightarrow \frac{\partial u}{\partial t} - \frac{k}{m_v \gamma_w} \times \frac{\partial^2 u}{\partial z^2} = 0 \quad -- (vii) \qquad [\text{Terzaghi diffn eqn}]$$

$$\Rightarrow \frac{\partial u}{\partial t} - C_v \times \frac{\partial^2 u}{\partial z^2} = 0 \quad -- (viii)$$

where, $C_v$ = coefficient of consolidation $= \frac{k}{m_v \times \gamma_w}$ -- (ix)

Solution of diffn eqn (viii) is,

$$T_v = \frac{C_v \times t}{d^2} \quad --- (x)$$

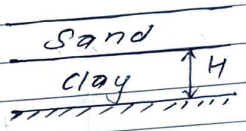
where, $T_v$ = dimensionless time factor.

$C_v$ = coefficient of consolidation $= k / m_v \gamma_w$

$t$ = time taken for consolidation

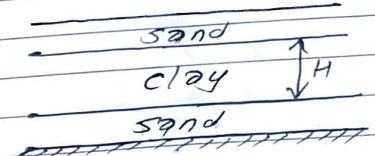
$d$ = drainage path.

For single drainage,




$d = H$ [H = thickness of soil]

For double drainage




$d = H/2$

For $U \leq 60\%$, $T_v = \frac{\pi}{4} \times (U\%)^2$ ----- (xi)

For $U > 60\%$, $T_v = 1.781 - 0.933 \log_{10}[100 - U\%]$ -- (xii)

* Degree of consolidation:-

→ The degree of consolidation is a number between zero & one, assigned to express the progress of consolidation at any particular stage.

→ In terms of settlement, degree of consolidation can be written as,

$$U = S/S_f \quad --- (xiii)$$

where, $S$ = ~~Initial~~ settlement at time 't'

$S_f$ = final settlement at end of consolidation.

$U = \frac{e_0 - e}{e_0 - e_f}$ ; in terms of void ratio

$U = \frac{\bar{\sigma} - \bar{\sigma_0}}{\bar{\sigma_f} - \bar{\sigma_0}}$ ; in terms of effective stress, $U = \frac{u_0 - u}{u_0}$ ; in terms of pore water pressure.

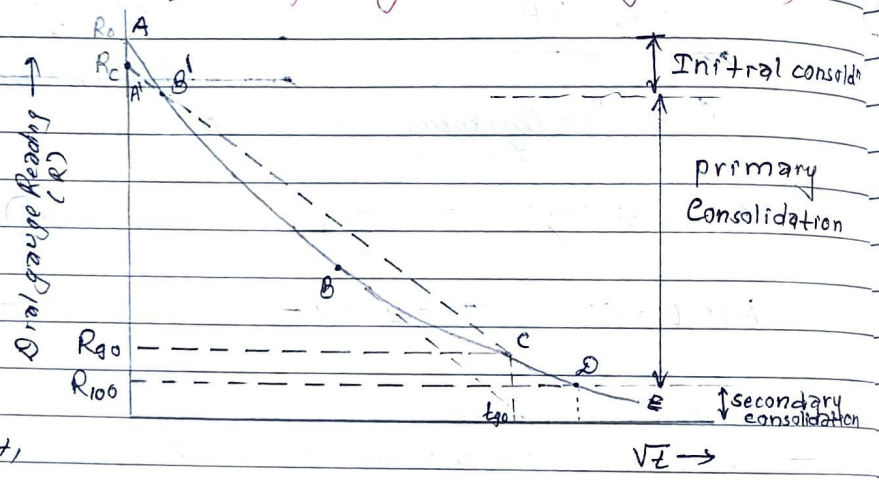
## * Determination of coefficient of consolidation:-

There are two methods of determing the coefficient of consolidation ($C_v$).

i) Square root of time fitting method <Taylor method>

ii) Logarithm of time method <Casgrande's>

### (i) Square root of time fitting method (Taylor method)




Let,

$R_0$ = Initial reading

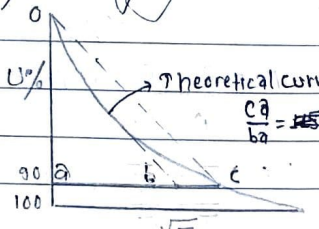
$R_c$ = corrected dial gauge reading

$R_0 - R_c$ → Initial consolidation

$R_c - R_{100}$ → Primary consolidation

below $R_{100}$ → Secondary consolidation.

→ Based on expression for 'U' as a function of $T_v$, the theoretical curve betn $\sqrt{T_v}$ and 'U' is straight upto U = 60% and the abscissa corresponding to U = 90% point on curve is 1.15 times the abscissa of point of intersection of straight line portion of curve




produced with the horizontal line at U = 90%. This property is used for fitting the theoretical curve to the laboratory curve.

→ In square root method, a plot of deformation against the square root of time ($\sqrt{t}$) is made for the incremental loading. other graphic constructions required are as follows.

- The plot of dial gauge reading versus $\sqrt{t}$ gives the curve ABCDE
- The curve begins at dial gauge reading $R_0$ at time $t_0$ indicated by point A
- Line A'C is drawn such that its abscissa is 1.15 times that of initial linear portion A'B of the curve.
- The intersection of this line with curve at point C' indicates 90% of 'U'.

→ $C_v$ can be calculated as,

$$C_v = \frac{T_v \times d^2}{t} \quad \text{---} (1)$$

For U = 90% ~~≠~~ 60%

$$T_v = (1.781 - 0.933) \log (100 - U\%)$$

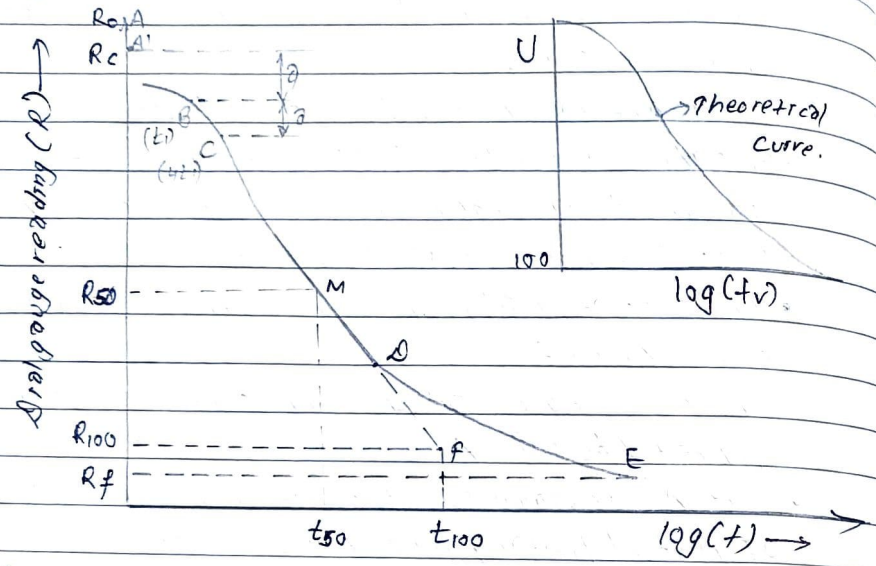
$$= 0.848$$

Then, from eqn (1),

$$C_v = \frac{0.848 \times d^2}{t_{90}}$$

→ The test is repeated for different load increments & average value of $C_v$ is obtained.

## (ii) Logarithm of time method <casgrande's>




→ This method is given by casgrande's to determine $C_v$.

→ For a given load increment, a curve is plotted betn dial gauge reading (R) & log (t) which is shown in figure.

→ Let $R_0$ be initial dial gauge reading before application of load increment. The corrected zero reading ($R_c$) is obtained using fact that, the initial portion of curve is parabolic.

→ Two points B & C are selected corresponding to some arbitary time t & 4t respectively having vertical intercept 'a'. The point A' is located such that the vertical intercept betn B & A' is equal to 'a'.

→ The final portion of experimental curve is linear. The point f corresponding to 100% consolidation is obtained from the intersection of two linear parts.

→ The point 'M' corresponding to 50% primary consolidation ($R_{50}$) is located midway betn $A_c$ & $R_{100}$ & the value of time $t_{50}$ is obtained.

For U = 50%, ≤ 60%

$$T_v = \frac{\pi}{4} \times (U\%)^2 = \frac{\pi}{4} \times 0.5^2 = 0.196$$

Then,

$$C_v = \frac{T_v \times d^2}{t}$$

$$C_v = \frac{0.196 d^2}{t_{50}} \quad \text{----(1)}$$

→ By using eqn (1), we can calculate coefficient of consolidation.

→ The test is repeated for different load increments & an average value of $C_v$ for the desired load range is determined.

Formula Used in this chapter

a) $T_v = \frac{C_v t}{d^2}$ b) $C_v = \frac{K}{m_v \gamma_w}$ c) $m_v = \frac{a_v}{1+e_0}$

d) $a_v = \frac{\Delta e}{\Delta \bar{\sigma}}$ e) $S = C_c \times \frac{H_0}{1+e_0} \log_{10}\left(\frac{\bar{\sigma_0} + \Delta\bar{\sigma}}{\bar{\sigma_0}}\right)$

f) $S = \Delta H = m_v H_0 \Delta \bar{\sigma}$

g) $U \leq 60\%$, $T_v = \frac{\pi}{4} \times (\%U)^2$

h) $U > 60\%$, $T_v = 1.781 - 0.933 \log(100 - U\%)$

**Q1** Two clay samples A and B have initial void ratio of 0.55 & 0.632 respectively under a pressure 100 kN/m². The pressure increased to 150 kN/m². With the application of the pressure, the void ratio decreases to 0.495 and 0.616 resp. The time taken by the specimen A to reach 50% consolidation is one third of that required by specimen 'B' for same consolidation. Find the ratio of coefficient of the permeability of the two specimens A & B if their thickness was 40mm & 30mm respectively.

→ Given,

| For clay Sample A | For clay sample B, |
|---|---|
| Initial void ratio ($e_0$) = 0.55 | $e_0$ = 0.632 |
| Final void ratio ($e_1$) = 0.495 | $e_1$ = 0.616 |

$\bar{\sigma_0}$ = 100 kN/m² & $\bar{\sigma_1}$ = 150 kN/m²

$t_A = \frac{1}{3} t_B$ for 50% consolidation.

$d_A$ = 40mm & $d_B$ = 30 mm.

Then, $\frac{k_A}{k_B}$ = ?

Now,

$$\frac{k_A}{k_B} = \frac{C_{vA}}{C_{vB}} \times \frac{m_{vA}}{m_{vB}} \quad \text{---} (1)$$

As we know,

$$m_{vA} = \frac{a_{vA}}{1+e_0} = \frac{1.1 \times 10^{-3}}{1+0.55} = 7.096 \times 10^{-4}$$

$$a_{vA} = \frac{\Delta e}{\Delta \bar{\sigma}} = \frac{0.55 - 0.495}{50} = 1.1 \times 10^{-3}$$

$$m_{vB} = \frac{a_{vB}}{1+e_0} = \frac{3.2 \times 10^{-4}}{1+0.632} = 1.96 \times 10^{-4}$$

$$a_{vB} = \frac{0.632 - 0.616}{50} = 3.2 \times 10^{-4}$$

Since, $T_{vA} = T_{vB}$

$$\Rightarrow \frac{C_{vA} \times t_A}{d_A^2} = \frac{C_{vB} \times t_B}{d_B^2}$$

$$\Rightarrow \frac{C_{vA}}{C_{vB}} = \frac{d_A^2}{d_B^2} \times \frac{t_B}{t_A} = 5.33$$

From eqn (1)

$$\frac{k_A}{k_B} = 5.33 \times \frac{7.096 \times 10^{-4}}{1.96 \times 10^{-4}}$$

$$\frac{k_A}{k_B} = 19.308$$

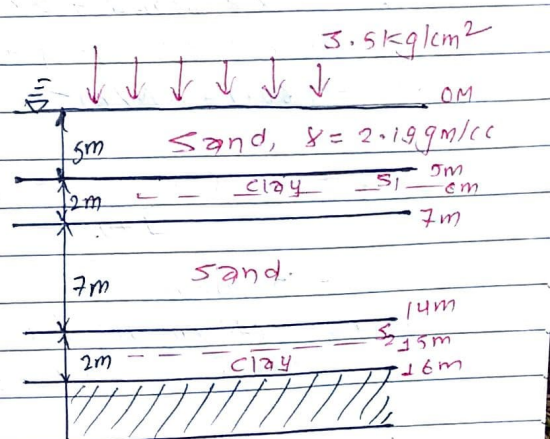
**Q2** The following information was obtained from the soil investigation work. Two layers of clay of thickness 2m each & sandwiched between two layers of dense sand was encountered at 5m & 14m depths respectively. The compression index, water content and specific gravity of both clays were 0.38, 30% & 2.7 respectively. The sand weighed 2.1 gm/cc and was completely submerged. Compute the total settlement under a UDL of 3.5 kg/cm² on a large area over the ground surface.

→ Given,

$C_c$ = 0.38

$w$ = 30%

$G$ = 2.7




Total settlement,

$S = S_1 + S_2$

Here,

$S \times e_0 = w \cdot G$

$e_0 = w \cdot G = 0.81$

$H_0$ = 2m for both layers.

$\gamma_{subsand} = \gamma_{sat} - \gamma_w = 21 - 10 = 11\ kN/m^3$

$\gamma_{subclay} = \left(\frac{G-1}{1+e}\right)\gamma_w = 9.39\ kN/m^3$

$\Delta\bar{\sigma} = 350\ kg/cm^2$

$= 350 \times 9.81 \times 10^{-3} \times 10^4\ kN/m^2$

$= 343.35\ kN/m^2$

Then,

At 6m from top,

$\bar{\sigma_0} = 5 \times \gamma_{subsand} + 1 \times \gamma_{subclay}$

$= 64.39\ kN/m^2$

$S_1 = C_c \times \frac{H_0}{1+e_0} \log\left(\frac{\bar{\sigma_0} + \Delta\bar{\sigma}}{\bar{\sigma_0}}\right)$

$= 0.38 \times \frac{2}{1+0.81} \log\left(\frac{64.39 + 343.35}{64.39}\right)$

$= 0.336m$

At 15m from top,

$\bar{\sigma_0} = 5 \times \gamma_{subsand} + 2 \times \gamma_{subclay} + 7 \times \gamma_{subsand} + 1 \times \gamma_{subclay}$

$= 5 \times 11 + 2 \times 9.39 + 7 \times 11 + 1 \times 9.39$

$= 160.17\ kN/m^2$

$S_2 = 0.38 \times \frac{2}{1+0.81} \log_{10}\left(\frac{160.39 + 343.35}{160.17}\right)$

$= 0.208\ m$

Total settlement $(S) = S_1 + S_2$

$= 0.5448\ m$

$= 54.48\ cm$ ✗✗

Ch-10

# Shear strength of soil:-

→ The maximum internal resistance of soil against shear stress.

→ The shear strength of soil is internal resistance that is offered by soil mass against sliding & plane failure inside it.

→ Shear strength is the most important engineering properties of soil which is contributing factor for stability of a cut, a foundation, slopes & embankments.

→ The soil mass posses shear strength due to following reasons

(i) Friction between particles ($\phi$)

ii) Cohesion betn particles (C)

iii) Interlocking betn particles.

→ A soil whose shearing strength is entirely due to friction is called $\phi$ soil. Eg:- sand

→ The soil whose shearing strength is entirely due to cohesion is called C-soil. Eg:- clay

→ The soil which posses friction & cohesion is called C-$\phi$ soil Eg:- silty sand, silty clay etc.

## ✻ Mohr & Coulomb failure criteria:-

Failure envelope is the line tangent to the Mohr circle at which failure occurs when the failure envelope just touches the Mohr's circle.

### (a) Mohr failure Criteria:-

According to Mohr, the soil fails when shear stress ($\tau$) on failure plane at failure is unique function of normal stress ($\sigma$) acting on the plane.

$\tau = f(\sigma)$

→ Also, the material fail due to critical combination of normal & shear stress, not due to max^m normal stress or maximum shear stress.

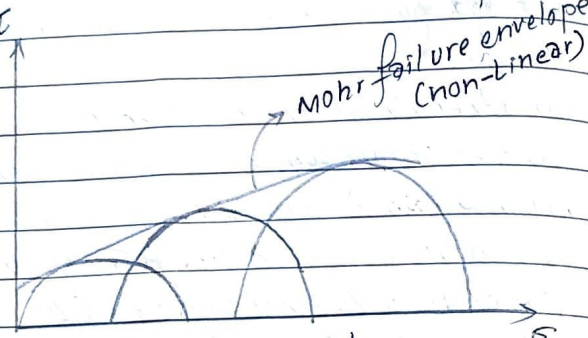



Fig: Mohr envelope.

(b) Coulomb failure Criteria:

→ According to coulomb, there is linear relationship betn normal stress and shear stress before failure.

→ At failure, shear strength is,

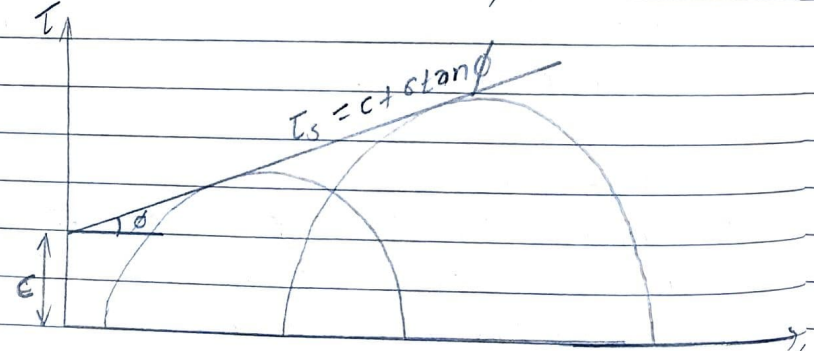
$$\tau = c + \sigma \tan\phi$$

Where,

$c$ = cohesion

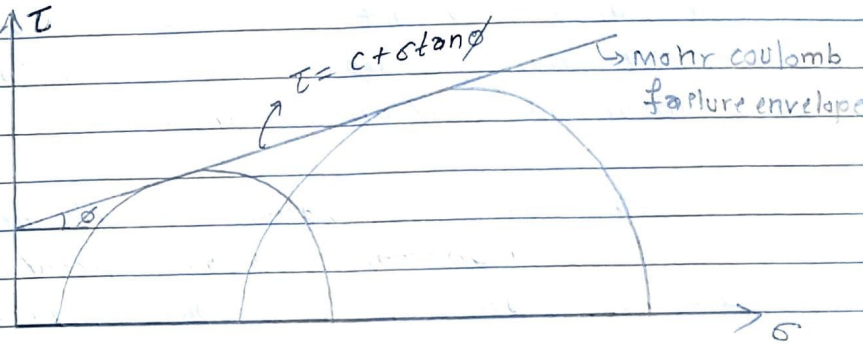
$\phi$ = internal frictional angle

$\sigma$ = Normal stress on failure plane.




✱ Mohr-coulomb failure criteria:-

→ Mohr-coulomb failure criteria is the combination of Mohr failure criteria & coulomb failure criteria.




✱ Modified Mohr-coulomb failure Criteria:-

Terzaghi modify the Mohr-columb theory. According to Terzaghi, shear strength of soil is function of effective stress not total stress.

$$\tau_s = c' + \bar{\sigma}_n \tan\phi'$$

Where,

$\bar{\sigma}_n = \sigma - u$ = effective normal stress

$c'$ = Effective cohesion

$\phi'$ = Effective angle of friction.

✱ Principle plane & Principal stresses:-

→ plane having zero shear stress is called principle plane.

→ Normal stresses acting on principal planes are called principle stresses.

→ At any point in soil mass, the applied stress system can be resolved into three mutually orthogonal principle planes. The ~~normal stresses~~. They are major principal plane, intermediate principal plane & minor principal plane.

→ The stress acting on these planes are called major principle stress ($\sigma_1$), intermediate principle stress ($\sigma_2$) & minor principle stress ($\sigma_3$) respectively.

→ Critical stress values & obliquities generally occur on major & minor principle planes. Therefore a two dimensional solution is possible.

## ✱ Inclination of plane of failure Caused by Shear:-

Let us consider an element which is subjected to two dimensional stress system as shown in figure. Let x-x be major principal plane & $\sigma_1$ be major principal stress acting normal to xx & yy be minor principal plane & $\sigma_3$ be the minor principal ~~plane~~ stress acting normal to y-y.

Consider any inclined plane AB inclined at an angle $\theta$ to the major principal plane xx. Let '$\sigma$' be normal stress & '$\tau$' be shear stress.

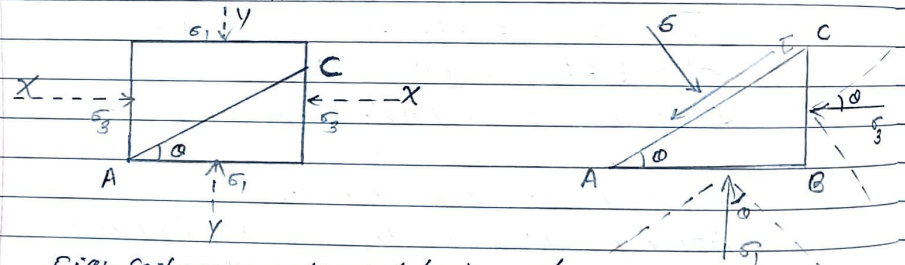



Fig: Soil mass subjected to two dimensional Stress System.

Considering equilibrium of wedge ABC,

$$\tau \times (AC \times 1) + \sigma_3 \cos\theta \times (BC \times 1) - \sigma_1 \sin\theta\,(AB \times 1) = 0$$

$$\tau = \sigma_1 \sin\theta \times \frac{AB}{AC} - \sigma_3 \times \cos\theta \times \frac{BC}{AC}$$

$$\tau = \sigma_1 \sin\theta \times \cos\theta - \sigma_3 \times \cos\theta \times \sin\theta$$

$$\tau = \left(\frac{\sigma_1 - \sigma_3}{2}\right) \sin 2\theta \quad \text{---} (1)$$

Also,

$$(AC \times 1)\sigma - \sigma_3 \sin\theta \times (BC \times 1) - \sigma_1 \cos\theta\,(AB \times 1) = 0$$

$$\sigma = \sigma_1 \cos\theta \times \frac{AB}{AC} + \sigma_3 \sin\theta \times \frac{BC}{AC}$$

$$\sigma = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$$

$$\sigma = \sigma_1 \left[\frac{1 + \cos 2\theta}{2}\right] + \sigma_3 \left[\frac{1 - \cos 2\theta}{2}\right]$$

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \left(\frac{\sigma_1 - \sigma_3}{2}\right) \cos 2\theta \quad \text{---} (1)$$

Eq$^n$ (1) & (1) give the tangential stress & normal stress on plane inclined at an angle '$\theta$' to the major principal ~~stress~~ plane.

## ✱ Mohr's Circle:-

→ Otto Mohr's gave a graphical solution for determination of normal & shear stresses on the given inclined plane.

→ He conveniently assumed that, compressive normal stress are positive & tensile normal stresses are negative.

→ The complete solution may be obtained from following steps.

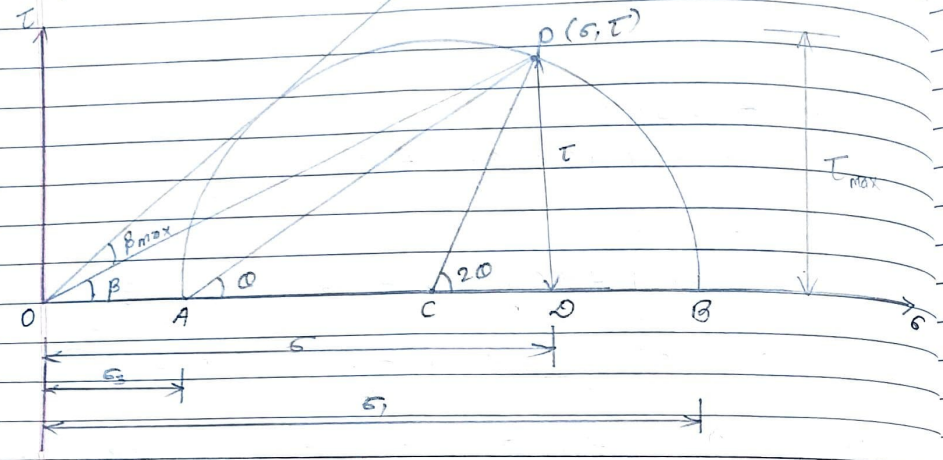
(i) Along x-axis plot normal stresses $\sigma_1 = OB$ & $\sigma_3 = OA$

(ii) Find midpoint of AB as 'c'.

(iii) Draw semi-circle having its centre at c & radius equal to AC or BC.

(iv) Through 'A' draw line AP which makes angle '$\theta$' with horizontal.

i) Now, the 'x' coordinate of 'p' is normal stress & 'y' coordinate of 'p' is shear stress on given plane.




Proof

Join p & C & drop perpⁿ pD to AB, from Δ pCD,

$\angle pCD = 2\theta$

Normal stress ($\sigma$) = $OD = OA + AC + CD$

$= \sigma_3 + \frac{\sigma_1 - \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$

$\therefore \sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$

Shear stress ($\tau$) = $pD = pC \sin 2\theta$

$\tau = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta$

The following relationship may be directly obtained from Mohr's circle as shown in figure.

(i) The maximum ~~principle~~ shear stress ($\tau_{max}$) is equal to radius of Mohr's circle, & it occurs on plane inclined at 45° to the principal planes.

$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$

ii) The normal stresses on the planes of maximum shear stress are equal to each other & equal to half the sum of principle stress.

$\sigma_{max} = \frac{\sigma_1 + \sigma_3}{2}$

(iii) The resultant stress ($\sigma_R$) on any plane is $\sqrt{\sigma^2 + \tau^2}$ & its angle of obliquity $\beta$ will be, $\tan^{-1}(\sigma/\tau)$

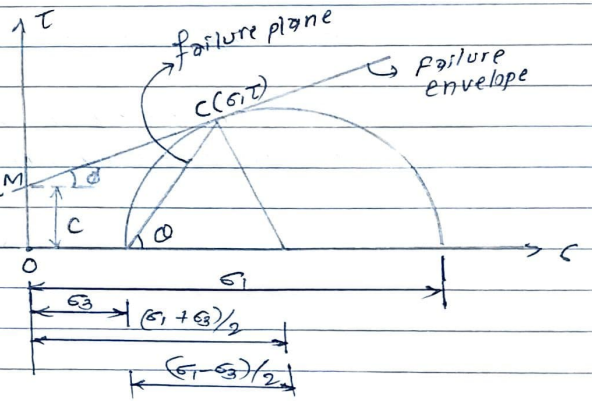
$\therefore \sigma_R = \sqrt{\sigma^2 + \tau^2}$

$\beta = \tan^{-1}\left(\frac{\sigma}{\tau}\right)$

(iv) The maximum angle of obliquity $\beta_{max}$ occurs on a plane inclined at an angle of, $\theta = 45° + \frac{\beta_{max}}{2}$ with respect to major principal plane.

## ✻ Relation b/t major ($\sigma_1$) & minor ($\sigma_3$) principal stresses at failure.

→ With refrence to figure, the relationship b/t major & minor principal stresses at failure may be derived from geometry of Mohr's circle.



In $\Delta AOC$

$$\cot\phi = \frac{OA}{C} \Rightarrow OA = C\cot\phi$$

Also, In $\Delta ABC$,

$$\sin\phi = \frac{BC}{AB} = \frac{BC}{OA+OB}$$

$$\sin\phi = \frac{(\sigma_1 - \sigma_3)/2}{C\cot\phi + \left(\frac{\sigma_1+\sigma_3}{2}\right)}$$

$$\Rightarrow \frac{\sigma_1 - \sigma_3}{2} = C\cos\phi + \frac{\sigma_1+\sigma_3}{2}\times\sin\phi$$

$$\Rightarrow \sigma_1 - \sigma_3 = 2C\cos\phi + (\sigma_1+\sigma_3)\sin\phi$$

$$\Rightarrow \sigma_1(1-\sin\phi) = 2C\cos\phi + \sigma_3(1+\sin\phi)$$

$$\Rightarrow \sigma_1 = \frac{2C\cos\phi}{1-\sin\phi} + \sigma_3 \times \frac{1+\sin\phi}{1-\sin\phi}$$

$$\Rightarrow \sigma_1 = 2C\tan\left(45^\circ + \frac{\phi}{2}\right) + \sigma_3\tan^2\left(45^\circ + \phi/2\right)$$

$$\Rightarrow \sigma_1 = \sigma_3 N_\phi + 2C\sqrt{N_\phi} \quad ---(A)$$

where, $N_\phi$ = Flow ratio = $\tan^2\left(45^\circ + \phi/2\right)$

Eqn (A) is obliquity relation which gives the relationship beth principal stresses at failure.

Ch 2

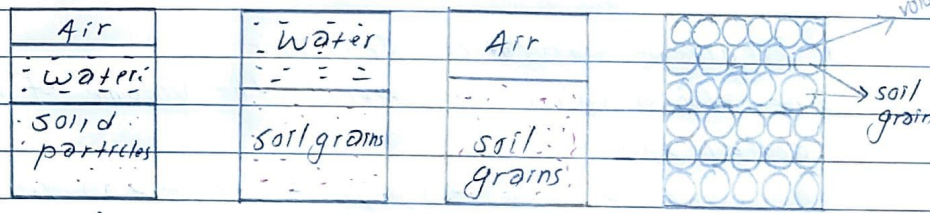
# Solid-Water-Air Relations & Index properties of soil:-

## * Phase diagram:-

→ Soil consists of three distinct phases such as soil grains, liquid (water) and gas (air). The space betn the soil particles is known as void & these voids may be filled either with air or water or both.

→ The soil will behave as a two phase system when its void space is filled with either water or air alone. When all void space is filled with water, the soil mass is fully saturated & when all void space is filled with air, the soil mass is dry soil.

→ The diagrammetic representation of the different phases in a soil mass is called the phase diagram.




Three phase diagram | Two phase diagram for saturated soil | Two phase diagram for dry soil. | Actual soil mass

## * Some definitions:-

Let, $V_a$ = Volume of air $W_a$ = Weight of air (taken as zero)

$V_w$ = Volume of water $W_w$ = Weight of water

$V_s$ = Volume of solid $W_s$ = Weight of solid

$V_v$ = Volume of voids $V$ = Total volume

$W$ = Total weight.

Then, some defn of the terms used in soil are,

a) Void ratio:- (e)
The ratio of volume of voids to the the volume of solid particles is called void ratio.

$$e = \frac{V_v}{V_s}$$

b) porosity (n)
The ratio of volume of voids to the total volume of soil mass.

$$n = \frac{V_v}{V}$$

c) Water content:- (w):-
The ratio of mass of water to the mass of solid in a given soil mass is called water content.

$$w = \frac{M_w}{M_s} \times 100\% = \frac{W_w}{W_s} \times 100\%$$

d) Degree of saturation ($S_r$ or s):-
The ratio of volume of water to the volume of voids is called DOS.

$$S = \frac{V_w}{V_v} * 100\% = \frac{V_w}{V_w + V_a} * 100\%$$

→ For fully saturated soil, $V_a = 0$, $S = 100\%$ or 1
→ For dry soil, $V_w = 0$, $S = 0$
∴ The value of s lies between '0' & '1'. $(0 < S < 1)$

(e) Air Content ($a_c$):-
It is defined as the ratio of air voids to the volume of void. Mathematically,

$$a_c = \frac{V_a}{V_v}$$

f) percentage of air void:- ($n_a$)
It is the ratio of volume of air to the total volume of soil mass.

$$n_a = \frac{V_a}{V}$$

g) Unit weight ($\gamma$)
It is the ratio of weight to the volume.

$$\gamma = \frac{W}{V}$$

h) Bulk Unit Weight:- ($\gamma_t$)
Ratio of total weight of soil mass to the total volume of soil.

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_w + V_a} \quad [W_a = 0]$$

i) Dry unit weight ($\gamma_d$)
Ratio of weight of solid to the volume of soil.

$$\gamma_d = \frac{W_s}{V}$$

j) Saturated Unit Weight ($\gamma_{sat}$):-
Ratio of weight of fully saturated sample to the volume of soil mass.

$$\gamma_{sat} = \frac{W_{sat}}{V}$$

k) Submerged unit weight:- ($\gamma_{sub}$)
Ratio of submerged weight of solid to the volume of soil mass.

$$\gamma_{sub} = \frac{W_{sub}}{V} = \frac{W_{sat} - W_w}{V} = \frac{W_{sat}}{V} - \frac{W_w}{V}$$

$$\Rightarrow \gamma_{sub} = \gamma_{sat} - \gamma_w$$

l) Unit weight of solid ($\gamma_s$):-

→ Ratio of weight of solid ($W_s$) to the volume of solid ($V_s$)

$$\gamma_s = \frac{W_s}{V_s}$$

## m) Unit weight of water:- ($\gamma_w$)

→ weight of water ($W_w$) to the volume of water ($V_w$)

$$\gamma_w = \frac{W_w}{V_w}$$

## n) Specific gravity ($G_T$)

It is defined as the ratio of mass of a given volume of solids to the mass of an equivalent volume of water at 4°c.

$$G_T = \frac{M_s}{M_w} = \frac{\rho_s}{\rho_w} = \frac{\gamma_s}{\gamma_w}$$

## ✻ Some important relations:-

### a) Relation between e, w, S & G.

We know,

$$e = \frac{V_v}{V_s} = \frac{V_v}{V_w} \times \frac{V_w}{V_s} = \frac{V_v}{V_w} \times \frac{W_w/\gamma_w}{W_s/\gamma_s} = \frac{V_v \times W_w \times \gamma_s}{V_w \times W_s \times \gamma_w}$$

$$e = w * G_T * \frac{1}{(V_w/V_v)} = w * G_T * \frac{1}{S}$$

$$\Rightarrow S.e = w.G_T$$

### b) Relation between 'e' and 'n':-

$$n = \frac{V_v}{V} = \frac{V_v}{V_v + V_s}$$

$$\Rightarrow \frac{1}{n} = \frac{V_v + V_s}{V_v}$$

$$\Rightarrow \frac{1}{n} = 1 + \frac{V_s}{V_v}$$

$$\Rightarrow \frac{1}{n} = 1 + \frac{1}{e}$$

$$\Rightarrow n = \frac{e}{1+e} \quad \& \quad e = \frac{n}{1-n}$$

### c) Relationship between unit weight '$\gamma$' in terms of water content (w), void ratio (e) & specific gravity 'G' & $\gamma_w$.

$$\rightarrow \gamma = \frac{w}{V} = \frac{W_s + W_w}{V_s + V_v} = \frac{W_s\left(1 + \frac{W_w}{W_s}\right)}{V_s\left(1 + \frac{V_v}{V_s}\right)} = \frac{\gamma_s(1+w)}{(1+e)}$$

$$\Rightarrow \gamma = \frac{\gamma_s(1+w)}{(1+e)}$$

Also, $G_T = \frac{\gamma_s}{\gamma_w} \Rightarrow \gamma_s = G_T \gamma_w$

Then,

$$\gamma = \frac{G_T \gamma_w (1+w)}{1+e}$$

### d) Relation bet$^n$ $\gamma$, $G_T$, S, e & $\gamma_w$.

We know,

$$\gamma = \frac{G_T \gamma_w (1+w)}{(1+e)}$$

$$\gamma = \frac{\gamma_w (G_T + Gw)}{1+e}$$

$$\Rightarrow \gamma = \frac{\gamma_w (G_T + S.e)}{1+e}$$

If soil is fully saturated, S = 1. Then, $\gamma_{sat} = \gamma_w \left(\frac{G+e}{1+e}\right)$

If soil is dry, S = 0, Then, $\gamma_d = \frac{\gamma_w G}{1+e}$

(e) Relation between dry unit weigh '$\gamma_d$', Bulk unit weight '$\gamma$' & water content 'w'.

→ we have,

$$w = \frac{W_w}{W_s}$$

$$\Rightarrow 1 + w = 1 + \frac{W_w}{W_s} = \frac{W_s + W_w}{W_s}$$

$$\Rightarrow 1 + w = \frac{W}{W_s}$$

$$\Rightarrow W_s = \frac{W}{1+w}$$

Also, $\gamma_d = \frac{W_s}{V}$

$$\gamma_d \times V = W_s$$

$$\Rightarrow \gamma_d * V = \frac{W}{1+w}$$

$$\Rightarrow \gamma_d = \frac{\gamma}{1+w} = \frac{\gamma_t}{1+w}$$ [$\gamma = \gamma_t$ = bulk unit weight]

(f) Relation betn submerged unit weight ($\gamma_{sub}$), G & e,

we know,

$$\gamma_{sub} = \gamma_{sat} - \gamma_w$$

$$\gamma_{sub} = \frac{\gamma_w (G+e)}{(1+e)} - \gamma_w$$

$$\Rightarrow \gamma_{sub} = \frac{\gamma_w (G-1)}{(1+e)}$$

(g) Relation between dry unit weight ($\gamma_d$), specific gravity of solid 'G', water content 'w' & percentage of air voids ($n_a$)

we know,

total volume of soil mass is,

$$V = V_a + V_w + V_s$$

$$\Rightarrow V = V_a + \frac{W_w}{\gamma_w} + \frac{W_s}{\gamma_s}$$

$$\Rightarrow 1 = \frac{V_a}{V} + \frac{W_w}{V * \gamma_w} + \frac{W_s}{V * \gamma_s}$$

$\Rightarrow$ we know that,

$$w = \frac{W_w}{W_s}$$

$$\Rightarrow W_w = w * W_s$$

Then,

$$1 = n_a + \frac{w * W_s}{V * \gamma_w} + \frac{W_s}{V \gamma_s}$$

$$\Rightarrow 1 = n_a + \frac{W_s}{V}\left(\frac{w}{\gamma_w} + \frac{1}{\gamma_s}\right)$$

$$\Rightarrow (1 - n_a) = \gamma_d \left(\frac{w}{\gamma_w} + \frac{1}{\gamma_s}\right)$$

$$\Rightarrow (1 - n_a) = \gamma_d \left(\frac{w\gamma_s + \gamma_w}{\gamma_s \gamma_w}\right)$$

$$\Rightarrow (1 - n_a) = \frac{\gamma_d}{\gamma_w}\left(w + \frac{\gamma_w}{\gamma_s}\right)$$

$$\Rightarrow (1 - n_a) = \frac{\gamma_d}{\gamma_w}\left(w + \frac{1}{G}\right)$$

$$\Rightarrow (1 - n_a) = \frac{(wG + 1)\gamma_d}{G\gamma_w}$$

$$\Rightarrow \gamma_d = \frac{G\gamma_w (1 - n_a)}{1 + wG}$$

When soil mass is fully saturated, $n_a = 0$.

| Relationship in terms of unit weight | Relationship in terms of unit mass. |
|---|---|
| 1. $n = \frac{e}{1+e}$ | $n = \frac{e}{1+e}$ |
| 2. $S.e = wG$ | $S.e = wG$ |
| 3. $n_a = n(1-S)$ | $n_a = n(1-S)$ |
| 4. $a_c = 1-S$ | $a_c = 1-S$ |
| 5. $n_a = n a_c$ | $n_a = n a_c$ |
| 6. $\gamma_d = \frac{G\gamma_w}{1+e}$ | $\rho_d = \frac{G\rho_w}{1+e}$ |
| 7. $\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$ | $\rho_{sat} = \frac{(G+e)\rho_w}{1+e}$ |
| 8. $\gamma = \frac{(G+S.e)\gamma_w}{1+e}$ | $\rho = \frac{(G+S.e)\rho_w}{1+e}$ |
| 9. $\gamma_{sub} = \frac{(G-1)}{1+e} \times \gamma_w$ | $\rho_{sub} = \frac{(G-1)}{1+e} \times \rho_w$ |
| 10. $\gamma_d = \frac{\gamma}{1+w}$ | $\rho_d = \frac{\rho}{1+w}$ |
| 11. $\gamma = \gamma_d + S(\gamma_{sat} - \gamma_d)$ | $\rho = \rho_d + S(\rho_{sat} - \rho_d)$ |
| 12. $\gamma_{sub} = \gamma_d - (1-\eta)\gamma_w$ | $\rho_{sub} = \rho_d - (1-\eta)\rho_w$ |
| 13. $\gamma_d = \frac{(1-\eta_a)G\gamma_w}{1+wG}$ | $\rho_d = \frac{(1-\eta_a)*G*\rho_w}{1+wG}$ |
| 14. $n_a = n - \frac{w\gamma_d}{\gamma_w}$ | $n_a = n - \frac{w\rho_d}{w}$ |

## * Index properties of soil:

The properties of the soil which help to assess its engineering behaviour & which assists in determining its classification accurately are termed as index properties of soils. They are,

a) Water content
b) Specific gravity of soil particle.
c) Field density
d) Particle size distribution
e) Consistency limits & indices.
f) Density index.

Q) A 50cm³ of moist soil ~~weighs~~ weighes 95 grams, its dry mass is 70 grams & specific gravity of solids is 2.68. Compute the water content, void ratio, porosity & unit weight of soil mass ($\gamma_t$).

→ Given,

Volume of soil mass (V) = 50 cm³

~~Weigh~~ Mass of dry soil ($M_s$) = 70 grams

Mass of soil mass (M) = 95 grams.

Specific gravity ($G_s$) = 2.68

(i) water content $(w) = \frac{W_w}{W_s} = \frac{M_w}{M_s} = \frac{95-70}{70} = 0.3571 = 35.71\%$

(ii) Void ratio $(e) = \frac{V_v}{V_s} = \frac{V-V_s}{V_s} = \frac{V}{V_s} - 1$

$$e = \frac{V \times \rho_s}{M_s} - 1 = \frac{V * \rho_w * G}{M_s} - 1$$

$$e = \frac{50 \times 1 \times 2.68}{70} - 1 = 0.914$$

(iii) porosity $(n) = e/1+e = 0.477$

iv) Unit weight of soil mass $(\gamma_t) = \frac{w}{V} = \frac{mg}{V}$

$$= \frac{95 * 10^{-3} * 9.81}{50 \times 10^{-6}\ m^3} = 18.639\ kN/m^3$$

## ※ Water Content determination:-

→ Oven drying method
→ Pycnometer method
→ Sand bath method
→ Calcium carbide method

### * Oven drying Method:-

It is the accurate method of determining the w.c present in soil sample. Firstofall, a clean container (can) is taken & weighed. Then, a soil sample is transferred into the container & the weight is taken. Then, the can with soil is kept on oven about 24 hours @ temp° of 110 ±5°c. The the weight of dry soil with container is taken.

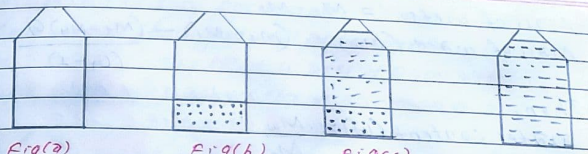
Let, $W_1$ = Weight of can (empty)
$W_2$ = Weight of can with wet soil.
$W_3$ = Weight of can + oven dried soil.

Then,

$$w = \frac{\text{Mass of water}}{\text{Mass of soil solid}} = \frac{M_2 - M_3}{M_3 - M_1} \times 100\%.$$

## ※ Pycnometer method:-

→ A pycnometer is a glass jar of about 1 litre capacity & fitted with a brass conical cap by means of screw type cover. The cap has a small hole of 6 mm diameter at its apex.
→ This method is used if the specific gravity of solid($G_1$) is known.
→ It is suitable for coarse grained soils.



Fig(a) Empty pycnometer | Fig(b) Pycnometer with moist soil | Fig(c) Pycnometer with moist soil + water | Fig(d) Pycnometer with water only.

Let, Mass of pycnometer = $M_1$
Mass of pycnometer + Mass of moist soil = $M_2$
Mass of pycnometer + moist soil + water = $M_3$
Mass of pycnometer + water only = $M_4$.

Here,

$$M_4 = M_3 - M_s + \left(\frac{M_s}{G_1 \rho_w}\right) * \rho_w \quad ---(1)$$

where, $M_s$ = Mass of solids,

We have to replaced the mass of solid by equt mass of water.

so, $M_w = \left(\frac{M_s}{G_1 \rho_w}\right)$ (volume of solid) $* \rho_w$

From (1)

$$M_4 = M_3 - M_s + \frac{M_s}{G_1}$$

$$M_4 = M_3 - M_s\left(1 - \frac{1}{G_1}\right)$$

$$\Rightarrow M_s\left(\frac{G_1 - 1}{G_1}\right) = M_3 - M_4$$

$$\Rightarrow M_s = \frac{(M_3 - M_4)G_1}{(G_1 - 1)}$$

Mass of wet soil = $M_2 - M_1$

Mass of water ($M_w$) = $(M_2 - M_1) - \frac{(M_3 - M_4)G}{(G-1)}$

Then,

Water content ($w$) = $\frac{M_w}{M_s}$

$$= \frac{(M_2 - M_1) - \frac{(M_3 - M_4)G}{(G-1)}}{\frac{(M_3 - M_4)G}{G-1}}$$

$$= \left[\frac{(M_2 - M_1)}{(M_3 - M_4)} \times \frac{(G-1)}{G} - 1\right] \times 100\%$$

## ✱ Specific Gravity Determination:-

→ Pycnometer method
→ Density bottle method.
→ Measuring flask method
→ Gas jar method
→ Shrinkage limit method

## ✱ Pycnometer method:-

→ A pycnometer is taken & weighted. The dry soil is added to it & again weight is taken. The water is added to the pycnometer containing dry soil & its weight also taken. A pycnometer with full of water is also taken & weighted.

→ It can be used for both coarse grained & fine grained soil.

→ Let, $M_1$ = Mass of pycnometer
→ $M_2$ = Mass of dry soil + pycnometer
→ $M_3$ = Mass of dry soil + water + pycnometer.
→ $M_4$ = Mass of pycnometer + water only.

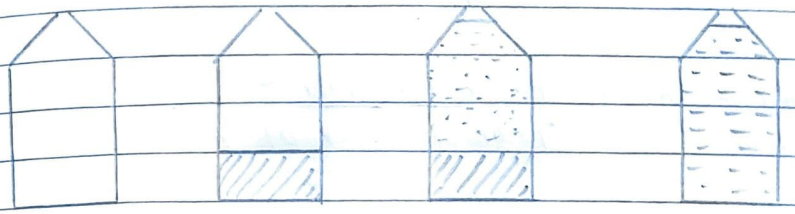


Fig (a) Empty pycnometer | Fig (b) Pycnometer with dry soil | Fig (c) Pycnometer with dry soil + water | Fig (d) pycnometer with water only

We know,

$$M_4 = M_3 - M_s + \left(\frac{M_s}{G\rho_w}\right) \times \rho_w$$

$$\Rightarrow M_4 = M_3 - M_s + \frac{M_s}{G}$$

$$\Rightarrow (M_4 - M_3) = -M_s\left(1 - \frac{1}{G}\right)$$

$$\Rightarrow \left(\frac{G-1}{G}\right) M_s = M_3 - M_4$$

$$\Rightarrow \left(1 - \frac{1}{G}\right) M_s = M_3 - M_4$$

$$\Rightarrow M_s - \frac{M_s}{G} = M_3 - M_4$$

$$\Rightarrow \frac{M_s}{G} = M_s - (M_3 - M_4)$$

$$\Rightarrow G = \frac{M_s}{M_s - (M_3 - M_4)} = \frac{M_2 - M_1}{(M_2 - M_1) - (M_3 - M_4)}$$

$$\therefore G_t = \frac{(M_2 - M_1)}{(M_2 - M_1) - (M_3 - M_4)} \quad \text{----(1)}$$

$$G_{27°C} = G_t * \frac{\text{Sp. gr of water at } t°C}{\text{Sp gr of of water at } 27°C}$$

$$G_{4°C} = G_t * \frac{\text{Sp. gr of water at } t°C}{\text{Sp. gr of water at } 4°C.}$$

N// During a test for water content determination on soil sample by pycnometer, the following observation were made.

Mass of wet soil sample = 1000 gm

Mass of pycnometer with soil & filled with water = 2000 gm

Mass of pycnometer filled with water only = 1480 gm

specific gravity of solids = 2.67.

Determine the water content. If the density of soil is 2.05 gm/ml. Determine degree of saturation.

→ Here,

$M_1$ = Mass of pycnometer.

$M_2$ = Mass of wet soil + pycnometer = $M_1 + 1000$

$M_3$ = Mass of pycnometer + soil + water = 2000 gm

$M_4$ = Mass of pycnometer with water = 1480 gm

$G_1$ = sp. gr of solids = 2.67.

Then,

$$w = \left[\frac{(M_2 - M_1)}{(M_3 - M_4)}\left(\frac{G_1 - 1}{G_1}\right) - 1\right] \times 100\%$$

$$= \left[\frac{(M_1 + 1000) - M_1}{2000 - 1480}\left(\frac{2.67 - 1}{2.67}\right) - 1\right] \times 100\%$$

$$= 20.28\%$$

Given, field density of soil ($\rho_t$) = 2.05 gm/ml

Degree of saturation (S) = ?

Then,

$$\rho_d = \frac{\rho_t}{1 + w} = \frac{2.05}{1 + \frac{20.28}{100}} = 1.704 \text{ gm/cc}$$

We know,

$$\rho_d = \frac{G \rho_w}{1 + e}$$

$$\Rightarrow 1.704 = \frac{2.67 \times 1}{1 + e}$$

$$\Rightarrow e = 0.566$$

Again, As we know, $S.e = w.G$

$$\Rightarrow S = \frac{wG}{e} = \frac{0.2028 \times 2.67}{0.566}$$

$$\Rightarrow S = 0.9566.$$

* A soil has porosity of 40%, the specific gravity of soil solid of 2.65 & water content of 12%. Determine the mass of water required to be added to 100 $m^3$ of this soil for full saturation.

→ Given,

$n = 0.4$

$G = 2.65$

$w = 0.12$

$V = 100\ m^3$

| | |
|---|---|
| Air | $V_v = 40 m^3$ |
| water | |
| Solid. | $V_s = 60 m^3$ |

We know that,

$$n = \frac{V_v}{V} \Rightarrow V_v = n \times V.$$

$$\Rightarrow V_v = 40 m^3 \quad \text{Then, } V_s = 60 m^3$$

Then, $w = \frac{M_w}{M_s}$

$\Rightarrow M_w = w M_s$

$\Rightarrow M_w = w \times (\rho_w \times G) \times V_s$

$\Rightarrow M_w = 0.12 \times 1000 \times 2.65 \times 60$

$\Rightarrow M_w = 19080\ kg$

Volume of water ~~reqd~~ $= \frac{19080}{1000} = 19.080\ m^3$

Volume of air $= 40 - 19.080$

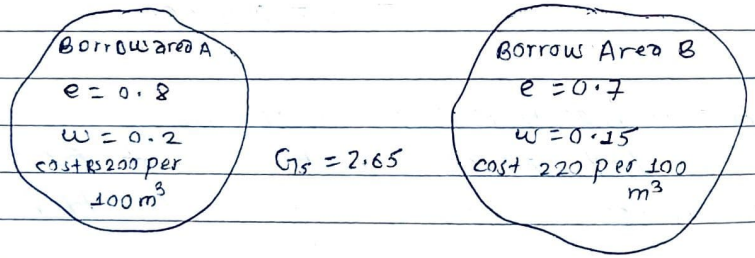
$= 20.92\ m^3$

Mass of water reqd to replace $20.92\ m^3$ vol. of air

$= 20.92 \times 1000$

$= 20920\ kg$

B.E 2.13




(a) For Borrow area A,

$\rho_d = \frac{G \rho_w}{1+e} = \frac{2.65 \times 1000}{1+0.8}$

$= 1472\ kg/m^3$

Let us consider $1\ m^3$ of burrow soil.

Then,

Mass of dry soil per $m^3 = 1472\ kg$

In embankment,

$\rho_d = \frac{\rho_t}{1+w} = \frac{2000}{1+0.22} = 1639.34\ kg/m^3$

Mass of dry soil reqd for $1\ m^3 = 1639.34\ kg$

$\therefore$ volume of soil required from A $= \frac{1639.34}{1472} = 1.113\ m^3$

Cost of soil from A, $= Rs\ \frac{200}{100} \times 1.113 = 2.23$ per $m^3$

(b) For Borrow Area 'B',

$\rho_d = \frac{G \rho_w}{1+e} = \frac{2.65 \times 1000}{1+0.7} = 1558.82\ kg/m^3$

Mass of dry soil per $m^3$ of volume $= 1558.82\ kg$

But,

Mass of dry soil per $m^3$ of volume for embankment $= 1639.34\ kg/m^3$

$\therefore$ volume of soil required from B $= \frac{1639.34}{1558.82} = 1.051\ m^3$

Then,

Cost of soil from B, $= Rs\ \frac{220}{100} \times 1.051$

$= 2.31/m^3$

Hence, Burrow area 'A' is economical.

* Index properties:-
  - Fine grained soil → sp. gravity, consistency limit
  - Coarse grained soil → sp. gr, grain size & shape

* Individual particles → size, shape
* Aggregate → Relative density

→ Index properties are just for classification of soils.

* Engineering properties:-
→ Used to find behaviour of soils.
- permeability
- shear strength
- Compressibility.

→ Time consuming & costlier to determine engineering properties of soil.

* Particle Size distribution:-
→ also known as mechanical analysis.
→ Method of separation of soils into different fractions based on particle size.
→ Two methods of analysis of particle size
  (a) Sieve Analysis [for gravel & sand]
  (b) Sedimentation Analysis. [for silt & clay]
→ Sieve analysis is for coarse grained soils (size > 75μ) which can easily pass through a set of sieves.
→ Sedimentation/wet analysis is used for fined grained soil (size < 75μ) [silt & clay]

[ Gravel: size > 4.75mm & Sand: 0.075mm < size < 4.75mm

* Sieve Analysis:-
→ Sieve is utensil made up of spun brass having size of square opening in mm or microns (80mm - 75μ) & diameter of sieve is 15-20 cm & used for separating coarse & fine grained soils.
→ Sieve analysis is done for coarse grained soils.
  (i) Gravel: size greater than 4.75mm
  (ii) Sand: size in between 75μ to 4.75mm.
→ The sieves are stacked one over the other, with decreasing size from top to bottom. The sieve of largest opening is kept at the top. A lid or cover is placed at the top of largest sieve. A receiver, known as pan, which has no opening is placed at the bottom of smallest sieve.
→ There are two types of sieve Analysis,
  ↳ Dry sieve Analysis
  ↳ Wet sieve Analysis.

(1) Dry Sieve analysis:-
→ We take roughly 500 gm of soil sample which must be free from lumps. We take total weight 'W'
→ Then, we sieved the sample through 4.75mm sieve. The sample retained on 4.75mm sieve is gravel fraction & sample passed is sand fraction.
→ For gravel fraction, Sieve size: 80mm, 40mm, 20mm, 10mm, 4.75mm
For Sand fraction, Sieve size: 2mm, 1mm, 600μ, 425μ, 212μ, 150μ & 75μ.
→ After stacking the sieve with decreasing size from top to bottom, whole arrangement is placed on mechanical Shaker & shaked for 10 minutes.
→ Amount of soil retained on each sieve is calculated.
[ soil retain = weight of soil with sieve - wt of empty sieve]

% retained on sieve = $\frac{\text{wt of soil retained on that sieve} \times 100}{\text{Total weight}}$

$P_1 = \frac{M_1}{M} \times 100$, $P_2 = \frac{M_2}{M} \times 100$ & so on.

Cumulative percentage retained,

$C_1 = P_1$, $C_2 = P_1 + P_2$

$C_3 = P_1 + P_2 + P_3$ & so on...

percentage finer,

$N_1 = 100 - C_1$, $N_2 = 100 - C_2$ & so on

percentage finer = 100 − Cumulative % retained.

(b) Wet Sieve Analysis:-

→ The purpose of wet sieve analysis is to remove all the particles which are finer than 75 μ particles from the correct sieve analysis results.

→ The sample is first washed over the 75 μ sieve to remove the fine particles sticking to the sand particles.

→ The wet sand retained on 75 μ sieve is dried in an ~~oven~~ Oven & after drying the dry sieve analysis is done in usual manner.

→ For finer fraction, hydrometer analysis is done.

* Particles Size distribution Curve:-

→ Also called gradation curve

→ This curve can be obtained by plotting percentage finer (N) in 'y' axis as normal scale & particle size (D) mm in 'x' axis as logarithmic scale.

∴ [semilog graph]

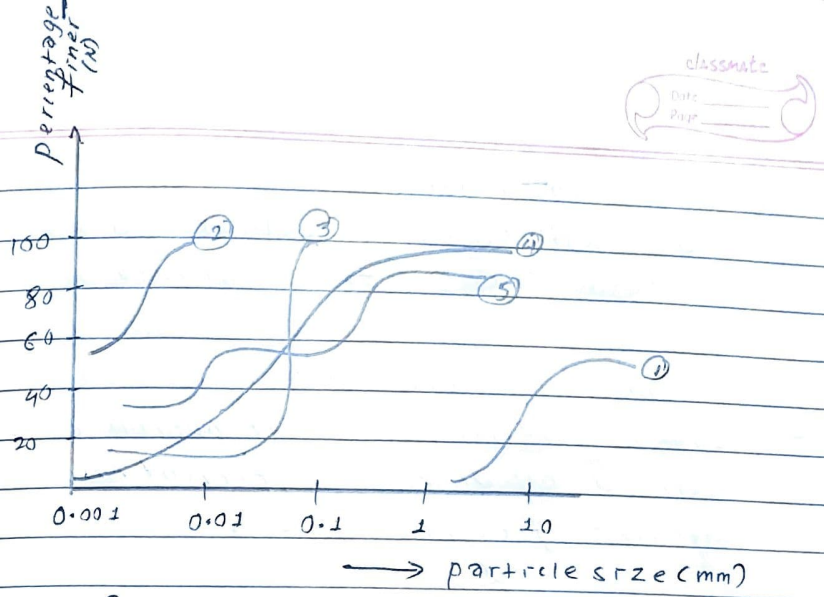



Fig:- Gradation curve.

→ Curve situated to the right (①) indicates coarse grained soils.

→ Curve situated to the left (②) indicates fine grained soils.

→ Curve (③) indicates the soils of almost same size. So, such soils are uniform soils.

→ The flat 's' curve (④) which represent a soil having particles of different sizes in good proportion. Such soil is called well graded soil.

→ A curve with hump (such as curve ⑤) represents the soil in which some of the intermediate size particles are missing. Such soils is called gap graded soils.

* Coefficient of uniformity:-

The uniformity of soil is expressed in terms of Coefficient of uniformity.

$C_u = \frac{D_{60}}{D_{10}}$ | $D_{10}$ = effective size.

$D_{60}$ = particle size such that 60% of soil is finer than this size.

$D_{10}$ = particle size such that 10% of soil is finer than this size.

→ $C_u < 2 \Rightarrow$ uniform soils.
Sands with $C_u \geq 6 \Rightarrow$ well graded
Gravel with $C_u \geq 4 \Rightarrow$ well graded.

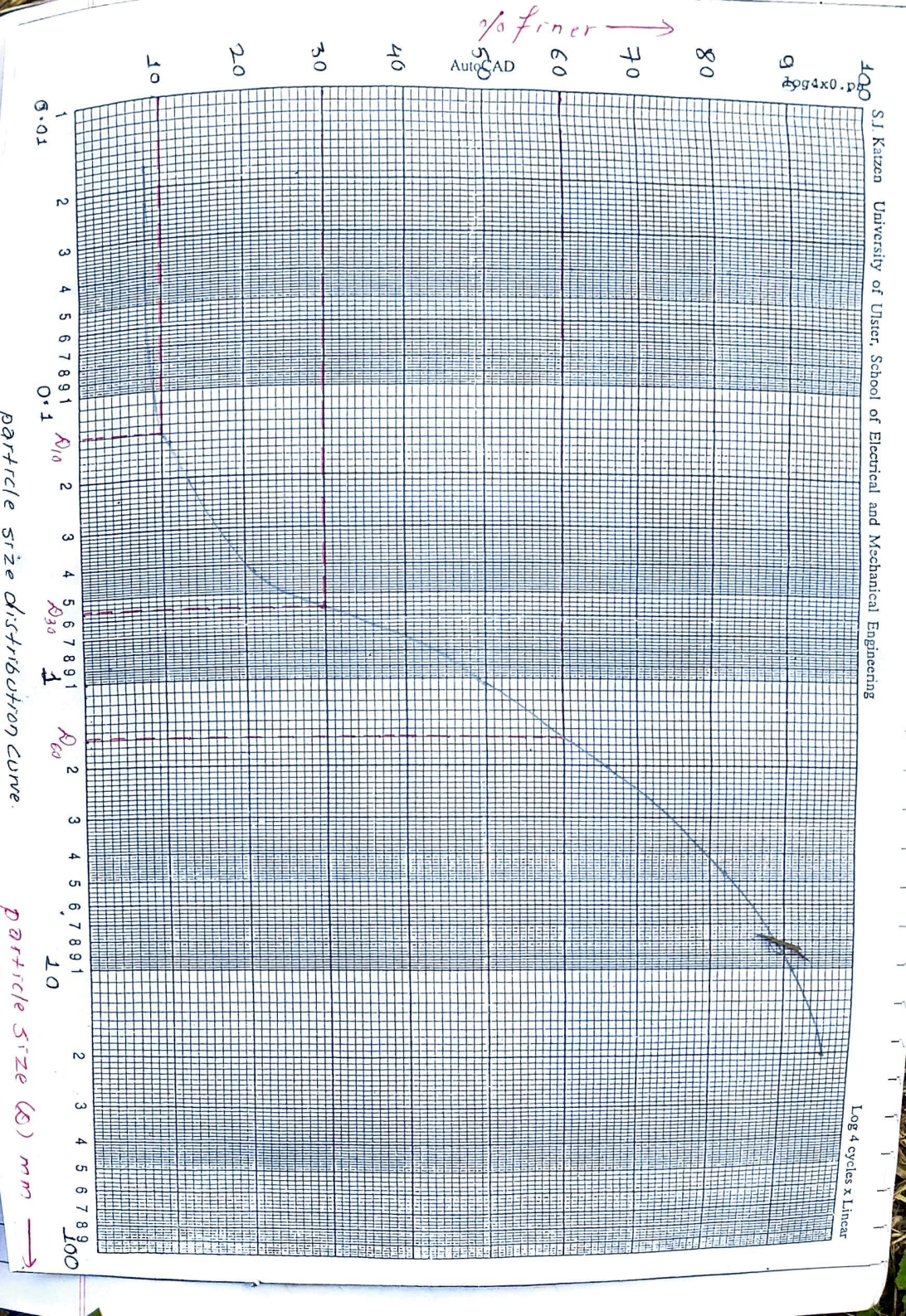
* Coefficient of Curvature:-
→ Shape of particle size distribution curve is described by another coefficient known as coefficient of curvature ($C_c$).

→
$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}}$$

Where,
$D_{30}$ = particle size corresponding 30% finer.
→ For well graded soil, coeff. of curvature lies bet$^n$ 1 & 3.
$(1 < C_c < 3)$

** Uses of particle size distribution curve:-
→ Classification of coarse grained soils.
→ Det$^m$ of coefficient of permeability
→ To know susceptibility of soil to frost action.
→ Design of drainage filter.
→ Det$^m$ of shear strength of soil.
→ Compressibility of soil
→ Used for soil stabilization
→ Design of pavement.
→ Indicate mode of deposition of soil.
→ Age of soil deposit.

Q The results of sieve analysis of soil are given below. Total mass of sample = 900gm.
Draw p.s.d.c & find $C_c$ & $C_u$.



## * Sedimentation Analysis:

Soil particles finer than 75 μ size cannot be sieved. The particle size distribution of such soils (silt, clayets) is determined by sedimentation Analysis. This analysis is based on stoke's law which gives the terminal velocity of a small sphere settling in a fluid of infinite extent.

We know,

$$\text{terminal velocity } (v) = \frac{1}{18} \frac{g D^2 (G-1) \rho_w}{\eta} \quad \text{--- (I)}$$

If a particle falls through height He in 't' minutes,

$$v = \frac{He}{60t} \quad \text{--- (II)}$$

Combining (I) & (II),

$$D = \sqrt{\frac{0.3\eta \times He}{g(G-1)\rho_w \times t}}$$

→ There are two methods of sedimentation analysis
  a) Pipette analysis
  b) Hydrometer analysis.

## * Preparation of suspension for sedimentation analysis:-

→ 50 gm oven-dried soil is taken in an evaporating dish.
→ Dispersion solution is prepared by adding 33 gm of sodium hexa-metaphosphate and 7 mgm of sodium carbonate to distilled water to make one litre of solution.
→ 100 ml dispersion solution is added to dish to oversoil & mixture is warmed gently for 10 minutes.
→ Transfer it to the mechanical stirrer cup & distilled water is added upto 3/4th part of cup.
→ Then, suspension is stirred for about 15 minutes & after completion, the suspension is then washed through a 75 μ sieve, using jets of distilled water. The portion of the suspension which has passed through the sieve is used for sedimentation analysis.

## * Hydrometer method for sedimentation Analysis:-

→ A hydrometer is an instrument used for determination of specific gravity of liquids.
→ As the specific gravity of soil suspension depends upon the particle size, a hydrometer can be used for the particle size analysis.
→ Hydrometer has two parts namely stem and bulb. Stem is marked from top to bottom as 0.995 to 1.030

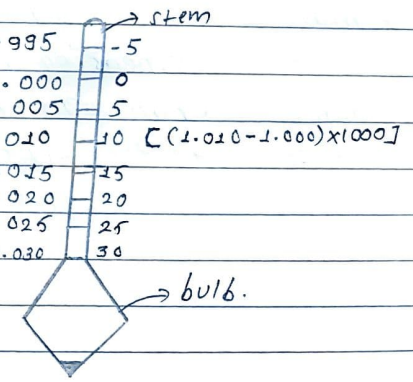
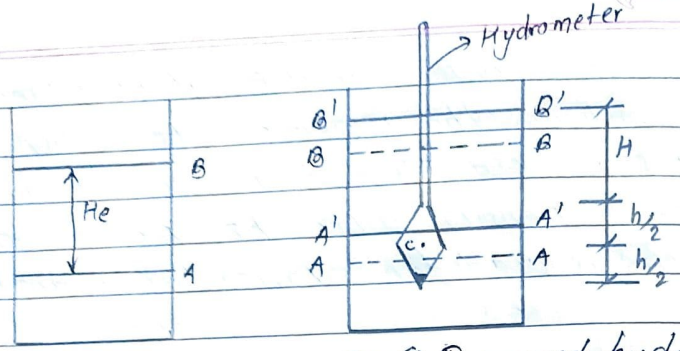



Fig.: Hydrometer.

### a) Callibration of Hydrometer:-

To determine the depth at which specific gravity is measured, callibration of hydrometer is done. The volume of hydrometer $V_H$ is first determined by immersing it in a graduated cylinder partly filled with water & note down the volume due to rise in water level.




Fig(a): Suspension with water




Fig(b): Immersed hydrometer in setup of Fig(a)

When hydrometer is inserted in cylinder, the layer of suspension A-A rises to level A'-A' and level B-B rises to B'-B'.

The effective depth He is,

$$He = H + \frac{h}{2} - \frac{V_H}{A} + \frac{V_H}{2A}$$

→ The marking on the hydrometer stem gives the specific gravity of the suspension at the centre of the bulb.

→ A calibration chart can be obtained by plotting graph of effective depth (He) versus hydrometer reading $R_h$.

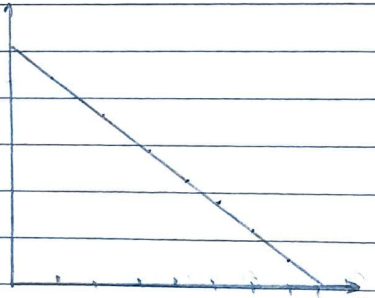



Fig: Callibration chart. $R_h$

(b) Test procedure:-

→ 1000 ml of suspension is taken & mixed throughly by placing palm on open end of jar & turning it up & down.

→ Then, hydrometer is inserted & reading ($R_h$) taken at 0.5, 1, 2 & 4 minutes.

→ Hydrometer is then removed from there & placed in jar with distilled water. The reading ($r_m$) is taken after 8, 15, 30 minutes & 1, 2, 4, 8 & 24 hours.

→ For each reading, hydrometer is inserted about 20 seconds before the reading.

(c) Correction of hydrometer reading:-

(i) Meniscus Correction:-

$$R_h = R_h' + C_m$$

(ii) Temp$^r$ correction:-

$$R_h = R_h' \pm C_t$$

+ → If temp$^r$ > 27°C

− → If temp$^r$ < 27°C

(iii) Dispersion agent correction,

$$R_h = R_h' - C_d$$

* Plasticity of soils:

→ plasticity of soil is its ability to undergo deformation without cracking or fracturing.

→ A plastic soil can be moulded into various shapes when it is wet.

→ Plasticity is an important index properties of fine grained soil (specially clay)

* Consistency limits:-

→ The consistency of fine grained soil is the physical

state in which it exists.

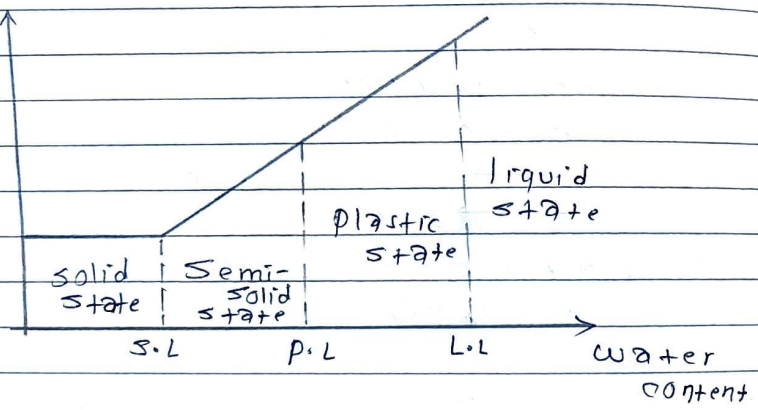
→ It is used to denote the degree of firmness of soil.

→ Consistency of soil is indicated by such terms as soft, firm or hard.

→ According to Atterberg, fine grained soil can exist in four states namely liquid, plastic, semi-solid or solid state.

→ The water content at which soil changes from one state to another state are known as consistency limits or atterberg's limit.

→




→ A soil containing high water is in liquid state. In this condition, shear strength of soil is zero.

→ As the water content is reduced, the soil becomes stiffer & starts developing resistance to shear deformation. At particular water content, soil becomes plastic.

→ The water content at which soil changes from liquid state to plastic state is known as liquid limit.

→ The soil in plastic stage can be moulded into various shapes.

→ As water content reduces, the soil changes from plastic state to semi-solid state. In semi-solid state, soil solids get cracked when moulded.

→ The water content at which soil changes from plastic state to semi-solid state is known as plastic limit.

→ With decrease in water content, volume decreases & a situation occured at which with decrease in water content, volume remains unchanged. This means, soil is said to have reach in solid state.

→ The water content at which soil changes from semi-solid state to solid state is known as shrinkage limit.

## * Liquid Limit:-

→ Water content at which soil changes from liquid state to plastic state is known as liquid limit.

→ The liquid limit is determined in the laboratory either by Cassagrande's apparatus or by cone penetration method.

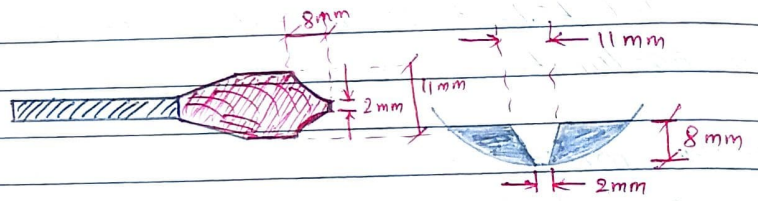
→ The device used in cassagrande's method consists of brass cup which drops through a height of 1cm on a hard base when operated by handle. The device is operated by turning the handle which raises the cup & lets it drop on the rubber base. The height of drop is adjusted with the help of adjusting screws.

→ About 120 gm of dry soil sample passing through 425 μ sieve is taken in a dish & mixed with distilled water to form a uniform paste.

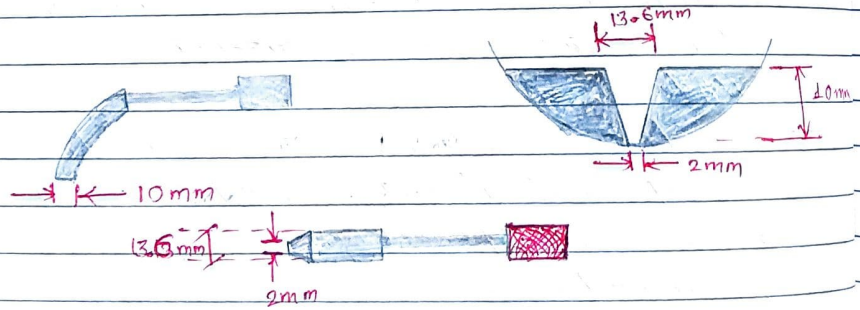
→ A portion of this paste is placed in the cup of liquid limit device & surface is smoothened & leveled with a spatula.

→ A groove is cut through the sample along the symmetrical axis of cup, preferably in one stroke, using standard grooving tool.

→ Grooving tool ┬→ Casagrande tool
└→ ASTM tool.




Casagrande tool.




ASTM tool.

→ The casagrande tool is recommended for normal fine grained soil.

→ After cutting the groove, the handle is turned at rate of rev/sec until two parts of soil sample come into contact at the bottom of groove along a distance of 12mm. The groove should close by flow of soil & not by slippage beth soil & cup.

→ Soil sample paste is taken for water content.

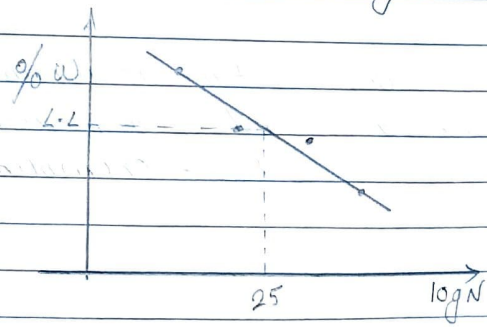
→ The test is repeated at least two times. Following data must be recorded.

| % water content | No. of blows |
|---|---|
| $w_1$ | $N_1$ |
| $w_2$ | $N_2$ |
| $w_3$ | $N_3$ |

→ The water content corresponding to the 25 number of blows gives value of liquid limit.

→ The test is conducted with different water contents to get number of blows in the range of 10 to 40.

→ The semilog graph is plotted beth water content as ordinate & No. of blows as abscissa in log scale.



## * Plastic Limit:-

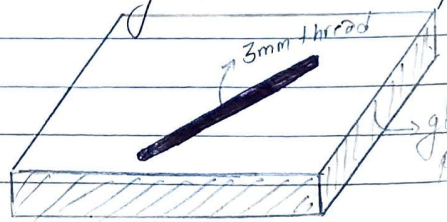
→ The water content at which soil changes from plastic stage to semi-solid stage denotes plastic limit.

→ For determination of plastic limit of a soil, an air dried 30gm soil sample passing through 425μ sieve is taken in dish. It is mixed throughly with distilled water till it becomes plastic & can be easily moulded with fingers.

→ About 10gm of soil paste is taken in one hand & a ball is formed. The ball is rolled with fingers on a glass plate to form a soil thread of uniform diameter.

→ If the diameter of thread becomes smaller than 3mm, without crack formation, it shows that the water content is more than the plastic limit. The soil is kneaded further. This results in the reduction of water content, as some water is evaporated due to heat of hand. The soil is re-rolled & procedure is repeated.

→ The water content at which the soil can be rolled into a thread of approximately 3mm in diameter without crumbling is known as plastic limit.




## * Some terminology

### a) plasticity index:-

→ Range of water content over which soil remains in plastic state

$$P.I = L.L - P.L$$

### b) Liquidity index:-

→ indicates the nearness of its water content to liquid limit.

→ $$L.I = \frac{W - P.L}{L.L - P.L} \times 100$$

→ When soil is at liquid state, L.I is 100% & when soil is in plastic state, liquidity index is 0.

### c) Consistency index:-

→ Indicates the consistency (firmness) of the soil.

→ It shows the nearness of the water content of soil to its plastic limit.

$$C.I = \frac{L.L - W}{P.I} \times 100$$

### d) Flow index:-

→ Flow index is the slope of flow curve obtained bet^n no. of blows & the water content in casagrande's method of determination of liquid limit.

$$I_f = \frac{W_1 - W_2}{\log_{10}(N_2/N_1)}$$

### e) Toughness index:-

→ Measure of shearing strength of soil at plastic limit.

$$I_t = \frac{I_p}{I_f}$$

## * Shrinkage limit:-

→ The water content at which soil changes from semi-solid state to solid state is called shrinkage limit.

→ When soil reaches to S.L, the change in volume does not occurs. This means volume is constant.

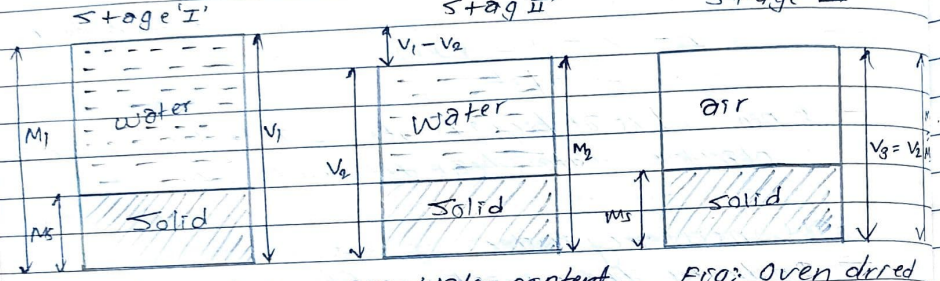



Fig: water content greater than shrinkage limit

Fig: water content at shrinkage limit

Fig: Oven dried soil.

Let,

$M_s$ = Mass of solids.

$M_1 - M_s$ = Mass of water in stage I

$(V_1 - V_2) \times \rho_w$ = Loss of mass of water from stage I to II

Mass of water in stage II $= (M_1 - M_s) - (V_1 - V_2)\rho_w$.

From defn,

$$\text{Shrinkage limit} = \text{water content in stage II} = \frac{(M_1 - M_s) - (V_1 - V_2)\rho_w}{M_s}$$

$$= w_1 - \frac{(V_1 - V_2)}{M_s} \times \rho_w$$

where,

$w_1$ = water content at stage I.

## * Shrinkage parameters:-

### (i) Shrinkage Index:-

The shrinkage index ($I_s$) is diff$^n$ betn L.L & S.L

$$I_s = L.L - S.L$$

### (ii) Shrinkage ratio:-

→ Ratio of given volume change expressed as a percentage of dry volume, to the corresponding change in water content.

$$S.R = \frac{(V_1 - V_2)/V_d \times 100}{w_1 - w_2}$$

where,

$V_1$ → volume of soil mass at water content $w_1$

$V_2$ → volume of soil mass at water content $w_2$

$V_d$ → volume of dry soil mass.

### (iii) Volumetric Shrinkage:-

Change in volume expressed as a percentage of dry volume when water content is reduced from a given value to shrinkage limit.

$$V.S = \frac{V_1 - V_d}{V_d} \times 100$$

### (iv) Linear shrinkage:-

Change in length divided by initial length when water content is reduced to shrinkage limit.

$$LS = \frac{\text{Initial length} - \text{Final length}}{\text{Initial length}} \times 100$$

Ch-8

# Soil Identification & its classification.

→ The process of distinguishing different types of soils by observing the index & engineering properties of soil either in lab or in field is known as soil identification.

→ Soil classification is the arrangement of soils into different groups such that soils in a particular group have same behaviour. The soil classification has following requirements.

1) It should have a limited number of group.
2) should be based on engineering properties.
3) should be simple & easily understood.

→ A soil classification system provides a common language between engineering dealing with soils. It is useful in exchange of information & experience betn the geotechnical engineers.

→ Soil classification is not exact analysis. so, for design of large structures, engineering properties should also be determined by conducting tests.

## * Purpose of soil classification:-

a) It provides general guidance about engineering properties of soils through accumulated experience.
b) The purpose of soil classification is to make possible the estimation of soil properties by association with soils of the same class whose ~~performance~~ properties are known.
c) Use data of others in predicting foundation performance.
d) Provides a common language betn engineers dealing with soil.
e) It is useful in exchange of information & experience betn the geotechnical engineers.

## * Field identification of soils:-

→ The sample is spread on a flat surface. If more than 50% particles are visible to the naked eye, the soil is coarse grained otherwise it is fine grained.

→ particle size ($D$) > 300 mm → Boulder
300 mm < $D$ < 80 mm → Cobble
80 mm < $D$ < 4.75 mm → Gravel
4.75 mm < $D$ < 0.075 mm → Sand
0.075 mm < $D$ < 2μ → Silt
particle size ($D$) < 2μ → clay

→ Boulder, cobble & Gravel can be separated by measuring particle size. We can also separate them from sand.

→ Fine sand cannot be easily distinguished from silt by simple visual examination. Silt may look a little darker in colour. However, it is possible to differentiate them by dispersion test.

• This test consists of pouring a spoonful of sample in a jar of water. If the material is sand, it will settle down in a minute or two, but if it is silt, it may take 15 min to 1 hour.

→ To separate silt from clay, many laboratory test are performed. But in field, we can separate them in following way.

### (a) Dilatency test:-

In this test, soil mixed with water to soft consistency is shaken after placing in the palm of hand. If the soil is silt, water will rise quickly to the surface & give it a shiny glistening appearance. If it is clay, the water cannot move easily & hence it continues to look dark.

b) Dry strength:-

It can be estimated by crushing 3mm size dried fragment betn thumb & forefinger. A clay fragment can be broken only with great effort whereas a silt fragment crushes easily.

c) Toughness test:-

A thread is made from moist soil sample with diameter of 3mm. If the material is silt, it is not possible to make such a thread without disintegration & crumbling. If material is clay, thread will be formed without cracking.

d) Dispersion test:-

A spoonful of soil is poured in a jar of water. If it is silt, the particle will settle down in 15 min to 1 hour but if it is clay, it will form a suspension which will remain as such for hours, even days, provided fluctuation does not take place.

## ✱ Soil classification:-

- particle size classification [MIT classification only]
- Textural classification
- USCS
- ISCS
- AASTHO classification

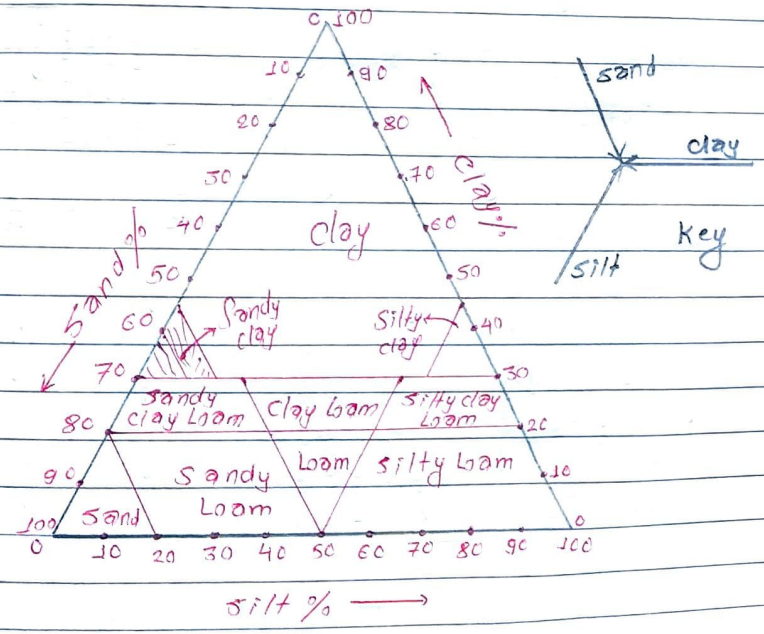
## ✱ MIT Classification:-

→ This classification is based on particle size only.
→ Massachusettes Institute of technology

| clay | ~~Sand~~ Silt | | | Sand | | | Gravel |
|---|---|---|---|---|---|---|---|
| | Fine | Medium | Coarse | Fine | Medium | Coarse | |
| | 0.002 mm | 0.006mm | 0.02mm | 0.06mm | 0.2mm | 0.6mm | 2mm |

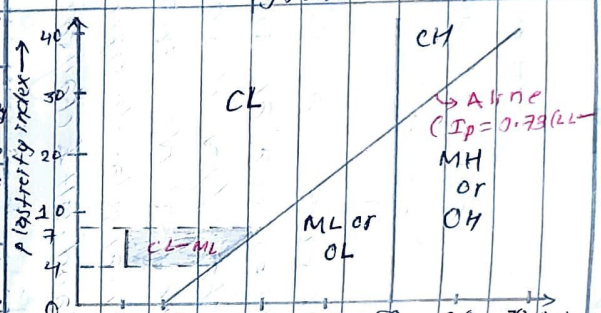
## ✱ Textural classification:-

→ The visual appearance of soil depends upon particle size, shape of particle, gradation of particle etc.
→ Textural classification is also based on particle size only.
→ Triangular classification system in which percentage of sand, silt & clay are expressed on three sides of triangle.



## * USCS classification: (particle size characteristics)

↳ Unified soil classification system.

| Major division | | | Group symbols | Typical names | Classification Criteria | |
|---|---|---|---|---|---|---|
| Coarse grained soils [More than 50% retained on No. 200 sieve (0.075mm)] | Gravel [50% or more of coarse fraction retained on No. 4 sieve (4.75mm)] | Clean Gravels | GW | Well graded gravels | percentage of fines (a) Less than 5% passing No. 200 GW, GP, SW, SP (b) More than 12% passing No. 200, GM, GC, SM, SC (c) 5 to 12% passing No. 200 use of dual symbols as GW-GM, SP-SC | $C_U > 4$ & $C_c$ = 1 to 3 |
| | | | GP | poorly graded gravels | | Not meeting both criteria for GW |
| | | Gravel With fines | GM | silty Gravels | | Atterberg limits below A-line or plasticity index less than 4 / Atterbergs Limit in hatched area GM-GC |
| | | | GC | clay Gravels. | | Atterberg limits above A-line & plasticity index greater than 7 |
| | Sand [More than 50% of coarse fraction passing No. 4 sieve (4.75mm)] | Clean Sands | SW | well graded sand | | $C_U > 6$ & $C_c$ = 1 to 3 |
| | | | SP | poorly graded sand | | Not meeting both criteria for SW |
| | | Sand with fines | SM | silty sands | | Atterberg limits below A line or plasticity index less than 4 / Atterberg Limits in hatched area SM-SC |
| | | | SC | clay sands | | Atterberg Limits above A-line & plasticity index greater than 7 |
| Fine grained soils [50% or more passing No. 200 sieve (0.075mm)] | Silts and clays Liquid Limit 50% or less | | ML | Inorganic silts of low plasticity |   plasticity chart (USC) | |
| | | | CL | Inorganic clays of low to medium plasticity | | |
| | | | OL | organic silts of low plasticity | | |
| | Silts & clays Liquid Limit greater than 50% | | MH | Inorganic silts of high plasticity | | |
| | | | CH | Inorganic clays of high plasticity | | |
| | | | OH | organic clays of medium to high plasticity | | |
| Highly Organic soils | | | Pt | Peat, muck & other highly organic soils. | visual manual identification. | |

## * USCS classification:

↳ Indian standard classification system

| Division | Subdivision | | Group symbol | Typical Names | Laboratory Criteria | | Remarks |
|---|---|---|---|---|---|---|---|
| (1) Coarse grained soils (More than half of material is larger than 75 μ IS sieve size) | Gravel (G) [More than half of coarse fraction is larger than 4.75mm IS sieve] | clean gravels [Fines less than 5%] | GW | well graded gravels | $C_U$ greater than 4, $C_c$ = 1 to 3 | | when fines are bet 5% to 12% boarder line cases requiring dual symbols such as GP-GM, SW-SC etc. |
| | | Gravels with appreciable amount of fines (fines more than 12%) | GP | poorly graded gravels | Not meeting all gradation requirements for GW | | |
| | | | GM | silty gravels | Atterberg limits below A-line with $I_p$ less than 4 | Atterberg limits plotting above A-line with $I_p$ between 4 & 7 are border line use of dual symbol GM-GC | |
| | | | GC | clay gravels | Atterberg limits above A-line & $I_p > 7$ | | |
| | Sand (S) [More than half of coarse fraction is smaller than 4.75mm IS sieve. | clean sands [Fines less than 5%] | SW | well graded sands | $C_U > 6$ $C_c$ = 1 − 3 | | |
| | | | SP | poorly graded sands | Not meeting all gradation requirement for SW | | |
| | | sands with appreciable amount of fines [Fines more than 12%] | SM | silty sands | Atterbergs limit below A-line, $I_p < 4$ | Atterbergs limit plotting above A line with $I_p$ between 4 & 7 are cases requiring double symbol SM-SC | |
| | | | SC | clay sands. | Atterbergs limits above A-line $I_p > 7$ | | |
| (2) Fine grained soils (more than 50% pass 75 μ IS sieve) | Low compressibility (L) [Liquid limit less than 35%.] | | ML | Inorganic silts with low plasticity | Atterberg limits plot below A-line $I_p < 4$ | Atterberg limits plotting above A-line with $I_p$ between 4 to 7 (ML-CL) | 1) Organic & inorganic soils plotted in the same zone in plasticity chart are distinguished by odour and color or liquid limit test after oven drying. A reduction in L.L after oven drying to a value less than three-fourth of L.L before oven drying is positive |
| | | | CL | Inorganic clays of low plasticity | Atterbergs limits plot above A-line $I_p > 7$ | | |
| | | | OL | Organic silts of low plasticity | Atterbergs limits not below A-line | | |
| | Intermediate compressibility (I) [35% < L.L < 50%] | | MI | Inorganic silts of medium plasticity | Atterbergs limits plot above A-line | | 2) Black cotton soils of India lie along a band partly above A-line & partly below A-line |
| | | | CI | Inorganic clays of medium plasticity | Atterbergs limit plot above A-line | | |
| | | | OI | organic silts of medium plasticity | Atterbergs limit plot below A-line | | |
| | High compressibility (H) [L.L > 50%] | | MH | Inorganic silts of high comp. | Atterbergs limit plot below A-line | | |
| | | | CH | Inorganic clays of high plasticity | Atterbergs limit plot above A-line | | |
| | | | OH | Organic clays of medium to high plasticity | Atterbergs limit plot below A-line | | |
| 3) Highly organic soils | | | Pt | Peat & other highly organic soils | Readily identified by colour, odour, spongy feel & fibrous texture. | | see plasticity chart |

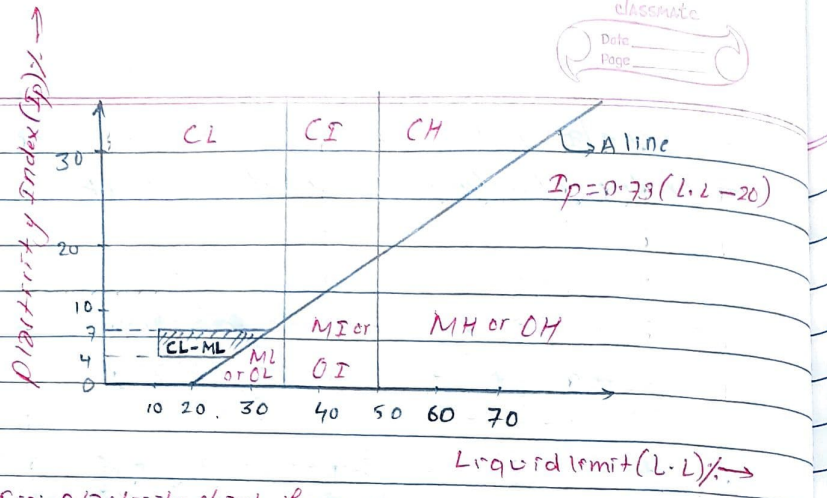



Fig: plasticity chart for ISCS.

## * AASTHO CLASSIFICATION SYSTEM:-

→ American Association of state highway & transportation Official (AASTHO) classification System.

→ Useful for classifying soils for highways.

→ The particle size and plasticity characteristics are required to classify the soils.

→ For both coarse grained & fine grained soils.

→ In this system, soils are divided into 7-types, designated as A-1 to A-7. The soils A-1 & A-7 are further subdivided into two categories.

→ The soil of group A-1 is most suitable for highways & A-7 is least stable.

→ Fine grained soils are further rated for their sustainability for highways by group index (GI), determined as,

$$GI = 0.2(F-35) + 0.005(F-35)(LL-40) + 0.01(F-15)(I_p-10)$$

→ smaller the value of group index, better is the soil in that category. A GI of zero indicates a good subgrade whereas a group index of 20 or greater shows a very poor subgrade.

F = percentage by mass passing American sieve no. 200 (size 0.075mm).

**AASTHO CLASSIFICATION System**

| General Classification | Granular materials (≤35% or less passing No.200 sieve (0.075mm)) | | | | | | | Silt clay Materials (More than 35% passing No. 200 sieve (0.075mm)) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Group classification | A-1 | | A-3 | A-2 | | | | A-4 | A-5 | A-6 | A-7 |
| | A-1-a | A-1-b | | A-2-4 | A-2-5 | A-2-6 | A-2-7 | | | | A-7-5, A-7-6 |
| (a) Sieve analysis, percent passing | | | | | | | | | | | |
| (i) 2.00 mm (No.10) | 50 max | | | | | | | | | | |
| (ii) 0.425 mm (No.40) | 30 max | 50 max | 51 min | | | | | | | | |
| (iii) 0.075 mm (No.200) | 15 max | 25 max | 10 max | 35 max | 35 max | 35 max | 35 max | 36 min | 36 min | 36 min | 36 min |
| (b) Characteristics of fraction passing 0.425 mm (No.40) | | | | | | | | | | | |
| (1) Liquid limit | | | | 40 max | 41 min | 40 max | 41 min | 40 max | 41 min | 40 max | 41 min |
| (ii) Plasticity index | 6 max | | N.P. | 10 max | 10 max | 11 min | 11 min | 10 max | 10 max | 11 min | 11 min |
| (c) Usual types of significant constituents | Stone fragments gravel & sand | | Fine sand | Silty or clayey gravel & sand | | | | Silty soils | | Clayey soils | |
| (d) General rating as subgrade | Excellent to good | | | | | | | Fair to poor | | | |

* If Ip is 9 less than (LL − 30) then soil is A-7-5 (i.e. Ip < LL − 30)
If Ip > (LL − 30) then soil is A-7-6 (i.e. Ip > LL − 30)

## * Comparison of AASTHO and USCS classification:-

→ AASTHO system is suitable for highways only whereas USCS is for determining suitability of soils for general use.

→ According to AASTHO system, a soil is termed fine grained if more than 35% passes NO.200 (0.075mm) sieve, whereas in USC system, if more than 50% passes that sieve.

→ In AASTHO system, sieve NO.10 (2mm size) is used to divide the soil into gravel & sand, whereas in USC system, sieve NO. 4 (4.75 mm is used).

→ In USC system, the gravel & sandy soil are clearly separated, whereas in AASTHO system, clear demarcation is not done.

→ Symbols used in USC system are more descriptive & are more easily remembered than those in AASTHO system.

→ In AASTHO system, there is no place for organic soils but In USCS, soil are classified as OL & OH.

→ USCS System is more convenient to use than AASTHO System.

**Q** A sample of soil was tested in a laboratory, & following observations were recorded

Liquid Limit = 45% plastic limit = 16%

| US Sieve NO. | NO.4 | NO.10 | NO.40 | NO.200 |
|---|---|---|---|---|
| % passing | 100 | 91.5 | 80 | 60 |

Classify the soil according to AASTHO System:

→ Here,

plasticity index = L.L - P.L = 45 - 16 = 29%

→ More than 35% [i.e 60%] passing through I.S sieve NO. 200., so it is silt & clay material.

→ From Liquid Limit, A-5 & A-7 is possible (41 minimum)

→ From plasticity index A-5 is not possible. So, given soil sample belong to A-7 group.

• Here, I.P = 29%

L.L - 30 = 45 - 30 = 15%

Since $I_p > L.L - 30$, so, soil is A-7-6

→ Group index,

$GI = 0.2(F-35) + 0.005(F-35)(LL-40) + 0.01(F-15)(I_p-10)$

Take, F = 60,

$GI = 0.2(60-35) + 0.005(60-35)(45-40) + 0.01(60-15) \times (29-10)$

$= 13.3 \approx 13$

→ ∴ Soil is classified as A-7-6 (13).

**Q** Classify the soils A and B with the properties as shown below according to USC system.

| Soil | Liquid Limit (%) | $I_p$ (%) | % passing NO.4 sieve | % passing NO.200 sieve |
|---|---|---|---|---|
| A | 45 | 29 | 100 | 59 |
| B | 55 | 15 | 100 | 85 |

→ (a) Soil A :- 50% more, particles passes through NO.200 sieve so, soil is fine grained. Silt & clays having L.L < 50%. From plasticity chart, the atterberg limit lies above A line. Also, from eqn of A line,

$I_p = 0.73(LL-20) = 0.73(45-20) = 18.25\%$

Therefore, soil is classified as CL.

(b) Soil B :- More than 50% passes through NO.400 sieve. So, soil is fine grained. Silt & clays having L.L > 50%.

→ The atterberg limit plots below A-Line. It can either MH or OH.

$I_p = 0.73(55 - 20)$

$= 25.55\% > 15\%$.

→ If the soil is OH, its L.L will decrease considerably on oven drying.

Q. Classify the soil with following properties according to ISCS System.

| Liquid limit | plasticity Index | % passing 4.75 mm sieve | % passing 75 μ sieve. |
|---|---|---|---|
| 40% | 10% | 60% | 45% |

→ More than 50% particles retained on IS 75 μ sieve. So, the soil is coarse grained.

→ coarse fraction = 55%

Gravel fraction = 40%

sand fraction = 15%.

→ As more than half the coarse fraction is larger than 4.75mm IS sieve, the soil is gravel

→ The soil has more than 12% fines, it can be either GM or GC.

→ As the Atterberg limits plot below A-line, the soil is GM.

## ch-11 Stability of slopes:-

→ In geotechnical engineering, the term slope commonly refers to a soil mass with its surface inclined to the horizontal, & there exists both natural & man-made slopes. Natural slopes are those that exist in nature & formed by natural causes. such slopes exist in natural area such as hills. The side cutting, slope of embankments constructed for roads, railway lines, canals etc are man made slopes.

→ The failure of earth surface occurs when the forces tending to slide are greater than those tending to restore or stabilize the soil mass - along the critical surface of failure.

→ The surface along which the soil mass slides when the failure of an earth surface occurs is known as critical surface of failure.

### * Necessity of slope stability:-

→ Important consideration in the design & construction of earth dams.

→ It should also considered in the management of many types of mining operations or civil engineering projects

For eg:
- Underground mines
- Landfills & excavation
- Tailing dams.
- Embankments
- Earthen dams

→ For road construction, slope stability factor plays important role. If it is not considered, landslide, slope failure etc occurs. Due to which there will be loss of property & life.

## * Causes of slope instability :-

There are two causes of instability of slopes.

(a) Increase in shear stress ($\tau$)

i) Due to increase in unit weight of soil by wetting.
ii) Increase in surcharge load.
iii) Due to increase in steepness of slopes (erosion or excavation)
iv) Due to dynamic shock (earthquake)

(b) Decrease in shear strength of soil ($s$) :-

i) Due to increase in water content
ii) Due to increase in pore water pressure
iii) Due to weathering & erosion.

## * Infinite & finite slopes :-

→ An infinite slope is one which has constant slope of infinite extent and having same soil properties for identical depth below the surface. so that slip surface will be plane parallel to the surface of slope. (Translational failure)




→ The slope that is of limited extent in which engineer deals is called finite slope. Slope length not infinite. The slope of embankments & earth dams are examples of finite slopes. Slope in which soil doesn't have identical properties at identical depth.
(Rotational failure)

## * Types of slope failure :- (Modes)

a) Rotational failure
b) Translational failure
c) Compound failure
d) Wedge failure
e) Miscellaneous failure.

### (a) Rotational failure :-

→ This type of failure occurs by rotation along a slip surface by downward & outward movement of soils.
→ It may be either circular or non-circular.
→ Rotational slips are further divided into three types.
(a) Toe failure → failure occurs along the surface that passes through toe.
(b) Slope failure → failure occurs along a surface that intersect the slope above the toe.
(c) Base failure → failure surface passes below the toe.

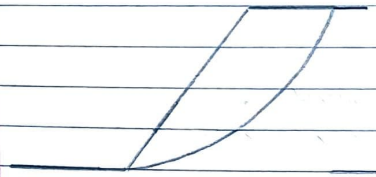


Fig: Toe failure,

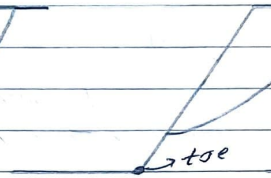



Fig: Slope failure

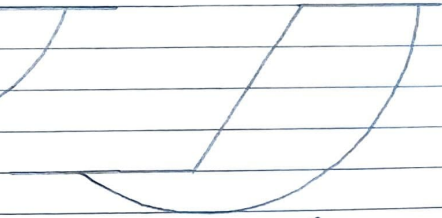
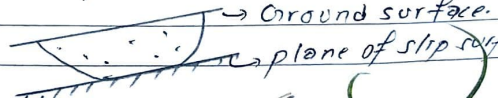


Fig: Base failure

### (b) Translational failure :-

Translational failure occurs in a infinite slope along a long failure surface parallel to the slope.



(c) Compound failures:-

A compound failure is the combination of rotational & translational failures. A compound failure surface is curved at the two ends and plane in the middle portion. A compound failure generally occurs when a hard stratum exists at considerable depth below the toe.

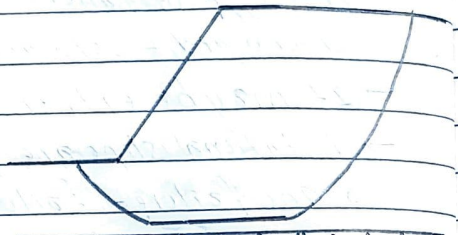


Fig: Compound failure.

(d) Wedge failure:-

A failure along an inclined plane is known as plane failure or wedge failure. It occurs when distinct blocks & wedges of the soil mass become separated. plane failure occurs in finite slope.

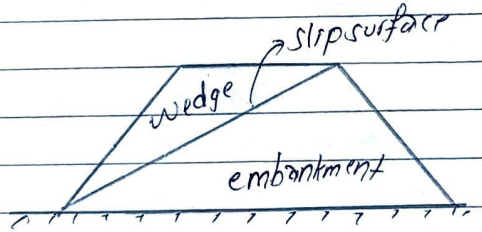



Fig: wedge failure.

(e) Miscellaneous failures:-

In addition to above four types of failures, some complex types of failures in the form of spread & flows may also occurs.

## Factor of safety:-

Three different defn of FOS are used in slope stability. They are

(a) Factor of safety wrt shear strength

Ratio of shear strength (s) to the shear stress ($\tau_m$) along surface of failure.

$$FOS = F_s = \frac{s}{\tau_m} = \frac{c + \bar{\sigma}\tan\phi}{c_m + \bar{\sigma}\tan\phi_m}$$

Where, $c$ = cohesion of soil

$c_m$ = Mobilized cohesion

$\bar{\sigma}$ = Effective stress

$\phi$ = Internal frictional angle of soil.

$\phi_m$ = Mobilized frictional angle of soil.

(b) Factor of safety wrt cohesion:-

Ratio of cohesion of soil to its mobilized cohesion.

$$F_c = \frac{c}{c_m}$$

(c) Factor of safety wrt friction:-

Ratio of available frictional strength to mobilized frictional strength

$$F_\phi = \frac{\bar{\sigma}\tan\phi}{\bar{\sigma}\tan\phi_m}$$

$$F_\phi = \frac{\tan\phi}{\tan\phi_m}$$

For small angles,

$$F_\phi = \frac{\phi}{\phi_m}$$

Generally, for analysis of stability of slope, all three FOS are taken equal. $F_s = F_c = F_\phi$

## * Stability of an infinite slope of cohesionless soils.

The stability criteria of an infinite slope of cohesionless soils will depend whether the soil is dry or submerged or steady seepage.

### (a) Dry soils:-

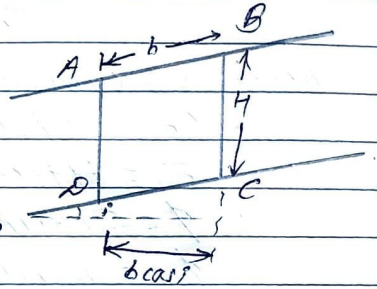



Figure shows a section of an infinite slope having inclination 'i'.

Let 'b' be the inclined length of soil section & 'H' be the height. Let width of soil mass be unity. Then,

The forces acting along slip surface CD are,

W = weight of prism ABCD

T = Tangential component of weight along CD.

N = Normal component of weight along CD.

Here,

$$W = \gamma * \text{volume of prism}$$
$$= \gamma * (Hb\cos i \times 1)$$

Then,

$$N = W\cos i = \gamma Hb\cos^2 i$$
$$T = W\sin i = \gamma Hb\sin i \cdot \cos i$$

Also,

$$\text{Normal stress } (\sigma) = \frac{N}{A} = \frac{N}{b \cdot 1} = \gamma H\cos^2 i$$

$$\text{Shear stress } (\tau) = \frac{T}{A} = \frac{T}{b \cdot 1} = \gamma H\sin i \cdot \cos i$$

We know,

$$F_s = \frac{\text{Shear strength}}{\text{Mobilized shear strength}} = \frac{c + \bar{\sigma}\tan\phi}{\tau_m}$$

For cohesionless soil, $c = 0$

Then,

$$F_s = \frac{\gamma H\cos^2 i \times \tan\phi}{\gamma H\sin i \cdot \cos i}$$

$$F_s = \frac{\tan\phi}{\tan i}$$

In similar way,

| (a) For cohesionless soils | (b) For cohesive soils |
|---|---|
| (i) Dry soil<br>$F_s = \frac{\tan\phi}{\tan i}$ | (i) Dry soil,<br>$F_s = \frac{c + \gamma H\cos^2 i \cdot \tan\phi}{\gamma H\sin i \cdot \cos i}$ |
| (ii) Submerged soil,<br>$F_s = \frac{\tan\phi}{\tan i}$ | (ii) Submerged soil<br>$F_s = \frac{c + \gamma' H\cos^2 i \cdot \tan\phi}{\gamma' H\cos i \cdot \sin i}$ |
| (iii) Steady seepage,<br>$F_s = \frac{\gamma'}{\gamma_{sat}} \times \frac{\tan\phi}{\tan i}$<br>$\gamma' = \gamma_{sat} - \gamma_w$ | (iii) Steady seepage,<br>$F_s = \frac{c + \gamma' H\cos^2 i\tan\phi}{\gamma_{sat} \times H\cos i\sin i}$ |

**Q.** A long natural slope in a clay (cohesion = 10kN/m²) $\phi = 25°$, saturated unit weight ($\gamma_{sat}$) = 20kN/m³ is inclined at 10° to the horizontal. The water table is at the surface & seepage is parallel to the slope. If the plane slip had developed at a depth of 5m below the ground surface. Determine the factor of safety. $\gamma_w = 10kN/m^3$.

→ Here, Given

cohesion ($c$) = 10 kN/m²
depth of slip surface ($H$) = 5 m
Internal friction angle of soil ($\phi$) = 25°
Inclination of natural slope with horizontal ($i$) = 10°
The seepage is steady. So,

$$F_s = \frac{c + \gamma' H \cos^2 i \tan\phi}{\gamma_{sat} \times H \times \cos i \times \sin i}$$

Here,
$\gamma_{sat} = 20\ kN/m^3$
$\gamma_{sub} = \gamma_{sat} - \gamma_w = 20 - 10 = 10\ kN/m^3$

Then,

$$F_s = \frac{10 + 10 \times 5 \times \cos^2 10° \times \tan 25°}{20 \times 5 \times \cos 10° \times \sin 10°}$$

$$= 1.907$$ ✱✱

✱ Stability Analysis of slope:-
a) Infinite slope
i) Cohesionless soil → dry soil condition
→ submerged soil
→ steady seepage condition
ii) Cohesive soil → dry soil condition
→ submerged soil
→ steady seepage condition

b) Finite Slope:-
i) Swedish circle method → $\phi = 0$ soil [clay]
→ $c - \phi$ soil [Method of slices]
ii) Friction Circle method → $\phi = 0$ Analysis
↳ saturated clay.

✱ $\phi = 0$ Analysis by Swedish Circle method:-
Let us consider finite slope in clay. Let ABC be trial slip surface. 'W' be the weight of soil mass above slip surface. 'd' be the horizontal distance betⁿ the centre of circular slip surface & C.G of soil mass above slip surface.

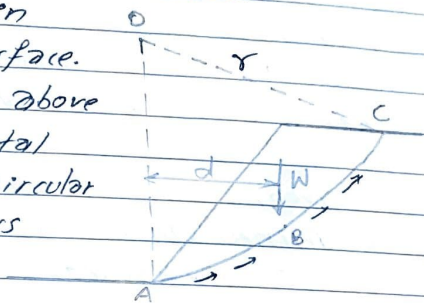


Fig: $\phi = 0$ Analysis.

We know,

$$\text{Factor of safety} = \frac{\text{shear strength of soil}}{\text{shear stress of soil}}$$

$$F = \frac{s}{\tau_m}$$

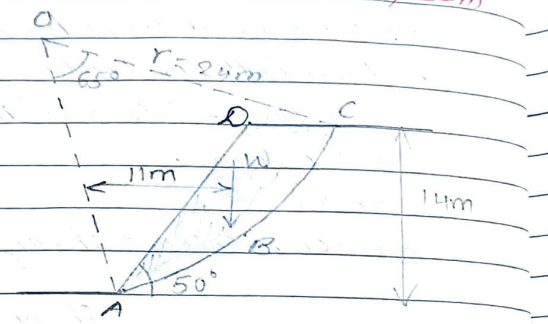
$$\Rightarrow F = \frac{c + \bar{\sigma}\tan\phi}{\tau_m}$$

Since, $\phi = 0$; $F = \frac{c}{\tau_m}$ $\Rightarrow \tau_m = \frac{c}{F}$ ----------①

For equilibrium,
$M_o = 0$
$\Rightarrow w \times d = (\tau_m \times L_a) \times r$ [Assuming unit width]
$\Rightarrow w \times d = \frac{c_u}{F} \times L_a \times r$ [$c = c_u$ = undrained cohesion]
$\Rightarrow F = \frac{c_u \times L_a \times r}{w \times d}$

where, $L_a$ = Length of arc,
$r$ = radius of slip surface
$d$ = Horizontal distance.

5. An unsupported slope is shown in figure. Determine the FOS against sliding for the trial slip surface. Take cohesion = 50 kN/m² & internal frictional angle of soil ($\phi$) = 0. The weight of wedge ABD is 2518 kN & acts at a horizontal distance of 11m from the vertical AC.




→ Given,

Undrained cohesion ($C_u$) = 50 kN/m²

Internal frictional angle of soil ($\phi$) = 0

Radius of slip surface, R = 24m

Horizontal distance centre of slip surface & CG of soil mass (d) = 11m.

Weight of wedge (W) = 2518 kN

NOW,

Area of wedge = (Arc length ABC × 1)

$= 2\pi \times 24 \times \frac{65°}{360°} \times 1$

$= 27.227 m^2$

$L_a = \frac{2\pi \times 24 \times 65°}{360°}$

$= 27.22m$

Then, the factor of safety against sliding for the trial slip surface is,

$$F = \frac{C_u \times L_a \times r}{W \times d}$$

$$= \frac{50 \times 27.227 \times 24}{2518 \times 11}$$

$$= 1.17959 kN$$

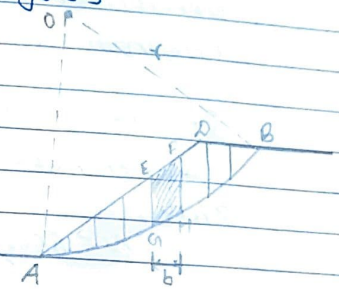
$$= 1179.59 N$$

## ✱ Method of Slices:- [C - $\phi$ Analysis]

Let AB be a slip surface of circular section, centre 'O' & radius 'r'.

Let, the slip surface ABDA be divided into number of vertical slices.

Consider a slice EFGH, then the forces acting on the slice are,



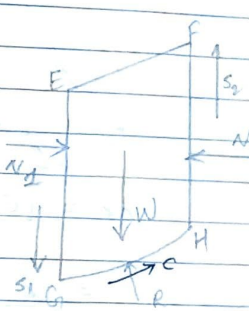

(i) Weight of slice, W

ii) Cohesive force along slip surface 'C'

iii) Rxn of soil below slip surface 'R'

iv) Shear forces due to adjacent slices ($S_1$, $S_2$)

v) Normal forces due to adjacent slices ($N_1$, $N_2$)




→ There are two methods to analyse it.

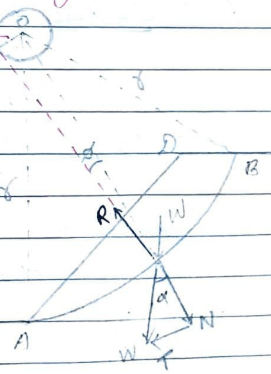
i) Neglecting effect of adjacent vertical slices (W, C & R only consider) [Neglect S & N]

ii) Considering W, C, R, S & N (Bishop's method).

### (i) Considering W, C & R only [Neglecting S & N]:-

Here, reaction 'R' acts at angle '$\phi$' to the normal. Rxn on two vertical sides of slice due to adjacent slides are equal & opposite & so neglected.

The weight 'W' is resolved into its normal component (N) & tangential component (T).



Overturning Moment = $T * r$

The moment due to 'N' component is zero. [passes through O]

Resisting Moment = $(C \times \Delta L) \times r + R * (r \sin\phi)$

Where, $\Delta L$ = Length of curved surface of slice.

Then,

$$N = R\cos\phi \Rightarrow R = \frac{N}{\cos\phi}$$

$$\Rightarrow R\sin\phi = N\tan\phi$$

The factor of safety is,

$$F_s = \frac{\text{Resisting moment}}{\text{Overturning Moment}}$$

$$F_s = \frac{(C \times \Delta L) \times r + R \times (r\sin\phi)}{T \times r}$$

$$= \frac{r\,[C \times \Delta L + N\tan\phi]}{T \times r}$$

$$= \frac{C\Delta L + N\tan\phi}{T}$$

The factor of safety of entire wedge is given by,

$$F_s = \frac{\Sigma C \times \Delta L + \Sigma N\tan\phi}{\Sigma T}$$

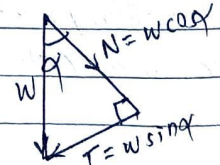
If C & $\phi$ are constant,

$$F_s = \frac{C L_a + \tan\phi \times \Sigma N}{\Sigma T}$$

where, $L_a$ = Length of entire slip surface = $\Sigma \Delta L$

→ The components N & T can be determined as,

$N = W\cos\alpha$ & $T = W\sin\alpha$




$$F_s = \frac{\Sigma C(\Delta L) + \Sigma[(N - u\Delta L)\tan\phi]}{\Sigma T}$$ [If pore water pressr is given]

The length $\Delta L$ of the arc is given by $b\sec\alpha$, where 'b' is width of slice.

Then,

$$F_s = \frac{\Sigma C b\sec\alpha + \Sigma (W\cos\alpha)\tan\phi}{\Sigma W\sin\alpha}$$

(ii) Considering W, R, C, N & S [Bishop's method]

(Derivation not necessary)

## ✻ Friction Circle Method:-

The friction circle method assume the failure surface as arc of circle. The sliding mass is assumed to be acted by three forces keeping it in equilibrium.

- Weight (W) of sliding mass (W)
- Cohesive force develop along slip surface (C)
- Reaction on slip surface (R).

Consider slip surface AB with centre at 'O' and radius 'r'. Let 'W' be the weight of sliding soil. The cohesive force $C_m$ will be,

$$C_m = c_m L_a \quad ---\ (1)$$

Where,

$c_m$ = mobilized cohesion

$L_a$ = Length of circular surface arc.

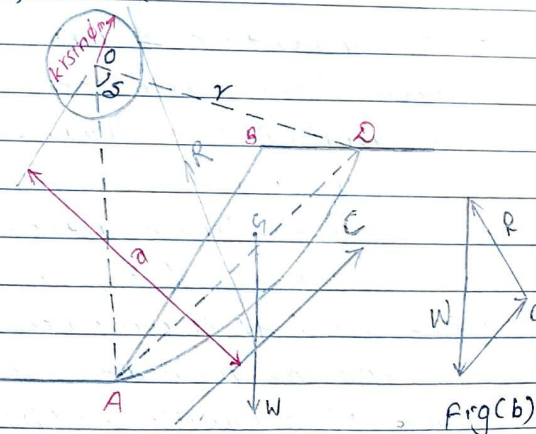



Fig (a) Fig (b)

Fig: Friction Circle Method.

→ It is convenient to replace this force acting along the arc by an equivalent force 'c' acting along a line. The force along arc AB is equal to force $c_m \times L_c$ where $L_c$ is length of chord AB.

$$C = c_m \times L_c \quad ---(ii)$$

C = equivalent force
$c_m$ = mobilized cohesion
$L_c$ = Length of chord.

→ For determining point of application of 'C',

$M_O = 0$

$\Rightarrow (C_m \times L_a) \times r = (C_m \times L_c) \times a$

$\Rightarrow a = \frac{L_a}{L_c} \times r$

Obviously distance a is greater than 'r', as $L_a > L_c$

Procedure

→ Taking centre 'O' & radius 'r', the slip surface AD is drawn. Then, taking centre 'O' & radius $r\sin\phi$ frictional circle is drawn. The value of 'r' is taken as unity unless it is specified.
→ To know the line of action of weight W, a vertical line is constructed through centroid of section ABDA.
→ The chord AD is constructed.
→ To find the line of action of resultant cohesive force $C_m L_c$, a line parallel to chord AD is drawn such that distance $a = r \times L_a/L_c$ from centre 'O'.
→ The length of arc AD is calculated as, $L_a = \frac{\pi R \theta}{180°}$
→ Chord length 'AD' ($L_c$) is found by measurement.
→ To know the line of action of resultant rxn 'R', a line which is tangential to friction circle is constructed from point of intersection of line of action of weigh (W) & cohesive force 'C'.
→ The weight 'W' of sliding mass ABDA is calculated & plotted in suitable scale as shown in figure(b).
→ Then, the value of cohesive force ($C_m L_c = C$) is calculated from triangle. To find value of $c_m$, cohesive force ($C = c_m \times L_c$) is divided by $L_c$.
→ The factor of safety with respect to cohesion, $F_c$ is given by,

$$F_c = \frac{C}{c_m}$$

→ Number of slip surface are taken & FOS for each is found. The circle giving the minimum FOS is critical slip circle.

$$[F_c = F\phi]$$

## * Taylor's Stability Number:-

Dimensionless number denoted by $S_n$ & defined as,

$$S_n = \frac{c_m}{\gamma H} \quad ---(i)$$

where, $c_m$ = mobilized cohesion
$\gamma$ = Unit weight of soil
H = Height of slope.

Also, Factor of safety wrt cohesion; $F_c = C/c_m \Rightarrow c_m = C/F_c$

From (i),

$$S_n = \frac{C}{F_c \times \gamma H}$$

where, C = Ultimate cohesion of soil.

→ Taylor determined the values of $S_n$ for finite slopes using friction circle method.

## Stability chart:-

→ Slopes that are of simple sections & of homogeneous soils may be analysed using slope stability charts given by Taylor.

→ The charts are prepared indicating stability number & slope angle 'i' for various values of $\phi_m$.

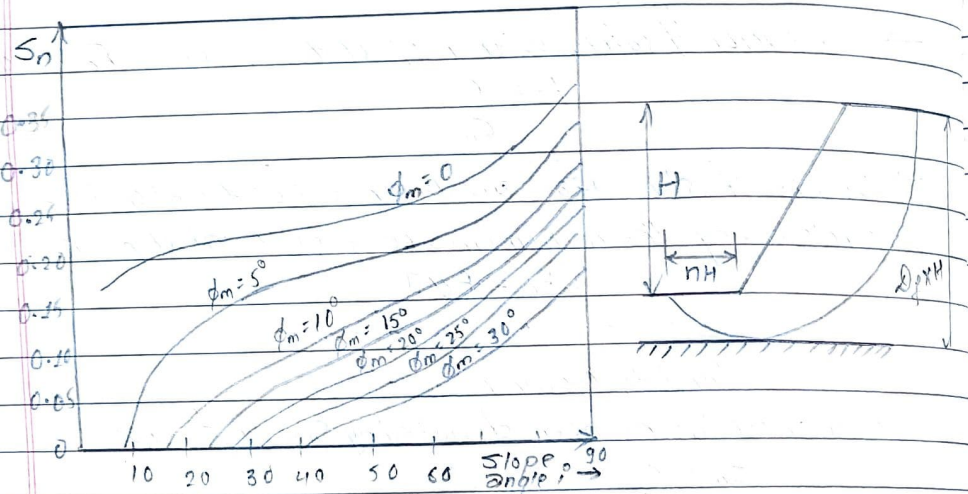



Fig:- stability chart.

→ Important parameter in stability chart is $D_f$.

$$D_f = \frac{\text{Depth of hard stratum below top of slope}}{\text{Height of slope}}$$

→ The stability number can be used to determine FOS for given slope. For known value of i & $\phi_m$, $S_n$ can be determined from chart & FOS is determined as,

$$F_c = \frac{c}{c_m} = \frac{c}{S_n \gamma H}$$

→ Stability chart can also be used to determine steepest slope for given value of $\phi_m$. ($S_n = c/\gamma H$) For known $\phi_m$ & calculated value of $S_n$, we can compute value of 'i'.

**Q** A vertical cut is made in clay deposit ($C = 30 kN/m^2$, $\phi = 0$, $\gamma = 16 kN/m^3$). Find the maximum height of cut which can be temporarily supported.

→ Given,

$C = 30 kN/m^2$

$\phi = 0$

$\gamma = 16 kN/m^3$

We have, stability number $(S_n) = \frac{c}{F_c \gamma H}$

Then, $H = \frac{c}{F_c \times \gamma \times S_n}$ ---(1)

Assume, FOS wrt cohesion ($F_c$) = 1,

From stability chart,

For $\phi = 0$ & $i = 90°$, $S_n = 0.261$

From (1),

$$H = \frac{30}{1 \times 16 \times 0.261} = 7.18 m.$$

**Q** Figure shows a slip surface with radius of 22m in a slope with a height of 14m & angle of inclination of 45°. If $\phi_m = 15$, $\gamma = 18 kN/m^3$ & $c = 40 kN/m^2$, determine the factor of safety wrt to cohesion using friction circle method. The weight of soil wedge is 1500kN & it acts at a horizontal distance of 10.3m from A.

→ Given,

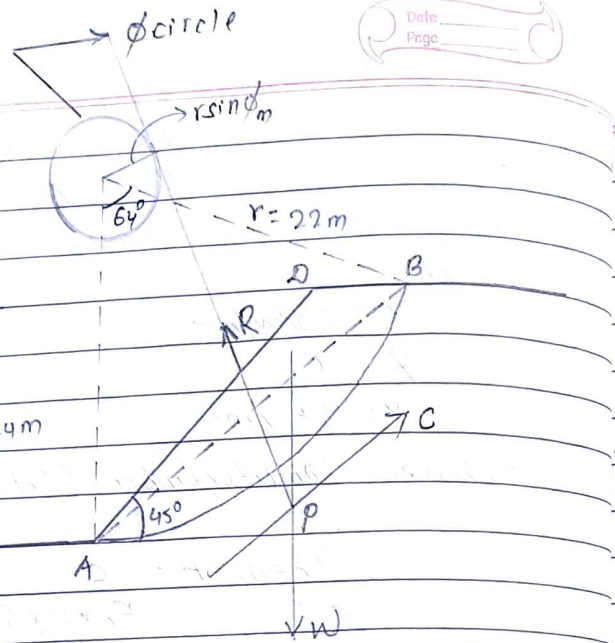
Radius of slip surface (r) = 22m

Height of slope (H) = 14m

Angle of Inclination (i) = 45°

$\phi_m = 15°$, $\gamma = 18 kN/m^3$ & $c = 40 kN/m^2$

$W = 1500 kN$, $d = 10.3 m$




Here,

radius of friction circle(r) = $r\sin\phi = 22 \times \sin 15° = 5.7m$

Then,

$$L_a = 2\pi \times 22 \times \frac{64°}{360^0} = 24.574m$$

$L_c = 2 \times [22 \times \sin 32°]$ [Bisector]

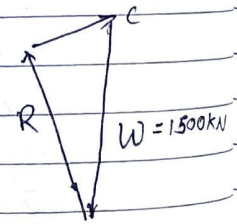
$= 23.316m$

Then, $a = r \times \frac{L_a}{L_c} = 22 \times \frac{24.574}{23.316} = 23.20m$

From force triangle,

$C = 600kN$

Then, $c_m = \frac{C}{L_c} = \frac{600}{23.316} = 25.7kN/m^2$




Then,

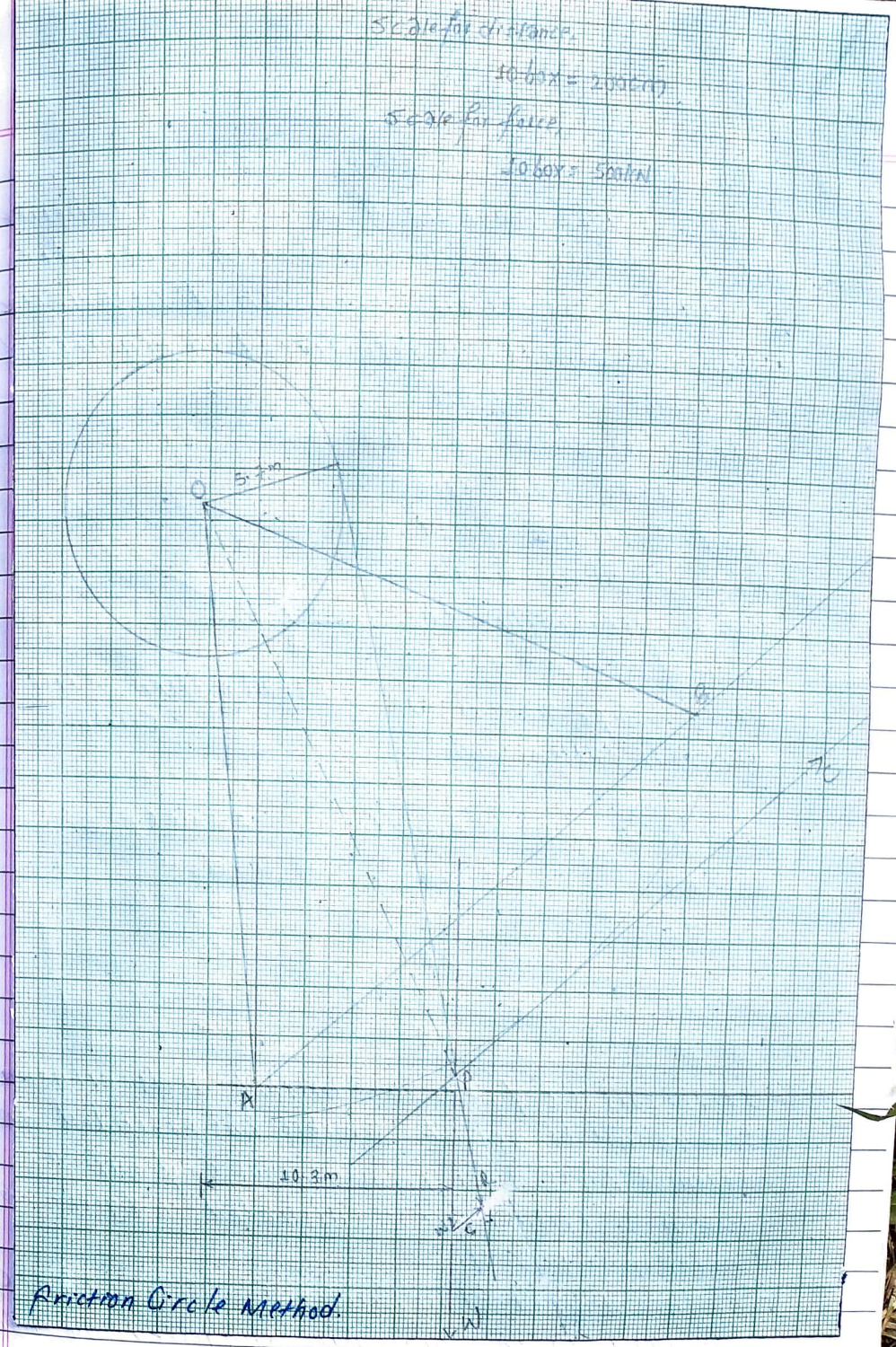
$$f_c = \frac{c}{c_m} = \frac{40}{25.7} = 1.56$$

Scale for distance,

10.6cm = 200cm

Scale for force,

10.6cm = 5mkN




Friction Circle Method.

## ⁂ Location of Critical failure Surface:-

→ In case of slope made out of homogeneous cohesive soil, it is possible to determine directly the centre of critical failure surface by a method proposed by Fellenius. The method is illustrated in figure (a).

→ The centre of critical arc is the intersection of two lines set off from the ~~point~~ bottom & top of the slope at an angle $\alpha$ & $\beta$ respectively. The values of $\alpha$ & $\beta$ for various slope angles are given in table below.

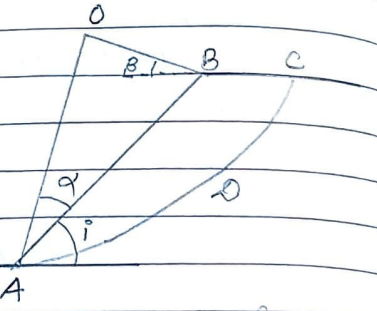



Fig a: Location of critical circle.

| Slope Angle (i) | Angle ($\alpha$) | Angle ($\beta$) |
|---|---|---|
| 60 | 29 | 40 |
| 45 | 28 | 37 |
| 34 | 26 | 35 |
| 27 | 25 | 35 |
| 18 | 25 | 35 |

→ If the soil is c-$\phi$, the above methods in its original from cannot be applied. Jumikis has extended the Fellenius method for homogeneous c-$\phi$ soil. The location of critical arc is illustrated in fig (b). The heigh of slope is H.

→ In this method, it is necessary to obtain centre $O_1$ according to the procedure suggested by Fellenius. Then, a point 'x' is found out such that 'x' is 2H below the top of slope and 4.5H horizontally from slope toe. points $O_1$ and x are joined. The centre of critical circle 'O' lies on the line Ox extended beyond $O_1$. The distance of 'O' beyond $O_1$ increases as the angle of internal friction increases.

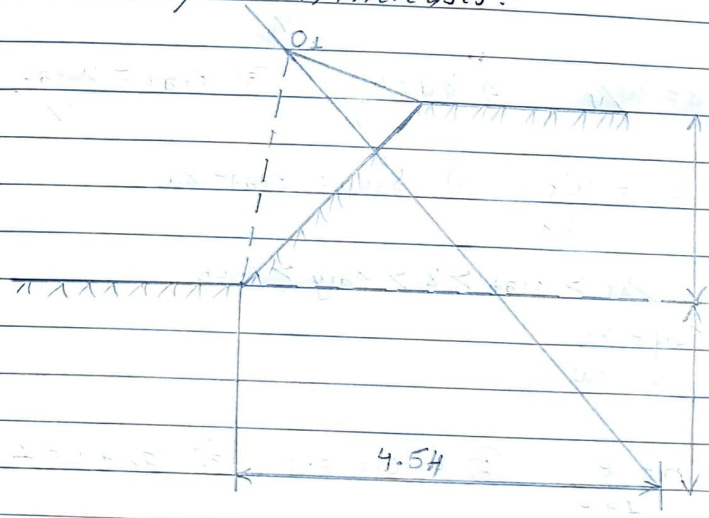



Fig: Critical Circle for c-$\phi$ soil.

* Formula Revision of ch-2:-

① $V = V_v + V_s$

② $V_v = V_a + V_s$

③ $w = w_w + w_s$

① $w = \frac{w_w}{w_s}$ ② $e = \frac{V_v}{V_s}$

③ $\eta = \frac{V_v}{V}$ ④ $S = \frac{V_w}{V_v}$ ⑤ $a_c = \frac{V_a}{V_v}$

⑥ $\eta_a = \frac{V_a}{V}$

① $\gamma = w/V$ ② $\gamma_d = \frac{W_s}{V}$ ③ $\gamma_{sat} = \frac{W_{sat}}{V}$

④ $\gamma_s = \frac{W_s}{V_s}$ ⑤ $\gamma_{sub} = \gamma_{sat} - \gamma_w$

$\gamma_s > \gamma_{sat} > \gamma > \gamma_{dry} > \gamma_{sub}$

$G_s = \frac{\gamma_s}{\gamma_w}$

① $\eta = \frac{e}{1+e}$ ② $w \cdot G = e \cdot S$ ③ $a_c + S = 1$

④ $\gamma_d = \frac{\gamma}{1+w}$ ⑤ $V_s = \frac{V}{1+e}$ ⑥ $W_s = \frac{W}{1+w}$

① $\gamma = \left(\frac{G + eS}{1+e}\right)\gamma_w$ ② $\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$

③ $\gamma_d = \left(\frac{G}{1+e}\right)\gamma_w$ ④ $\gamma_{sub} = \left(\frac{G-1}{1+e}\right)\gamma_w$

⑤ $\gamma_d = \frac{(1-\eta_a) G \gamma_w}{1 + wG}$

① $w = \frac{w_2 - w_3}{w_3 - w_1} \times 100\%$, $w = \left[\frac{(w_2 - w_1)}{(w_3 - w_4)}\left(\frac{G-1}{G}\right) - 1\right] \times 100\%$

$G = \frac{(w_2 - w_1)}{(w_2 - w_1) - (w_3 - w_4)}$

$C_u = \frac{D_{60}}{D_{10}}$, $C_c = \frac{D_{30}^2}{D_{60} \times D_{10}}$

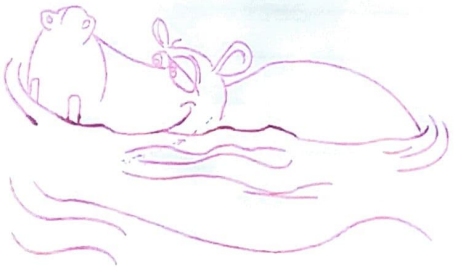
## Remedial Measures for Slope Stability Problems:-

a) Slope reduction
b) providing the Berm at the Toe of slope.
c) providing drainage
d) Densification (Hardening of soil)
e) Vegetation
f) Construction of earth Retaining Structure.

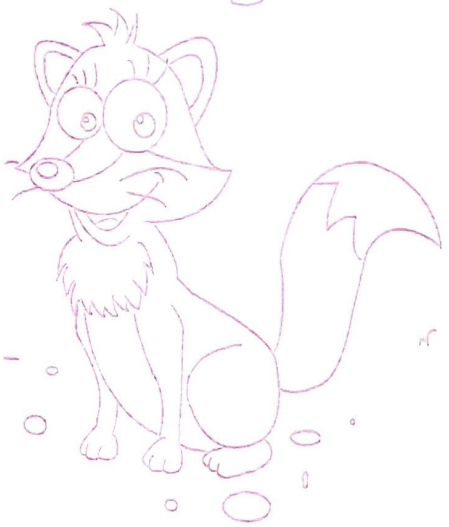


Both black and white rhinoceroses are actually gray!

The lips of a hippopotamus are about 2 feet wide!



# Did you know?



A fox uses its tail as a signal flag to communicate with other foxes.